

# A COOPERATIVE MODEL FOR MULTI-CLASS PEER-TO-PEER STREAMING NETWORKS

Pablo Romero, María Elisa Bertinat, Darío Padula, Pablo Rodríguez-Bocca  
and Franco Robledo Amoza

Laboratorio de Probabilidad y Estadística, Facultad de Ingeniería, Universidad de la República, Montevideo, Uruguay

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**Abstract:** Peer-to-peer networks are strongly based on cooperation. The users, called *peers*, communicate basically in a three-level based policy. In the first one, peers discover others interested in the same content, and is called swarm selection strategy (or *swarming*). Then, peers must select the best ones to cooperate, what is called *peer selection strategy*. Finally, peers cooperate sending pieces to each other, and the planning must attend the *piece selection strategy*.

In this paper we propose an extension of a simple model based on cooperation for peer-to-peer video streaming networks. We assume that the swarming classifies peers according to their bandwidth. In this model we meet both the peer and the piece selection strategies, for simplified scenarios. The aim is to design network policies in order to achieve the highest continuity of video reproduction when peers reach a stationary state. We show that under full knowledge, the network can scale even under free-riding effects. At the same time, we provide theoretical results that reveal Rarest First has a poor performance in comparison with other techniques. Finally, we analyze the scalability in a worst-case scenario when a variable amount of special peers are included in the network.

## 1 INTRODUCTION

An important amount of today's Internet traffic is due to live video streaming (Bertinat et al., 2009c). For this reason, several peer-to-peer streaming networks were developed in the last years. The most successful ones are PPlive(Liu et al., 2009; Huang et al., 2008), TVUnetwork(TVUnetworks home page, 2007), SopCast(SopCast - Free P2P internet TV, 2007), with proprietary and unpublished mesh-based protocols (Rodríguez-Bocca, 2008). Mesh-based P2P networks are virtual unstructured networks developed at the application layer, over the Internet infrastructure. Bittorrent is the best known mesh-based P2P protocol, originally created for file sharing purposes (Cohen, 2003). The users, called *peers*, offer their resources (bandwidth in a streaming application) to others, basically because they share common interests. They can connect and disconnect freely. This makes P2P networks an attractive tool for them, but increases P2P's design challenges, because the resource availability depends on them.

In P2P, the cooperation is the key element in order to assure a certain quality of experience to end-users (Rodríguez-Bocca, 2008). There are three main

steps in all mesh-based P2P protocols for cooperation. First, when a peer enters the net it should discover other peers sharing the same content, which is called *swarm selection strategy*. Once a new peer knows other peers in his swarm, he must select the best ones to cooperate, what is called *peer selection strategy*. Once a new peer handshakes other peers, it should decide which pieces of the streaming content should be asked first, called the *piece selection strategy* (Bertinat et al., 2009b).

The research literature related on Peer-to-Peer networks focused, from the beginnings, in system design and traffic measurements for file sharing (Ripeanu, 2001; Cohen, 2003). The new challenges adopted for streaming purposes inspired the scientific community to elaborate diverse mathematical models to understand the behavior and scalability of streaming systems, including Markov Chains, Fluid Models, Branching Processes and Marginal Probabilities (Zhou et al., 2007; Zhao et al., 2009). In this work we propose an extension of the simple model for cooperation defined in (Zhou et al., 2007). There, a pull process is considered, in which peers cooperate with each other in order to recover a video streaming content which is delivered by a server. The aim

is to find an optimal permutation, that dictates the order in which pieces must be downloaded in order to achieve high continuity of video reproduction. This model has been extensively analyzed in (Bertinat et al., 2009a; Bertinat et al., 2009b; Romero et al., 2010; Rodríguez, 2009; Zhao et al., 2009). Specifically, in (Zhao et al., 2009) the authors state properties of the optimal permutation for highly populated scenarios of identical peers, and a server with bounded uploading capacity.

This paper is structured as follows. Section 2 details the simple model for cooperation in peer-to-peer streaming networks originally defined in (Zhou et al., 2007). In Section 3 we summarize the main results of combinatorial problems concerning the simple model for cooperation. The contributions of this paper are the introduction of an extension of this mathematical model considering different peer classes (free-riders, normal peers, double-bandwidth peers and super-peers), an analysis of this new model under different scenarios and discussion of this results. At the same time we provide theoretical results that confirm super-peers can highly outperform the Rarest First strategy, which is widely used nowadays for file sharing purposes. We will work between the performance of the Rarest First and super-peer strategy, given that the performance of super-peers is not achievable in practice. This latter fact will be also proved.

Section 4 contains a generalization of the simple model, and an analysis of different scenarios. More specifically, Subsection 4.1 presents the Extended Model (EM for short). In Subsection 4.2 we show that super-peers achieve an upper bound in the performance of every possible piece selection strategy in the simple model. Super-peers play a prestigious role in the new model here proposed. Subsection 4.3 shows that the EM is in fact an extension of the simple model, and consequently the computational complexity of its resolution is higher than the complexity of the simple model. Subsection 4.4 shows how to deal with free-riders under the extended model. We define a natural hypothesis of full knowledge in the network, in which the network scales even under presence of free-riders. However, we show that the performance dramatically decreases if peers (or the server) cannot recognize the different classes of peers in the network, unless super-peers are included in the network. In fact, under full knowledge (when the server and peers can recognize the different classes of peers), the network always scales, meaning that the continuity of the video reproduction will remain high independently of the number of peers in the network. This fact is proved in Subsection 4.5.

The most complex interaction is between normal

and double-bandwidth peers. We analyze this scenario in Section 5. Specifically, the study is focused on the scalability of the network when a variable amount of superpeers is present. Finally, Section 6 contains the main conclusions of this work.

## 2 A SIMPLE MODEL FOR COOPERATION

Consider a static network with  $M$  identical peers with buffer size  $N$ , and one server that contains the original video content. The server cuts the video into pieces and shares them in order. In each time slot, it chooses only one peer randomly to send one piece. That peer places the piece in the first buffer position, and that piece will advance one buffer position in each time slot, until it reaches position  $N$ . Pieces from position  $N$  are displayed at the screen of that user. It is assumed that all peers are synchronized with time (i.e. every peer plays the piece at position  $N$ , see Figure 1 for a graphical description).

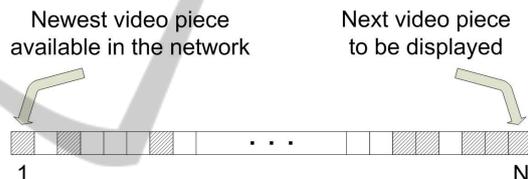


Figure 1: Buffer model for each peer. Position 1 represents the newest video piece in the network, and  $N$  the next piece to be displayed.

All peers can also communicate with each other in order to ask for pieces. In each time slot, every peer chooses one of the  $M - 1$  other peers at random, and can get no more than one piece during that time slot. The piece selection strategy works as follows: each peer chooses a permutation  $\pi$  of the set  $B = \{1, \dots, N - 1\}$ . Each element of  $B$  represent one position of the buffer size, without regarding the last (position  $N$ ), which is expected to be played on that time slot. Suppose peer  $P_1$  chooses peer  $P_2$ . Then  $P_1$  looks for position  $\pi_1$  of its own buffer. If it has that piece, it looks for  $\pi_2$  and so on. Otherwise, it checks if peer  $P_2$  has that piece, and takes it. This process is repeated until either  $P_1$  gets one piece during that time slot or it checks with no success every place of its buffer. In the former (latter) case we say peer  $P_1$  had a *good* (resp. *bad*) time slot.

In (Zhou et al., 2007), a symmetric strategy is considered, which means that every peer follows the same piece selection strategy  $\pi$ . Let us call  $p_i$  the probability that a peer has piece  $i \in \{1, \dots, N\}$  under stationary state (this probability is the same for differ-

ent peers because of the symmetry). It can be proved that (Zhou et al., 2007):

$$\begin{cases} p_1 &= \frac{1}{M} \\ p_{i+1} &= p_i + (1 - p_i)p_i s_i, \forall i \in B. \end{cases} \quad (1)$$

Clearly, the probability of a given peer to be chosen by the server is one out of  $M$ , so  $p_1 = \frac{1}{M}$ . We know that under steady state we have that  $p_i(t) = p_i$ , so the probability vector does not depend on time. As a consequence, the piece at position  $i + 1$  can be obtained in two different ways. The first one is by promotion with time of piece at position  $i$  (with probability  $p_i$ ). The second is because the peer did not have the piece  $i$  (event with probability  $1 - p_i$ ) but the requested peer did (probability  $p_i$ ) and piece at position  $i$  was chosen to be downloaded in the previous time slot (event with probability  $s_i$ ). The *strategic sequence*  $s_i$  represents the probability of taking the piece at position  $i$ , given that the requesting peer does not have that piece and the requested does. Note that  $s_i$  will depend on the permutation, and under this model we identify piece selection strategies with one permutation of the elements in the set  $B$ .

### 3 ANALYSIS OF THE SIMPLE MODEL

The previous model was originally proposed in (Zhou et al., 2007), and was extensively analyzed in (Bertinat et al., 2009a; Bertinat et al., 2009b; Romero et al., 2010; Rodríguez, 2009). Here, we summarize the main results, and provide a new pessimistic one, that assures the Rarest First strategy (a widely developed technique used in BitTorrent (Cohen, 2003)) has a low (linear) convergence rate to the perfect video quality, when the buffer size tends to infinity. At the same time, we will show that is impossible to achieve more than a quadratic convergence rate. Consequently, we will work between linear and quadratic convergence to the perfect video quality (outperforming Rarest First), whenever the buffer tends to be unlimited. We will formalize these ideas next. We suppose a symmetric network in steady state:

**Definition 3.1.** *The continuity of the video reproduction is measured by  $c = p_N$ .*

**Definition 3.2.** *The buffering time is  $L = \sum_{i=1}^N p_i$*

Definition 3.1 is clear: count the number of pieces showed at the screen and divide it by the total number of time slots. When the number of time slots tends to infinity, we have the continuity of the video reproduction.

Definition 3.2 deserves an explanation. Suppose a new peer  $P$  enters to that static network with an empty buffer. In the next time slots, it will get many pieces with high probability via requests. More precisely, the expected number of time slots needed to reach the steady state is the expected number of pieces of a peer, or  $p_1 + \dots + p_N$ .

In (Zhou et al., 2007), the performance of classical strategies are studied, named Rarest First, the Greedy strategy, and a Mixture of them. The name Rarest First is inspired by BitTorrent (Cohen, 2003). This piece selection policy tries to achieve uniform distribution, copying the rarest pieces in the network. In this way, it assures that rarest pieces are easier to find via requests. Under this model, observe that the vector  $(p_i)_{1 \leq i \leq N}$  is monotonous increasing. Then, Rarest First takes the identity permutation  $\pi_i = i, \forall i \in B$  (ask for the first piece in the buffer, then the second and so on until either downloading a piece or completing a bad time slot). The strategic sequence in Rarest First is:

$$s_i = (1 - 1/M) \prod_{j=1}^{i-1} [p_j + (1 - p_j)^2] \quad (2)$$

Expression 2 has a simple interpretation. In order to download the piece at position  $i \in B$ , the peer must fail in all previous positions  $j = 1, \dots, i - 1$  and must not be chosen by the server (with probability  $1 - 1/M$ ). A fail at position  $j$  occurs when the peer already has a piece at position  $j$  (with probability  $p_j$ ) or it does not have that piece but neither the requested peer (event with probability  $(1 - p_j)^2$ ). A direct induction for  $i \in \{1, \dots, N\}$  shows that  $s_i = 1 - p_i$  holds for Rarest First (Zhou et al., 2007).

On the other hand, the *greedy* notion of the problem is to ask first for the nearest piece to be played (i.e. the piece at position  $N - 1$ , because that one at position  $N$  is being played). Then, the Greedy strategy considers the permutation  $\pi_i = N - i, \forall i \in B$ . Its strategic sequence is:

$$s_i = (1 - 1/M) \prod_{j=i+1}^{N-1} [p_j + (1 - p_j)^2] \quad (3)$$

The interpretation is analogous to that of Rarest First, but reading the buffer in the opposite way. A mixture of both strategies can be obtained reading the buffer in the increasing way (using Rarest First) until a certain buffer position  $m : 1 < m < N$ , and then completing the buffer using Greedy from position  $N - 1$  down-to  $m + 1$ . The "Mixture" strategies are defined depending on  $m$ .

There are piece selection strategies that outperform classical strategies, as well as the Mixture strategy.

In fact, in (Bertinat et al., 2009a) a subfamily of permutation strategies was defined with polynomial size. That subfamily contains the classical strategies, as well as their mixtures, and has a polynomial cardinality in terms of the buffer size. An exhaustive search in this family permits to achieve higher continuities, keeping at the same time reduced buffering times. A more sophisticated design of piece selection strategies can be found in (Romero et al., 2010). There, a single objective function was defined, which captures the continuity and latency:

**Theorem 3.3.** *If  $\pi$  is an arbitrary permutation of  $B$  and  $X_\pi$  is the random variable that counts the number of steps in a good time slot, then its expected value is:*

$$E(X_\pi) = \frac{M}{M-1} \sum_{i=1}^{N-1} \pi_i(p_{i+1} - p_i) \quad (4)$$

Note that in Rarest First we get  $E(X_\pi) \propto Np_N - L$ . The convex combination of continuity and latency takes a natural form in this combinatorial problem (maximize  $E(X_\pi)$  choosing the best permutation  $\pi$ ). We translated this problem in a second opportunity into a suitable Asymmetric Traveling Salesman Problem (ATSP). Finally, an Ant Colony-based search (Dorigo and Stutzle, 2004) permits to find cheap tours (or equivalently permutations) and this permutations had a direct interpretation in the simple model, outperforming again classical selection strategies. We refer the reader to (Romero et al., 2010) for an overview.

In the next section, we will enrich the simple model by considering different classes of peers, according to a swarm selection policy based on bandwidth. The analysis is primarily focused on extreme scenarios, attending the interaction of four classes of peers: free-riders (Class 0 with zero upload bandwidth), normal peers with unit bandwidth (Class 1), double-peers with double bandwidth (Class 2) and super-peers with unlimited bandwidth (Class 3). All peers have unlimited downloading capacities (so, the limitation is in the uploading bandwidth). The main issue is to plan the network in order to maximize the continuity of reproduction, in a more realistic model.

## 4 A GENERALIZATION OF THE SIMPLE MODEL

### 4.1 Definition of the Extended Model

Suppose a static network that has  $M$  peers of Class  $X$ ,  $M'$  peers of Class  $Y$  and a server that has the original video content (where  $X, Y \in \{0, 1, 2, 3\}$ ). The server

cuts the video into small pieces, and shares them in turns. In each time slot, the server chooses one peer at random from Class  $X$  with probability  $\alpha$ , or one peer at random from Class  $Y$  with probability  $1 - \alpha$ , and sends one piece to that peer. As in the simple model, peers can cooperate. More precisely, one peer from Class  $X$  either chooses with probability  $\beta$  another peer at random from its own class or a peer from Class  $Y$  at random with probability  $1 - \beta$ . Symmetrically, peers from Class  $Y$  can request other peers from their own class (chosen at random) with probability  $\beta'$ , or from Class  $X$  (with probability  $1 - \beta'$ ). Every peer tries to download the highest number of pieces during each time slot, and that number will depend on the uploading bandwidth of the contacted class. For example, if a peer requests a double-peer (with double bandwidth), it will be able to download two pieces during the same time slot. The process of the request is identical to that of the simple model, but it may continue after one piece is obtained.

**Definition 4.1.** *A free-rider is a peer that has infinite downloading bandwidth, but no uploading bandwidth. When a peer requests a free rider, it will get no piece on that time slot.*

In other words, it is a selfish peer, that asks for pieces but does not share them.

**Definition 4.2.** *A normal peer has infinite downloading bandwidth and unit uploading bandwidth. When a peer requests a normal peer, the time slot works as in the simple model.*

**Definition 4.3.** *A double-bandwidth peer has infinite downloading bandwidth and double uploading bandwidth. When a peer requests a double-bandwidth peer, it can get zero, one or two pieces.*

For example, if one peer follows the Rarest First strategy and requests a double-bandwidth peer, then the request works as in the simple model. However, if a download occurs, the peer goes on asking for the next pieces, until downloading another one or reaching position  $N - 1$  of its buffer. An analogous request occurs when the piece selection strategy is identified with an arbitrary permutation.

**Definition 4.4.** *A super-peer has both infinite downloading and uploading bandwidth. When a peer requests a super-peer, it will take all pieces in only one time slot.*

The optimization problem is specified next. The two classes  $X$  and  $Y$ , the number of peers  $M$  and  $M'$  and the buffer size  $N$  are given. We want to plan the network by choosing the parameters  $\alpha$ ,  $\beta$  and  $\beta'$  as well as the permutation strategy  $\pi$ , in order to maximize the average continuity of reproduction in the network. More specifically, the Extended Model (from

now on the EM) is captured with the next optimization problem:

$$\max f(\pi, M, N, \alpha, \beta) = \frac{Mp_N + M'p'_N}{M + M'} \quad (5)$$

s.t.

$$\begin{cases} p_1 = \frac{\alpha}{M} \\ p'_1 = \frac{1-\alpha}{M'} \\ p_{i+1} = p_i + (1-p_i)[\beta p_i s_i^{(X,X,\pi)} + (1-\beta)p'_i s_i^{(X,Y,\pi)}] \\ p'_{i+1} = p'_i + (1-p'_i)[\beta' p'_i s_i^{(Y,Y,\pi)} + (1-\beta')p_i s_i^{(Y,X,\pi)}] \\ \alpha, \beta, \beta' \in [0, 1] \end{cases}$$

where  $s_i^{(X,Y,\pi)}$  is the probability that a peer from class  $X$  is using the permutation strategy  $\pi$  (which is a permutation of the set  $\{1, \dots, N-1\}$ ) and requesting a peer from class  $Y$  takes piece at position  $i$ . These expression will be analyzed for each possible scenario. The objective is to maximize the average quality of experience of all peers in the network (identifying quality with continuity of reproduction). If we recall that the server sends with probability  $\alpha$  one peer from Class  $X$  at random, then obviously  $p_1 = \alpha/M$  and  $p'_1 = (1-\alpha)/M'$  hold. The next equations are correct under steady state, and take into account the fact that the requested peer can be from their own class or the foreign class. We shall fix the parameters  $\beta$  and  $\beta'$  according to random peer selection (i.e.  $\beta = M/(M+M')$  and  $\beta' = M'/(M+M')$ ). In fact, we will show that under a full knowledge assumption, the network can work in optimal conditions and the combinatorial problem is reduced to the simple model, which has been extensively analyzed in previous works (Bertinat et al., 2009a; Bertinat et al., 2009b; Romero et al., 2010; Rodríguez, 2009). The intuition here is that if the server as well as the peers can discover which peers have the highest bandwidth, then the server will send pieces to them, and all peers will direct requests to this powerful peers (which play the role of intermediate nodes of a tree-like structure).

There are exactly  $4^2 - C_2^4 = 10$  different interaction of pairs of the four classes (we are considering only once the cases of interaction between classes  $X$  and  $Y$ , when  $X \neq Y$ ). Moreover, the cases of self-interaction can be reduced to the simple model. More precisely, the self-interaction between free-riders is strictly inadmissible, and does not deserve our attention. The interaction between normal peers behaves exactly as in the simple model, and between double-bandwidth peers translates proportionally to the case of the simple model (in fact, cut the time slot into two half). There is something to say for the case of self-interaction between super-peers. As a consequence, we will focus on 7 scenarios: the six different pairs of classes, and the simple model with infinite bandwidth.

## 4.2 The Best Strategy for the Simple Model

Certainly, the best piece selection strategy for the simple model occurs when all peers in the network have infinite bandwidth, and they can download all pieces in only one time slot. In fact, in steady state the strategic sequence for this case is  $s_i = 1$ . This means that if one peer does not have a piece and the requested does, the peer always downloads that piece. Naturally, the sequence  $p_i$  is the highest possible, because:

$$p_{i+1} = p_i + (1-p_i)p_i \geq p_i + (1-p_i)p_i s_i, \quad (6)$$

whenever  $s_i \leq 1$ , which is obvious ( $s_i$  is a probability for every  $i \in B$ ). Hence, the probability  $p_N$  is never exactly 1 (as a trivial induction can show), but the highest possible. By technological reasons, it is natural to ask what happens in the case of unlimited storage (when the buffer size  $N$  tends to infinity).

**Theorem 4.5.** *Under the simple model, the super-peers tend to experiment perfect continuity when the buffer tends to infinity:*

$$\lim_{N \rightarrow \infty} p_N = 1. \quad (7)$$

Moreover, the convergence order is quadratic.

*Proof.* Super-peers are characterized by  $s_i = 1$ . Substituting in (1) we have that:

$$p_1 = \frac{1}{M}$$

$$p_{i+1} = p_i(2-p_i), i \in B$$

Taking  $i = N-1$  we have that  $p_N = p_{N-1}(2-p_{N-1})$ . The sequence  $(p_i)_{1 \leq i \leq N}$  is monotonous increasing and bounded by 1; hence it has a limit  $a$ . Taking  $N$  tending to infinity:

$$a = a(2-a) \quad (8)$$

So  $a = 0$  or  $a = 1$ . But  $p_1 = 1/M > 0$  and  $(p_i)_{1 \leq i \leq N}$  is monotonous increasing. Consequently  $a = 1$ , and

$$\lim_{N \rightarrow \infty} p_N = 1. \quad (9)$$

Finally, its convergence order can be found easily:

$$\lim_{N \rightarrow \infty} \frac{1-p_N}{(1-p_{N-1})^2} =$$

$$\lim_{N \rightarrow \infty} \frac{1-p_{N-1}(2-p_{N-1})}{(1-p_{N-1})^2} = 1.$$

Hence, its convergence order is 2, and the result holds.  $\square$

Observe also that super-peers achieve the smallest buffering times, because if one super-peer enters the network in steady state, then in one time slot reaches the state of another super-peer.

It is interesting to compare this performance with respect to the one obtained following Rarest First:

**Theorem 4.6.** *The Rarest First strategy tends to have perfect continuity when the buffer size tends to infinity, but its convergence order is linear.*

*Proof.* In Rarest First  $s_i = 1 - p_i$  holds for all  $i \in B$ . As a consequence:

$$\begin{cases} p_1 &= \frac{1}{M} \\ p_{i+1} &= p_i + (1 - p_i)^2 p_i \end{cases}$$

Again, the limit of the sequence  $(p_i)_{1 \leq i \leq N}$  exists when  $N$  tends to infinity (it is a bounded increasing real sequence). The limit  $a$  must comply that  $a = a + (1 - a)^2 a$ . Then  $a = 0$  or  $a = 1$ . The null limit is discarded because the sequence  $(p_i)_{1 \leq i \leq N}$  is monotonous increasing and  $p_1 > 0$ . Hence,  $a = 1$ . Finally, the convergence order is linear, given that:

$$\begin{aligned} \lim_{N \rightarrow \infty} \frac{1 - p_N}{1 - p_{N-1}} &= \\ \lim_{N \rightarrow \infty} \frac{1 - p_{N-1} - p_{N-1}(1 - p_{N-1})^2}{1 - p_{N-1}} &= 1. \end{aligned}$$

As a consequence, the piece selection strategies will always work with convergence order  $p$  such that  $1 < p < 2$  when the buffer increases. In fact, we know there are better strategies than Rarest First, and that there is no strategy better than the one of super-peers (download the whole buffer of the requested peer).

### 4.3 The EM under Full-Knowledge

From now on, we study the EM (Extended Model) when different classes interact (i.e.  $X \neq Y$ ).

**Definition 4.7.** *We say that the network in the EM has full knowledge, when the server can recognize the different classes of peers in the network, and peers can deduce the best class-request (if it is better to ask one peer from its own class or the foreign class).*

**Definition 4.8.** *A peer-class has higher level than other when it has higher uploading bandwidth.*

**Definition 4.9.** *We say that the server is fair when each peer in the network has the same probability of getting a piece from it.*

**Definition 4.10.** *We say that the network is balanced when the peer selection strategy is at random.*

**Theorem 4.11.** *The EM is computationally more complex than the Simple Model.*

*Proof.* We will prove that the EM is trivially reduced to the simple model under full knowledge and fairness. Without loss of generality, suppose  $X$  has higher class than  $Y$ . Then clearly a peer has more

chances to download piece at position  $i$  requesting peers from class  $X$  rather than from class  $Y$ , and  $s_i^{(X,X,\pi)} \geq s_i^{(X,Y,\pi)}$ . Given that peer can recognize the highest class, then they will always choose peers from class  $X$  to ask for pieces, so  $\beta = 1$  and  $\beta' = 0$ . By symmetry, observe that  $s_i^{(Y,X,\pi)} = s_i^{(X,X,\pi)}$ . Denote this number with  $s_i^\pi$  for brevity. Substituting in the EM we have that:

$$\begin{aligned} p_1 &= \frac{\alpha}{M} \\ p'_1 &= \frac{1 - \alpha}{M'} \end{aligned}$$

$$\begin{aligned} p_{i+1} &= p_i + (1 - p_i)[p_i s_i^\pi] \\ p'_{i+1} &= p'_i + (1 - p'_i)[p_i s_i^\pi] \\ \alpha &\in [0, 1] \end{aligned}$$

Assuming fairness, the server will send pieces with probability  $\alpha = M/(M + M')$ . As a consequence,  $p_1 = p'_1 = 1/(M + M')$ . Hence, both recursive expressions are the same, and the sequences  $p_i$  and  $p'_i$  coincide. Moreover, the problem was reduced to:

$$\begin{cases} p_1 &= \frac{\alpha}{M} \\ p_{i+1} &= p_i + (1 - p_i)p_i s_i^\pi \end{cases} \quad (10)$$

being  $\pi$  a permutation, which is exactly the simple model with  $M + M'$  peers.  $\square$

So far, we know that the peers with higher class perform better under the simple model, and super-peers achieve the best performance, with unit strategic sequence ( $s_i = 1$ ).

### 4.4 Dealing with Free-riders

As we said before, the self-interaction of free-riders is not admissible (it is evident that without cooperation the network does not work). The reader can check that if all peers are free-riders then  $p_i = p_1 = \beta/M < 1/M, \forall i$ , and this performance is not acceptable since the network normally works with hundreds or thousands of peers. Similar results are obtained for the second class:  $p'_i = (1 - \beta)/M'$  is constant.

The interaction between free-riders and other classes has a special treatment. Particularly, suppose that  $X = 0$  (free-rider class) and  $Y \neq 0$ . Under full knowledge, the server will always choose to send pieces to peers from class  $Y$ , so  $\alpha = 0$ . Moreover, free-riders will choose to complete requests considering peers from Class  $Y$ , which will prefer to do self-requests, so  $\beta = 0$  and  $\beta' = 1$ . Substituting in the EM:

$$\begin{cases} p_1 &= 0 \\ p'_1 &= \frac{1}{M} \\ p_{i+1} &= p_i + (1 - p_i)p'_i s_i^{(X,Y,\pi)} \\ p'_{i+1} &= p'_i + (1 - p'_i)p'_i s_i^{(Y,Y,\pi)} \end{cases}$$

As a consequence, the quality of all non-free-riders in the network is equivalent to that of the simple model. Note that  $p_1 = 0$  but  $p_2 > 0$ . For example, if the Rarest First strategy is applied, then the sequence  $\{p_i\}_{1 \leq i \leq N}$  converges to 1 as  $N$  tends to infinity, and behaves exactly the same as  $\{p'_i\}_{1 \leq i \leq N}$  but with a shift. In this way, the free-riders follow the performance of the other class, and the network scales.

The previous discussion shows that with full knowledge, the planning of the network reduces to choose a piece selection strategy, or a permutation  $\pi$ , as in the case of the simple model (which has been extensively analyzed already). However, if the server cannot identify classes, it will tune  $\alpha \neq 0$ , and the performance of the network dramatically decreases, because pieces given to free-riders will be missing for all but only one peer. Hence, the network scales if and only if  $\alpha = 0$ . This results outstand the importance of the recognition of free-riders, under this new extension of the simple model. The full-knowledge hypothesis is strictly necessary in this case. This is an evidence of the empirical complexity of designing a scalable streaming network: normally the broadcaster does *not* have full knowledge, and neither peers do.

#### 4.5 The Network with Super-peers

Naturally, when one of the classes working in the network are super-peers, the cooperation is easier. With full knowledge of the network (i.e. the server as well as peers can recognize classes of different peers), the server will always send pieces to super-peers, and the other class will be pleased to complete full requests to them, making the network scalable. The quality of experience of every peer in the network follows, under these circumstances, the one of super-peers (as if there were no other class) in the simple model. As a consequence, all peers will have (discarding the small initial shift) the next probabilities:

$$\begin{cases} p_1 &= 1/M \\ p_{i+1} &= p_i(2 - p_i), \forall i \in B, \end{cases} \quad (11)$$

being  $M$  the number of super-peers in the network and  $p_N$  the continuity of video reproduction of each peer. When free-riders or super-peers are present inside the

network, the analysis of the EM is trivial (because the strategic sequence is reduced to 0 or 1 respectively). In the next section we analyze the most complex interaction.

## 5 INTERACTION BETWEEN NORMAL AND DOUBLE-BANDWIDTH PEERS

### 5.1 Presentation of the Problem

This case is clearly the most complex to analyze. Intuitively, the server should send pieces to the double-bandwidth peers, and the request always directed to them. Under full knowledge this will happen, and normal-peers will tend to follow the quality of double-bandwidth peers.

Let us focus on a more realistic scenario. Choose  $X$  as normal peers and  $Y$  double-bandwidth peers. Now, we will find an expression for the sequences  $s_i^{(X,Y,\pi)}$  and  $s_i^{(Y,Y,\pi)}$  (the other two cases are self-requests, and expressed as in the simple model). For brevity,  $s_i$  denotes the probability that normal-peers have to take *the first piece* from a double-bandwidth peer. If  $k$  is such that  $\pi_{k+1} = i$  then:

$$s_i = (1 - \alpha/M) \prod_{j=1}^k [1 - (1 - p_{\pi_j})p'_{\pi_j}] \quad (12)$$

Expression (15) deserves an explanation. One peer from class  $X$  will download the first piece at position  $i$  from class  $Y$  following the permutation strategy  $\pi$  whenever it fails in all previous positions (and success at position  $i$ ) and is not chosen by the server (with probability  $p_1 = 1 - \alpha/M$ ). Hence, a fail at all positions  $\pi_j, j = 1, \dots, k-1$  such that  $\pi_k = i$  must occur. Moreover, a fail at position  $\pi_j$  occurs when it is not the case that the requesting peer does not have that piece (with probability  $1 - p_{\pi_j}$ ) and the requested peer does (event with probability  $p'_{\pi_j}$ ). Then, a fail at position  $\pi_j$  has probability  $1 - (1 - p_{\pi_j})p'_{\pi_j}$ .

Now, we are ready to express the sequence  $s_i^{(X,Y,\pi)}$ :

$$s_i^{(X,Y,\pi)} = s_i + s_i \sum_{j=1}^{k-1} \frac{(1 - p_{\pi_j})p'_{\pi_j}}{1 - (1 - p_{\pi_j})p'_{\pi_j}} \quad (13)$$

When asking a double-bandwidth peer, we can download piece at position  $i$  in the first chance (the first term) or we downloaded a previous position  $\pi_j, j = 1, \dots, k-1$  with success. The factor

$$(1 - p_{\pi_j})p'_{\pi_j} / [1 - (1 - p_{\pi_j})p'_{\pi_j}],$$

represents a *replace* of a success instead of a fail at position  $\pi_j$  in the expression  $s_i$ .

In a similar way, the strategic sequence  $s_i^{(Y,Y,\pi)}$  is:

$$s_i^{(Y,Y,\pi)} = s_i^* + s_i^* \sum_{j=1}^{k-1} \frac{(1-p'_{\pi_j})p'_{\pi_j}}{1-(1-p'_{\pi_j})p'_{\pi_j}}, \quad (14)$$

where

$$s_i^* = (1 - (1 - \alpha)/M) \prod_{j=1}^{k-1: \pi_k=i} [1 - (1 - p'_{\pi_j})p'_{\pi_j}]. \quad (15)$$

The EM can be obtained for this interaction by substitution.

## 5.2 Empirical Results

We will concentrate on a *worst case* scenario, by taking the Rarest First strategy (i.e.  $\pi_i = i$ ), and analyzing the scalability of the network under different mass of double-bandwidth peers, with no knowledge of the network, which implies that the peer selection is balanced:  $\beta = M/(M+M')$  and  $\beta' = M'/(M+M')$ . Consider the common-network values  $M+M' = 1000$  and  $N = 40$ . Table 1 presents the objective function  $f(\alpha, M) = (Mp_N + M'p'_N)/(M+M')$  when the mass of double-bandwidth peers is variable accordingly with  $M' \in \{350, 250, 150, 100, 0\}$  double-bandwidth peers and correspondingly  $M = 1000 - M'$  normal peers. Table 2 contains the function  $p_N - p'_N$  taking the same set for  $M$  and probability  $\alpha$ .

Table 1: Expected continuity  $f(\alpha, M)$  for a balanced network with different number of double-bandwidth peers.

$\alpha$	350	250	150	100	0
0.0	1.0000	0.9998	0.9979	0.9941	0.9666
0.1	1.0000	0.9998	0.9979	0.9940	0.9665
0.2	1.0000	0.9998	0.9978	0.9939	0.9663
0.3	1.0000	0.9998	0.9978	0.9938	0.9661
0.4	1.0000	0.9998	0.9977	0.9937	0.9658
0.5	1.0000	0.9998	0.9977	0.9936	0.9655
0.6	1.0000	0.9998	0.9976	0.9934	0.9651
0.7	1.0000	0.9997	0.9975	0.9932	0.9646
0.8	1.0000	0.9997	0.9974	0.9929	0.9638
0.9	1.0000	0.9997	0.9972	0.9925	0.9625
1.0	1.0000	0.9997	0.9970	0.9918	impossible

It can be appreciated from Table 1 that the network always scales, although the server cannot recognize peers and tunes incorrectly the parameter  $\alpha$ . Certainly, the performance is the best when  $\alpha = 0$  (that is, to choose always double-bandwidth peers to send pieces). It can be noticed that the average continuity is higher than 96% in all instances, so the video quality is high. It is interesting to analyze if the video quality of normal peers is similar to double-peers or not. Table 2 contains the difference of continuity  $p_N - p'_N$ .

Table 2: Difference in continuity  $p_N - p'_N$  between double-bandwidth peers and normal peers, with different number of normal peers.

$\alpha$	350	250	150	100	0
0.0	0.0002	0.0013	0.0067	0.0117	0.0006
0.1	0.0001	0.0010	0.0055	0.0094	0.0005
0.2	0.0001	0.0008	0.0041	0.0071	0.0004
0.3	0.0001	0.0005	0.0028	0.0048	0.0002
0.4	0.0000	0.0003	0.0014	0.0024	0.0001
0.5	0	0	0	0	0
0.6	-0.0000	-0.0003	-0.0015	-0.0025	-0.0001
0.7	-0.0001	-0.0006	-0.0030	-0.0051	-0.0003
0.8	-0.0001	-0.0009	-0.0047	-0.0077	-0.0004
0.9	-0.0002	-0.0013	-0.0065	-0.0106	-0.0006
1.0	-0.0002	-0.0017	-0.0086	-0.0140	impossible

It is obvious that when the parameter  $\alpha$  increases, the quality of normal peers is increased as well. Moreover, in the case  $\alpha = 0.5$  both classes of peers experiment the same video quality, and there is a symmetry in the instances  $\alpha = i/10$  and  $\alpha = (10 - i)/10$ . It is evident that peers can follow double-bandwidth peers, and peers have better continuity than super-peer when  $\alpha > 0.5$ . This empirical analysis shows that the network scales when peers and double-peers interact, even under pessimistic scenarios. A further experiment with the balanced case of  $\alpha = 0.5$  shows the scalability property of this network when the storage size increases. Figure 2 reveals the average continuity of normal (and double-bandwidth) peers as a function of the buffer size  $N$ , considering again different amounts of double-bandwidth peers. It can be appreciated that the average continuity is higher than 90% when the storage capacity is higher than 25, even when the number of double-bandwidth peers is small.

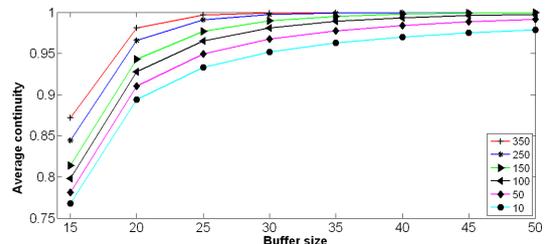


Figure 2: Evolution of the average continuity of peers as a function of the buffer storage capacity  $N$ .

## 6 CONCLUSIONS AND FUTURE WORK

This paper proposes an extension of a simple model for cooperation in peer-to-peer streaming networks. This model assumes a swarming policy based on uploading bandwidth, classifying peers as free-riders, normal, double-bandwidth and super-peers with infinite bandwidth. A primitive analysis demonstrates the strength of the full knowledge hypothesis in the net-

work. In fact, the scalability of the network is guaranteed whenever the server as well as peers can recognize different classes. When free-riders interact with other classes, peers will always experiment cuts in the video content, unless the server sends pieces to non-free-riders. On the other hand, when super-peers take part of it, the network scales.

The performance of the Rarest First strategy was contrasted with the one of super-peers. Particularly, the convergence to the perfect probability is faster for super-peers. Moreover, there is no real strategy that can achieve quadratic convergence to the perfect continuity, even with high buffer size. Finally, the most complex scenario considered the interaction between normal and double-bandwidth peers, and was analyzed via simulations. Although there exist many piece selection strategies with higher performance than Rarest First, the results show that the network scales using the latter strategy. This is an encouraging result, which motivates to apply different piece selection strategies. We have extensively analyzed the simple model, and we are currently approaching the general EM via metaheuristics. As a future work, we point to apply the results in a real peer-to-peer platform named GoalBit, an open source real platform that widely offers live video-streaming to end users (Bertinat et al., 2009c). It is worth to mention that we do not know a global model for peer-to-peer streaming networks that integrates swarming, peer selection and piece selection strategies. It sounds ambitious, and the model here proposed takes into account peers as well as piece selection strategies, in a cooperative simplified environment.

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