EMODS: A NOVEL EVOLUTIONARY METAHEURISTIC BASED IN THE AUTOMATA THEORY FOR THE MULTIOBJECTIVE OPTIMIZATION OF COMBINATORIALS PROBLEMS

Elias David Nino Ruiz[1,2] and Anangelica Isabel Chinchilla Camargo[2]

[1]*Virginia Polytechnic Institute and State University, Blacksburg, Virgina, U.S.A.*

[2]*Universidad del Norte, Barranquilla, Atlantico, Colombia*

Keywords: Combinatorial optimization, Metaheuristic, Evolutionary rules, MultiObjective optimization, Traveling salesman problem.

Abstract: This paper states a novel Evolutionary Metaheuristic based in the Automata Theory for the Multiobjective Optimization of Combinatorial Problems named EMODS. The proposed algorithm uses the natural selection theory to explore the feasible solutions space of a Combinatorial Problem. Due to this, local optimums are avoided. Also, EMODS takes advantage in the optimization process from the Metaheuristic of Deterministic Swapping to avoid finding unfeasible solutions. The proposed algorithm was tested using well known instances from the TSPLIB with three objectives. Its results were compared against four Multiobjective Simulated Annealing inspired Algorithms using metrics from the specialized literature. In every case, the EMODS results on the metrics were always better and in some of those cases, the distance from the Real Solutions was 4%.

1 INTRODUCTION

Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution (Yong-fa and Ming-yang, 2004). In this field it is possible to find a lot of problems denominated NP - Hard such as Multi-depot vehicle routing problem (Lim and Wang, 2005), delivery and pickup vehicle routing problem with time windows (Wang and Lang, 2008), multi-depot vehicle routing problem with weight-related costs (Fung et al., 2009), Railway Traveling Salesman Problem (Hu and Raidl, 2008), Heterogeneous, Multiple Depot, Multiple Traveling Salesman Problem (Oberlin et al., 2009) and Traveling Salesman with Multi-agent (Wang and Xu, 2009).

One of the most classical problems in combinatorial optimization is the Traveling Salesman Problem and it has been analyzed for years (Sauer and Coelho, 2008) either in a mono or multi - objective way. Although several algorithms have been implemented to solve TSP, there is no one that optimal solves it in a polynomial time.

This paper is structured as follows. In Section 2 some fundamentals concepts such as Multiobjective Optimization and Genetic algorithms are reviewed. In Section 3, a novel evolutionary metaheuristic is defined on the MODS template. Lastly, in Section 4, the metaheuristic proposed is tested and its results are analyzed.

2 PRELIMINARIES

2.1 Multi - objective Optimization

The multi - objective optimization consists in two or more objectives functions to optimize and a set of constraints (Glover and Laguna, 1997):

$$Opt. \quad F(X) = \{f_1(X), f_2(X), f_3(X), ..., f_n(X)\} \quad (1)$$

Subject to

$$H(X) = 0 \quad (2)$$

$$G(X) \leq 0 \quad (3)$$

$$X_l \leq X \leq X_u \quad (4)$$

X is the set of desition variables of the problem. $F(X)$ is the set of objective functions.n is the number of objective functions. $H(X)$ and $G(X)$ are the constraints of the problem. Finally, X_l and X_u are the lower and upper bound, respectively, of X.

Nino Ruiz E. and Chinchilla Camargo A..
EMODS: A NOVEL EVOLUTIONARY METAHEURISTIC BASED IN THE AUTOMATA THEORY FOR THE MULTIOBJECTIVE OPTIMIZATION OF COMBINATORIALS PROBLEMS.
DOI: 10.5220/0003754003990404
In *Proceedings of the 1st International Conference on Operations Research and Enterprise Systems* (ICORES-2012), pages 399-404
ISBN: 978-989-8425-97-3

2.2 Pareto Front

As well known, a Pareto Front is a set of nondominated solutions; it means that all the solutions of the PF are optimal. In the particular case of the three - objective optimization of a combinatorial problem, the PF will be in $\Re^3$ as can be seen in figure 1. Each point of this set represents a solution for the problem. Therefore, the dimension of the Pareto Front depends on the number of objectives of the problem.

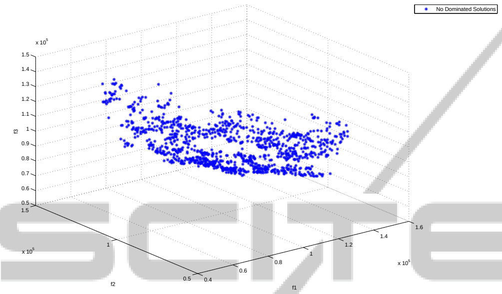

Figure 1: Pareto Front for a particular three - objective problem.

2.3 Genetic Algorithms

Genetic Algorithms (GA) are algorithms inspired in the natural selection theory. They consist in three steps as follows:

Step 1. Selection. Select solutions from a population.

Step 2. Crossover. Cross the selected solutions avoiding local optimums.

Step 3. Mutation. Perturbs the new solutions found for increasing the population.

The most known Genetic Algorithms from the literature (Glover and Laguna, 1997) are the Non-Dominated Sorting Genetic Algorithm (NSGA - II) and the Strength Pareto Evolutionary Algorithm 2 (SPEA2). NSGA II use a no - dominated sort for sorting the solutions in different Pareto Sets. Consequently, it demands a lot of time, but it allows a global verification of the solutions for avoiding the Local Optimums. On the other hand, SPEA2 is an improvement of SPEA. The difference with the first version is that SPEA2 works using strength for every solution according to the number of solutions that it dominates. Consequently, at the end of the iterations, SPEA2 has the non dominated solutions stronger avoiding Local Optimums.

2.4 Metaheuristic Of Deterministic Swapping

Metaheuristic Of Deterministic Swapping (MODS) (Niño et al., 2011) is a local serach strategy that explores the Feasible Solution Space of a Combinatorial Problem supported in a data structure named Multiobjective Deterministic Finite Automata (MDFA) (Niño et al., 2010). A MDFA is a data structure that allows the representation of the feasible solution space of a Combinatorial Problem. Formally, a MDFA is defined as follows:

$$M = \{Q, \Sigma, \delta, Q_0, F(X)\} \tag{5}$$

Where Q represents all the set of states of the Automata (Feasible Solution Space),Σ is the input alphabet that is used for δ (transition function) to explore the feasible solution space of a Combinatorial Problem, in other words δ perturbs the solutions for finding news, Q_0 contains the Initial set of States (Initial Solutions) and $F(X)$ are the Objectives to optimize.

The main algorithm set the MDFA to the Combinatorial Problem and explores it using a search direction based in the elitist set of solutions (Q_*). The elitist solutions are solutions that, when were found, dominated at least one solution from Q_ϕ.

The template algorithm of MODS is defined as follow:

Step 1. Create the initial set of solutions Q_0 using a heuristic relative to the problem to solve.

Step 2. Set Q_ϕ as Q_0.

Step 3. Select a random state $q \in Q_\phi$ or $q \in Q_*$

Step 4. Explore the Neighborhood of q using δ and Σ. Add to Q_ϕ the solutions found that are not dominated and add to Q_* those solutions that dominated at least one element from Q_ϕ.

Step 5. Check stop condition, go to 3.

3 EVOLUTIONARY METAHEURISTIC OF DETERMINISTIC SWAPPING

EMODS, Evolutionary Metaheuristic of Deterministic Swapping, is a framework that allows the Multiobjective Optimization of Combinatorial Problems. Its framework is based on MODS template therefore its steps are the same: create Initial Solutions, Improve the Solutions (Optional) and Execute the Core Algorithm.

Alike MODS, Q_0 has the Initial Solutions (states) of the Combinatorial Problem. Each state has a vector solution. Those are created using the well known Nest Neighbor Heuristic. Hence, a new function is created based in the Weighted Sum Metric therefore a weight is assigned to each Objective Function of the problem (This is a classic manner for multiobjective optimization(Pretorius and Helberg, 2004)) as follows:

$$F(X)=\sum_{i=1}^{n} \alpha_i \cdot f_i(X) \tag{6}$$

Subject to

$$\sum_{i=1}^{n} \alpha_i = 1 \tag{7}$$

Where n is the number of objective functions. The weights (α_i) values are randomly assigned to each function. Once this step has been concluded, the Nest Neighbor Heuristic is applied to (1) for creating the Initial Solutions. The Core Algorithm is defined as follows:

Step 1. Set θ as the maximum number of iterations, β as the maximum number of state selected in each iteration, ρ as the maximum number of perturbations by state and Q_ϕ as Q_0

Step 2. Selection. Randomly select a state $q \in Q_\phi$ or $q \in Q_*$

Step 3. Mutation. Set N as the new solutions found as result of perturbing q. Add to Q_ϕ and Q_* according to the next equations:

$$\left(Q_\phi = Q_\phi \cup \{q\}\right) \Longleftrightarrow \left(\nexists r \in Q_\phi / q \prec r\right) \tag{8}$$

$$\left(Q_* = Q_* \cup \{q\}\right) \Longleftrightarrow \left(\exists r \in Q_\phi / r \prec q\right) \tag{9}$$

Remove the states with dominated solutions for each set.

Step 4. Crossover. Randomly select states from Q_ϕ and Q_*. Generate a random value k, cross the solutions in a k-position as can be seen in figure 2.

Step 5. Check stop condition, go to 3.

One of the most important steps in the EMODS algorithm is step 4. There, the algorithm applies an Evolutionary Strategy based in the crossover step of Genetic Algorithms for avoiding Local Optimums as can be seen in 2. Due to the crossover is not always made in the same point (the k value is randomly generated in each state analyzed) the variety of solutions found are diverse avoiding local solutions.

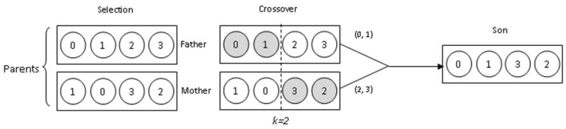

Figure 2: Crossover step from EMODS. Cross in the k^{th}-position.

4 EXPERIMENTAL STUDIES

4.1 Experimental Settings

4.1.1 Test Instances and Parameters

EMODS was tested using the Three Objective Traveling Salesman Problem (3-TSP). Formally, TSP is defined as follows:

$$min \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} \cdot X_{ij} \tag{10}$$

Subject to:

$$\sum_{j=1}^{n} X_{ij} = 1, \forall i = 1, \ldots, n \tag{11}$$

$$\sum_{j=1}^{n} X_{ij} = 1, \forall j = 1, \ldots, n \tag{12}$$

$$\sum_{i \in \kappa} \sum_{j \in \kappa} X_{ij} \leq |\kappa| - 1, \forall \kappa \subset \{1, \ldots, n\} \tag{13}$$

$$X_{ij} = 0, 1 \forall i, j \tag{14}$$

Where C_{ij} is the cost of the path X_{ij} and κ is any nonempty proper subset of the cities $1, \ldots, m$. (10) is the objective function. The goal is the optimization of the overall cost of the tour. (11), (12) and (14) fulfills the constrain of visiting each city only once. Lastly, Equation (13) set the subsets of solutions, avoiding cycles in the tour.

The test was made using well known Three - Objective Traveling Salesman Problem (3TSP) instances from from TSP LIB(Heidelberg,). The instances contains problems of 100 cities. Each city is represented as a point in the space, so the distance is computed using the euclidean distance between each pair of points. Each algorithm was run 10 times, the best nondominated solutions were selected for each of one. The true solution was constructed using the best nondominated solutions of all the sets.

4.1.2 Algorithms in Comparison

EMODS was compared against four MOSA algorithm, the algorithms are: Multiobjective Simulated

Annealing (CMOSA), Ulungu Multiobjective Simulated Annealing (UMOSA), Search Multiobjective Simulated Annealing (SMOSA) and Evolutionary Multiobjective Simulated Annealing (EMOSA). The most good-perfomance of them is for Evolutionary Multiobjective Simulated Annealing as can be seen in(Li and Landa-Silva, 2008).

4.2 Experimental Results

4.2.1 Performance Metrics

There are metrics that allow measuring the quality of a set of optimal solutions and the performance of an Algorithm(Jingyu et al., 2007). Most of them use two Pareto Fronts. The first one is PF_{true} and it refers to the real optimal solutions of a combinatorial problem. The second is PF_{know} and it represents the optimal solutions found by an algorithm.

Generation of Nondominated Vectors (GNDV). It measures the number of Nondominated Solutions generated by an algorithm.

$$GNDV = |PF_{know}| \tag{15}$$

A higher value for this metric is desired.

Generational Distance (GD). This metric measures the distance between PF_{know} and PF_{true}. It allows to determinate the error rate of a set of solutions relative to the real solutions.

$$GD = \left(\frac{1}{|PF_{know}|}\right) \cdot \left(\sum_{i=1}^{|PF_{know}|} d_i\right)^{(1/p)} \tag{16}$$

Where d_i is the smallest Euclidean distance between the solution i of FP_{know} and the solutions of FP_{true}. p is the dimension of the combinatorial problem, it means the number of objective functions.

Inverse Generational Distance (IGD). This is another distance measurement between FP_{know} and FP_{true}

$$IGD = \left(\frac{1}{|PF_{true}|}\right) \cdot \left(\sum_{i=1}^{|PF_{know}|} d_i\right) \tag{17}$$

Where d_i is the smallest Euclidean distance between the solution i of PF_{know} and the solutions of PF_{true}.

Spacing (S). It measures the range variance of neighboring solutions in PF_{know}

$$S = \left(\frac{1}{|PF_{know}|-1}\right)^2 \cdot \left(\sum_{i=1}^{|PF_{know}|} (\overline{d} - d_i)^2\right)^{(1/p)} \tag{18}$$

Where d_i is the smallest Euclidean distance between the solution i of PF_{know} and the rest of solutions of PF_{know}. $\overline{d}$ is the mean of all d_i. p is the dimension of the combinatorial problem.

A value closer to 0 for this metric is desired. A value of 0 means that all the solutions are equidistant.

4.2.2 Results Analysis

Figures 3 and 4 show a graphical comparison between EMODS Pareto Front and the rest of Compared Algorithms Pareto Fronts for the instances KROABC100 and KROBCD100 respectively. In addition, in tables 3 and 4 is measured the performance of the algorithms for each mentioned instance respectively.

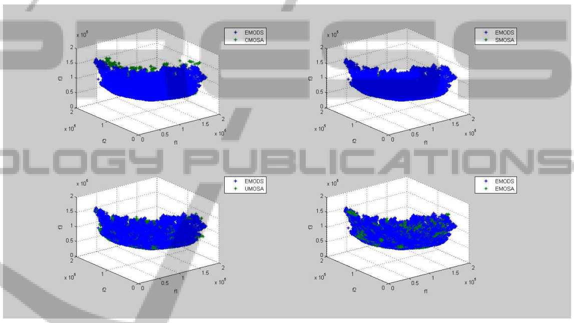

Figure 3: Graphical comparison between EMODS Pareto Front and the rest of Algorithms Pareto Fronts for the KROABC100 instance.

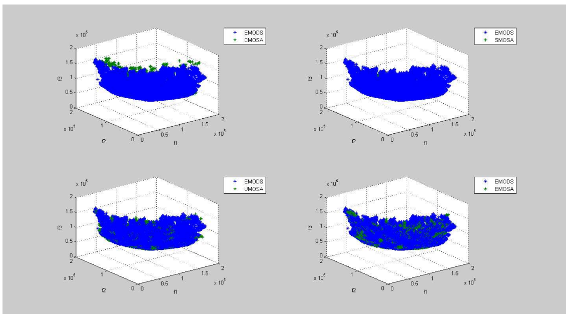

Figure 4: Graphical comparison between EMODS Pareto Front and the rest of Algorithms Pareto Fronts for the KROBCD100 instance.

In the first instance, as can be seen in the figure 5, most of the times, the EMODS solutions dominated the compared algorithms solutions. It can be corroborate in table 3 where EMODS generated 34077 solutions in its Pareto Front having the lowest GD only of 0,05 (5%). In the second case, EMODS had the best performance as can be seen in figure 6 and Table 4. EMODS generated 34824 solutions in its Pareto Front having a distance from the Real Pareto Front

Table 1: Measuring algorithms performance for the KROABC100 instance with multi - objective optimization metrics.

	GVND	*SPACING*	*GD*	*IGD*
SMOSA	1095	0,0599588	26,5312636	36172,0838
CMOSA	1817	0,04035959	14,3588074	29172,6591
UMOSA	2564	0,03498623	3,4396281	3333,40773
EMOSA	3194	0,03144919	2,38276567	2482,34369
EMODS	34077	0,01365555	0,05108237	129,865643

Table 2: Measuring algorithms performance for the KROBCD100 instance with multi - objective optimization metrics.

	GVND	*SPACING*	*GD*	*IGD*
SMOSA	1097	0,05749539	25,2829385	33752,4241
CMOSA	1795	0,04647346	14,1155524	28168,342
UMOSA	2472	0,03581236	3,46956584	3227,63056
EMOSA	3143	0,03150265	2,3144672	2321,81408
EMODS	34824	0,01307551	0,04979096	131,915227

Table 3: Average measuring algorithms performance for the KROABC100 and KROBCD100 instances with multi - objective optimization metrics.

	GVND	*SPACING*	*GD*	*IGD*
SMOSA	1097	0,05749539	25,2829385	33752,4241
CMOSA	1795	0,04647346	14,1155524	28168,342
UMOSA	2472	0,03581236	3,46956584	3227,63056
EMOSA	3143	0,03150265	2,3144672	2321,81408
EMODS	34824	0,01307551	0,04979096	131,915227

only of 0.4 (4%).

Lastly, the metrics values in the table 5 are averaged. It can be seen the superiority of EMODS solutions having the lowest error distance (GD) value of 4.6%. In addition, in all the cases, the values of the metrics applied to EMODS solutions are distant from the others and those show the best performance.

5 CONCLUSIONS

A novel metaheuristic named EMODS was proposed. EMODS is an Evolutionary Metaheuristic to solve Combinatorial Problems. It is based in the Natural Selection Theory for avoiding Local Optimums. Besides, it is based in MODS algorithm to represent the feasible solution space under the Automata Theory. Hence, a search direction is used to explore the feasible solution space (Multiobjective Deterministic Finite Automata). The proposed algorithm was tested against metaheuristic from the specialized literature using well known instances from the TSPLIB. EMODS showed the best performance in all the metrics worked and in some cases the error rate of EMODS was 4%.

REFERENCES

Fung, R., Tang, J., and Zhang, J. (2009). A multi-depot vehicle routing problem with weight-related costs. In *Computers Industrial Engineering, 2009. CIE 2009. International Conference on*, pages 1028 –1033.

Glover, F. and Laguna, M. (1997). *Tabu Search*. Kluwer Academic Publishers, Norwell, MA, USA.

Heidelberg, U. O. Tsplib - office research group discrete optimization - university of heidelberg. http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/.

Hu, B. and Raidl, G. (2008). Solving the railway traveling salesman problem via a transformation into the classical traveling salesman problem. In *Hybrid Intelligent Systems, 2008. HIS '08. Eighth International Conference on*, pages 73 –77.

Jingyu, Y., Chongguo, L., Zhi, W., Lei, D., and Demin, S. (2007). Diversity metrics in multi-objective optimization: Review and perspective. In *Integration Technology, 2007. ICIT 07. IEEE International Conference on*, pages 553–557.

Li, H. and Landa-Silva, D. (2008). Evolutionary multi-objective simulated annealing with adaptive and competitive search direction. In *Evolutionary Computation, 2008. CEC 2008. (IEEE World Congress on Computational Intelligence). IEEE Congress on*, pages 3311 –3318.

Lim, A. and Wang, F. (2005). Multi-depot vehicle routing problem: a one-stage approach. *Automation Science and Engineering, IEEE Transactions on*, 2(4):397 – 402.

Niño, E. D., Ardila, C., Donoso, Y., and Jabba, D. (2010). A novel algorithm based on deterministic finite automaton for solving the mono-objective symmetric traveling salesman problem. *International Journal of Artificial Intelligence*, 5(A10):101 – 108.

Niño, E. D., Ardila, C., Donoso, Y., Jabba, D., and Barrios, A. (2011). Mods: A novel metaheuristic of deterministic swapping for the multi objective optimization of combinatorials problems. *Computer Technology and Application*, 2(4):280 – 292.

Oberlin, P., Rathinam, S., and Darbha, S. (2009). A transformation for a heterogeneous, multiple depot, multiple traveling salesman problem. In *American Control Conference, 2009. ACC '09.*, pages 1292 –1297.

Pretorius, W. and Helberg, A. (2004). Application of an adapted evaluation process using numerical amp; qualitative weighted sum techniques. In *AFRICON, 2004. 7th AFRICON Conference in Africa*, volume 1, pages 367 –372 Vol.1.

Sauer, J. and Coelho, L. (2008). Discrete differential evolution with local search to solve the traveling salesman problem: Fundamentals and case studies. In *Cybernetic Intelligent Systems, 2008. CIS 2008. 7th IEEE International Conference on*, pages 1 –6.

Wang, S.-Q. and Xu, Z.-Y. (2009). Ant colony algorithm approach for solving traveling salesman with multi-agent. In *Information Engineering, 2009. ICIE '09. WASE International Conference on*, volume 1, pages 381 –384.

Wang, Y. and Lang, M. (2008). Study on the model and tabu search algorithm for delivery and pickup vehicle routing problem with time windows. In *Service Operations and Logistics, and Informatics, 2008. IEEE/SOLI 2008. IEEE International Conference on*, volume 1, pages 1464 –1469.

Yong-fa, Q. and Ming-yang, Z. (2004). Research on a new multiobjective combinatorial optimization algorithm. In *Robotics and Biomimetics, 2004. ROBIO 2004. IEEE International Conference on*, pages 187 –191.