STRATEGIES TO MODEL SCHEDULING DECISIONS TO PLAN THE SOFT DRINK PRODUCTION PROCESS

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Abstract: In this paper we present a mixed integer model that integrates lot sizing and lot scheduling decisions for the production planning of a soft drink company. The main contribution of the paper is to present a model that differ from others in the literature for the constraints related to the scheduling decisions. The proposed strategy is compared to other strategies presented in the literature.

1 INTRODUCTION

The lot sizing and scheduling problems has received a lot of attention given its relevance to the industrial process. A recent trend has been on mathematical models that capture the relationship between both problems (Clark et al (2011)). Integrated models have been proposed for several industrial contexts. For example, Almada Lobo et al (2007) study the problem for the glass container industry, Toso et al (2009) for the animal feed supplements industry, and Ferreira et al (2009, 2010) for the softdrink industry. Two main strategies have been used to model the scheduling decisions. The first one is a small bucket strategy in which each period of the planning horizon is divided into subperiods. For each subperiod only one item can be produced. This strategy is based on the GLSP model ((*General Lotsizing and Scheduling Problem*) (Fleischmann and Meyer (1999)). The second strategy is a bigbucket one and allows the production of several items in a given period. To obtain the production sequence constraints based on the asymmetric traveling salesman problema (ATSP) are added to the lot sizing formulation.

The paper is organized as follows. In Section 2 a brief description of the production process of soft drinks according to visits to small and medium scale soft drinks plants in Brazil and of the one stage one machine model P1S1MTS given in the literature is presented. In Section 3 a alternative to strengthen the P1S1MTS model is proposed. Section 4 presents final remarks.

2 BRIEF DESCRIPTION OF PREVIOUS WORK FOR PLANNING THE SOFT DRINK PRODUCTION PROCESS

In this section we review the mathematical model P1S1MTS proposed by Defalque *et al.* (2010) to represent the production process of small scale soft drink plants. The production process of soft drinks in different sizes and flavours is carried out in two stages: liquid flavor preparation (Stage I) and bottling (Stage II). The model P1S1MTS considers that there are J soft drinks (items) to be produced from L liquid flavors (syrup) on one production line (machine). To model the decisions associated with Stage I, it is supposed that there are several tanks to store the syrup and that it is ready when needed. Therefore, it is not necessary to consider the scheduling of syrups in the tanks, nor the changeover times since it is possible to prepare a new lot of syrup in a given tank, while the machine is bottling the syrup from another tank. However, the syrup lot size needs to satisfy upper and lower bound constraints in order to not overload the tank and to guarantee syrup homogeneity. In Stage II, the machine is initially adjusted to produce a given item. To produce another item, it is necessary to stop the machine and make all the necessary adjustments (another bottle size and/or syrup flavor). Therefore, in this stage, changeover times from one product to another may affect the machine capacity and thus have to be taken into account. The P1S1MTS model addresses the problem of defining the lot size and lot schedule taking into account the demand for items

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and the capacity of the machine and syrup tanks, minimizing the overall production costs. It assumes that there is an unlimited quantity of other supplies (*e.g.* bottles, labels, water).

2.1 The P1S1MTS Model

In the P1S1MTS model the decisions associated with lot sizing are based on the Capacitated Lot Sizing Problem (CLSP) (*e.g.* Karimi et al (2003)). The scheduling decisions use the ATSP approach with the MTZ constraints to eliminate subtours. Some simplifications of the production process have been made. Only one production line (machine) is considered and it is also supposed that there are several tanks dedicated to it. Therefore, it is not necessary to consider the scheduling of liquid flavor in the tanks, nor the changeover times. It is possible to prepare a new lot of liquid flavor in a given tank, while the machine is bottling the liquid flavor from another tank. However, the stage I constraints cannot be completely discarded. The liquid flavor lot size needs to satisfy upper and lower bound constraints in order to not overload the tank and to guarantee liquid homogeneity. The problem considered in this paper can thus be stated as: define the lot size and lot schedule of the products taking into account the items demands and the capacity of the production line and syrup tanks, minimizing the overall production costs. It is also supposed that there is an unlimited quantity of bottles, labels and water.

To present the model, let the following parameters define the problem size: J is the number of softdrinks (items); L is the number of syrup flavors and T is the number of periods; in the planning horizon. Let (i,j,k,l,t) be the index set defined as: $i,j,k \in \{1,\ldots,J\}; l \in \{1,\ldots,L\}; t \in \{1,\ldots,T\}$. The data and variables are described in Table 1. The superscript I relates to Stage I (syrup preparation) and with superscript II relates to Stage II (bottling).

The optimization criterion (1) is to minimize the overall costs taking into account inventory, backorder and machine changeover costs.

$$\text{Min } Z = \sum_{j=1}^{J}\sum_{t=1}^{T}(h_j I_{jt}^{+} + g_j I_{jt}^{-}) + \sum_{t=1}^{T}\sum_{i=1}^{J}\sum_{j=1, j\neq i}^{J} s_{ij}^{II} z_{ijt}^{II} \tag{1}$$

The lotsizing decisions in Stage I, as defined by constraints (2)-(5) control the syrup production. Constraints (2) guarantee that if the tank is ready for production of syrup l, then there will be production of item j and the quantity produced uses all the syrup prepared in that period. The variables n_{lt} allow partial use of the tank and is controlled to respect the minimum amount needed to ensure syrup homogeneity, as specified by constraints (3). Constraints (4) ensure that there is production of the syrup l only if the tank is prepared. According to constraints (5), the total number of tanks produced in period t is limited by the maximum number of tank setups.

Stage I: Syrup Preparation

$$\sum_{j\in\gamma_l} r_{lj} x_{jt}^{II} = K^{I}(w_{lt} - n_{lt}), \qquad \forall l, \forall t. \tag{2}$$

$$n_{lt} \leq 1 - \left(\frac{q_l}{K^I}\right), \qquad \forall l, \forall t. \tag{3}$$

$$y_{lt}^{I} \leq w_{lt} \leq S_t y_{lt}^{I}, \qquad \forall l, \forall t. \tag{4}$$

$$\sum_{l\in L} w_{lt} \leq S_t; \qquad \forall t. \tag{5}$$

The lotsizing decisions in Stage II are defined by constraints (6)-(9). Constraints (6) represent the flow conservation of each item in each time period. Constraints (7) represent the machine capacity in each time period. Constraints (8) guarantee that there is production of item j only if the machine is prepared. Note that the setup variable is considered implicitly in terms of the changeover variables and that production may not occur although the machine might be prepared. Constraints (9) control the maximum number of setups in each period.

Stage II (bottling) - Lot Sizing:

$$I_{j(t-1)}^{+} + I_{jt}^{-} + x_{jt}^{II} - I_{jt}^{+} - I_{j(t-1)}^{-} = d_{jt}, \qquad \forall j, \forall t \tag{6}$$

$$\sum_{j=1}^{J} a_j^{II} x_{jt}^{II} + \sum_{i=1}^{J}\sum_{j=1, j\neq i}^{J} b_{ij}^{II} z_{ijt}^{II} \leq K_t^{II}, \qquad \forall t \tag{7}$$

$$a_j^{II} x_{jt}^{II} \leq K_t^{II} \sum_{i=1, i\neq j}^{J} z_{ijt}^{II}, \qquad \forall j, \forall t \tag{8}$$

$$\sum_{i=1}^{J}\sum_{j=1, j\neq i}^{J} z_{ijt} \leq S_t, \qquad \forall t \tag{9}$$

Constraints (10)-(14) model the order in which the items will be produced in a given period t. They are based on the ATSP model. Constraints (10) consider that in each period the machine is initially setup for a ghost item i_0. The changeover costs associated with the ghost item are zero and do not interfere in total solution cost. Constraints (11) guarantee that each item j is produced at maximum once in each period t. Constraints (12) conserve flow and ensure that if there is a changeover from an item i to any item k then there is a changeover from that item k to an item j.

Constraints (10) and (12) alone might generate subtours, that is disconnected subsequences, and thus

Table 1: Model data and variables.

Data Name	Meaning
J	number of soft-drinks (items)
L	number of syrup flavors
T	number of periods
a_j^{II}	machine production time for one lot of item j;
b_{ij}^{II}	machine changeover time from item i to j;
d_{jt}	demand for item j in period t;
g_j	non-negative backorder cost for item j;
h_j	non-negative inventory cost for item j;
I_{j0}^{+}	initial inventory for item j;
I_{j0}^{-}	initial backorder for item j;
K_t^{II}	total time capacity of the machine in period t;
s_{ij}^{II}	machine changeover cost from item i to j;
S_t	maximum number of tank setups in perod t;
K^I	total capacity of the tank, in liters of syrup;
q_l	minimum quantity of syrup l to guarantee homogeneity;
r_{lj}	quantity of syrup l necessary for the production of one lot of item j;
γ_l	set of items that need syrup l;
Variable Name	**Meaning**
I_{jt}^{+}	inventory for item j at the end of period t;
I_{jt}^{-}	backorders for item j at the end of period t;
x_{jt}^{II}	production quantity of item j in period t;
z_{ijt}^{II}	changeover on machine (stage II) from item i to item j in period t.
u_{jt}	auxiliary variable - might be used to indique the production order of item j in period t;
w_{lt}	number of tanks to be prepared with syrup l in period t;
n_{lt}	fraction of tank capacity used to produce syrup l in period t;
y_{lt}^{I}	is equal to 1 if the tank is setup for syrup l in period t;

do not guarantee a proper sequence of the items. The MTZ type subtour elimination constraints (13) avoid this situation. With the inclusion of constraints (14) the variable u_{jt} gives the order position in which item j is produced. Finally constraints (15) define the variables' domain.

Stage II (bottling) - Scheduling:

$$\sum_{j=1, j\neq i_0}^{J} z_{i_0 jt}^{II} \geq \sum_{i=1, i\neq k}^{J} z_{ikt}^{II}, \forall k, \forall t \tag{10}$$

$$\sum_{j=1, j\neq i}^{J} z_{ijt}^{II} \leq 1, \qquad \forall i, \forall t \tag{11}$$

$$\sum_{i=1, i\neq k}^{J} z_{ikt}^{II} = \sum_{j=1, j\neq k}^{J} z_{kjt}^{II}, \forall k, \forall t \tag{12}$$

$$u_{jt} \geq u_{it} + 1 - (J-1)(1 - z_{ijt}^{II}); \quad \forall i, \forall j; i \neq j; \forall t \tag{13}$$

$$1 \leq u_{jt} \leq J - 1 \forall j, \forall t \tag{14}$$

$$x_{jt}^{II} \geq 0,\ z_{ijt}^{II},\ y_{lt}^{I} = 0/1,\quad w_{lt} \in \mathbb{Z}_+,\ n_{lt} \geq 0,\quad \forall i, j; \forall t; \forall l. \tag{15}$$

The complete description of the P1S1MTS model is given by expressions (1)-(15). More details on the P1S1MTS model can be obtained from Defalque *et al.* (2011). Other formulations of the soft drink production process can be found in Toledo *et al* (2007), Ferreira *et al* (2009 and 2010).

3 THE MULTICOMMODITY FLOW BASED MODEL

In the model P1S1MTS the constraints associated with the scheduling decisions are formulated based on the constraints proposed by Miller, Tucker and Zemlim (MTZ) to eliminate subtours, constraints (13). These constraints are of polynomial order, thus allowing their inclusion *a priori*. However, the MTZ constraints produce a weak linear relaxation of the associated formulation. Motivated by this fact, several authors have proposed different approaches to strengthen the ATSP mathematical formulation. Oncan et al (2009) reviews and compares several mathematical models for the ATSP. The review focuses on

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how the formulations compare to one another as regard to the strengthen of the associated linear relaxation.

The main difference among the various formulations for the ATSP relate to the constraints used to eliminate subtours. The multi-commodity-flow formulation proposed by Claus (1984) has been used by Clark et al (2011) to model the scheduling decisions in the presence of non-triangular setups times. The main idea of the proposed formulation is to ensure that, in any period t, there is always a path from the initial product s to any other product r in the period t's sequence. In this work the multi-commodity-flow formulation is also used to eliminate subtours. However, the objective is to obtain a formulation that is stronger than others from the literature, and therefore might have a better computational behavior when solved by a general purpose software.

To obtain the new formulation, it is necessary to define a new index $r = \{1,\dots,J\}$, and a new set of variables. The continuous variables, m^{II}_{rijt}, are used to formulate subtour elimination constraints based on the multi-commodity-flow formulation for the ATSP. The idea behind this formulation is that there are J commodities available at node i_0 and a demand of one unit of commodity j at node j. If $m^{II}_{rijt} = 1$ then the flow of commodity r flows from node i_0 no node r through arc (i, j). In terms of the items sequence in period t, it means that if product r is included in the production sequence, then product j follows product i in such sequence. The constraints (16)-(19) eliminates disconnected subsequence of items.

Since only the items which are produced (*i.e.* $x^{II}_{rt} > 0$) should be sequenced, constraints (16) and (17) take place only when the machine is prepared for item r. These constraints guarantee that if product r is included in the sequence at least one other item should be also included.

$$\sum_{j=1}^{J} m^{II}_{ri_0jt} - \sum_{j=1}^{J} m^{II}_{rji_0t} = \sum_{j=1,j\neq r}^{J} z^{II}_{jrt}, \qquad \forall r, \forall t \quad (16)$$

$$\sum_{j=1}^{J} m^{II}_{rjrt} - \sum_{j=1}^{J} m^{II}_{rrjt} = - \sum_{j=,j\neq r}^{J} z^{II}_{jrt}, \qquad \forall r, \forall t \quad (17)$$

Constraints (18) are the flow conservation constraints, for all but product r in node r. And constraints (19) states that item j should follow item i in the sequence that includes item r only if there is a changeover from product i to product j.

$$\sum_{i=1}^{J} m^{II}_{rijt} = \sum_{i=1}^{J} m^{II}_{rjit}, \qquad \forall r, \forall j; j \neq r, \forall t \quad (18)$$

$$m^{II}_{rijt} \leq z^{II}_{ij} \qquad \forall i, j, r; \forall t \quad (19)$$

The multi-commodity-flow model for the single stage single machine lot scheduling problem (MM1S1M) is defined by the objective function (1), the Stage I constraints (2)-(5), the Stage II constraints (6)-(12), the subtour elimination constraints (16)-(19), and the domain constraints (20).

$$x^{II}_{jt} \geq 0,\ m^{II}_{rijt} \geq 0,\ z^{II}_{ijt},\ y^{I}_{lt} = 0/1,\ w_{lt} \in \mathbb{Z}_+,$$
$$n_{lt} \geq 0,\ \forall i, j, r; \forall t; \forall l. \quad (20)$$

4 CONCLUSIONS

In this paper a new formulation for the single stage, single machine lotscheduling problem has been proposed. This model might be useful for building decision support systems for the production planning that arises in the soft drink production of small and medium sized plants. The main feature of the the model MM1S1M proposed in Section 3 is that it includes multi-commodity-flow constraints to model the sequence at which the items should be produced. In spite of the fact that the MM1S1M model has a higher number of constraints to eliminate subtours than the model P1S1MTS, the number of these constraints is still polynomial. Moreover, these constraints provide a stronger formulation since the ATSP formulation using these type of constraints is stronger than the MTZ formulation that is used in the P1S1MTS model. A computational experiment using data from the literature is being prepared to compare the MM1S1M model with other models from the literature and to evaluate its computational behavior when solved by general purpose software (e.g. Cplex (IBM, 2011), Gurobi (Gurobi, 2011)).

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