COPING WITH LONG TERM MODEL RISK IN MARKET RISK MODELS

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Abstract: The recent financial crisis has shown that most market risk models – even if they deliver sufficiently accurate risk figures over short time horizons – are not able to provide reliable forecasts for risk figures over longer time horizons like three, twelve or 36 months, which are the basis for both limit management and economic capital planning. As a potential remedy the concept of *potential future market risk* can be used to deal with such long term model risks in market risk measurement. Based on a toy example we will outline how this concept can be applied for new business planning or for limit setting and capital buffer definitions.

1 DEFINITION OF MODEL RISK

In the context of market risk measurement, *model risk* is usually understood as the risk that the model used for market risk measurement is specified wrongly and does therefore not or not fully capture the risks it was designed to measure (Jorion, 2007), Section 21.2.6. In the following, however, we focus on a different point of view along the lines of (Jorion, 2007), Chapter 9, and thus we need to distinguish between different kinds of model risk.

Short Term Model Risk. Short term model risk arises from the fact that model assumptions may be violated, which also includes parameter uncertainty (i.e. *estimation risk*). Taking the example of a delta-normal VaR model, the linear impact of risk factor changes on the portfolio value as well as the normal distribution assumption with constant volatility for risk factor changes are such assumptions. Short term model risk is usually an issue for the daily risk measurement process, see for example (Figlewski, 2003) or (Hendricks, 1996). (Berkowitz and OBrien, 2002) analyse the accuracy of VaR forecasts for banks' trading desks based on the models used in practice. There is vast literature on how short term model risk can be identified and controlled via back-testing procedures, see e.g. (Kupiec, 1995), (Christoffersen et al., 2001), (Christoffersen and Pelletier, 2004) or more recently (Berkowitz et al., 2011); or how it can be handled via more sophisticated models, see for instance (Kamdem, 2005) for an extension to elliptical distributions, (Alexander, 2001) for a non-parametric linear historical or Monte-Carlo VaR or (Bams and Wielhouwer, 2001) for adjustment factors for estimation risk. (Alexander and Sarabia, 2011) quantify VaR model risk and derive an add-on factor for market risk capital. Similarly, (Kerkhof et al., 2010) derive an add-on to capital reserves which accounts for VaR model risk and distinguishes between estimation and misspecification risk.

Long Term Model Risk. This kind of model risk covers the risk that the reported daily risk figures change in an adverse fashion over longer time scales, although the portfolio itself remains unchanged. This means that even if the risk is reasonably measured and predicted for small time horizons by the model, the market risk number might change on a daily basis and, therefore, cannot be used for longer term planning. Delta-normal VaR figures, for example, are highly impacted by (i) changing volatilities and correlations, and (ii) changing portfolio sensitivities. Before the beginning of the financial crisis, long term model risk was not considered to be an issue for banks or financial institutions[1], as any unwanted shift or increase in VaR figures could be easily countered by hedging or risk reduction actions. Since the second half of 2007, however, significant parts of trading

[1]As one of a few exceptions, let us mention the expositions by (Jorion, 2007), Section 9.5 or (Danielsson, 2002), who point out that VaR figures are volatile and not reliable in general.

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portfolios became more and more illiquid, risks could no longer be hedged adequately, and risk figures increased in an unpredicted fast and threatening fashion. Potential consequences were limit breaches, or in worst case situations, additional capital requirements to keep the financial institution solvent. In the following, we therefore discuss long term model risk only, with a special focus on the implications for limit management and economic capital planning.

As long term model risk is a rather novel issue, there is not much literature available. (Christoffersen and Goncalves, 2005) or (Jorion, 1996), who investigate the statistical properties of VaR figures in detail, propose confidence intervals around VaR estimates to cover model risk. Their expositions are, however, mainly centred around short term model risk. To quantify long term model risk we are more interested in the extent to which VaR figures may change in future. Therefore, we need to take into account potential future evolutions of market environments. Taking the ideas by Christoffersen, Goncalves and Jorion further, this immediately leads to the concept of *potential future value-at-risk* (PFVaR), developed by (Spangler and Werner, 2010). There, a detailed explanation of the concept is given together with a specific example on the computation of the corresponding risk figures. Here, we briefly recall the main definitions of PFVaR from (Spangler and Werner, 2010), before we focus on the application of the concept in risk management.

2 THE CONCEPT OF POTENTIAL FUTURE VALUE-AT-RISK

For the proper definition of PFVaR, let us fix a static portfolio, a reference time t_R (i.e. today) and let us denote the future (random) VaR figure at time $t > t_R$ with $\mathrm{VaR}_\alpha(t)$ for a given VaR level α.[2] Similarly to the potential future exposure concept in counterparty credit risk (see (Pykhtin, 2005) for more details), a few versions of PFVaR have been introduced by Spangler and Werner:

- The *expected VaR* at time $T > t_R$ is the average of the potential future VaR at time T:
$$\mathrm{EVaR}_\alpha(T) := \mathbf{E}\left[\mathrm{VaR}_\alpha(T) \mid \mathcal{F}_{t_R}\right]. \tag{1}$$
- The *peak VaR* at time $T > t_R$ is the maximum VaR that is expected to occur at time T at a given confidence level (quantile) $\beta \in (0,1)$:
$$\mathrm{PVaR}_{\alpha,\beta}(T) := q_\beta\left[\mathrm{VaR}_\alpha(T) \mid \mathcal{F}_{t_R}\right]. \tag{2}$$
- The *maximum peak VaR* until time $T > t_R$ is the maximum VaR that is expected to occur in $[t_R, T]$ at a given confidence level $\beta \in (0,1)$:
$$\mathrm{MPVaR}_{\alpha,\beta}(T) := q_\beta\left(\max_{t\in[t_R,T]}\left[\mathrm{VaR}_\alpha(t) \mid \mathcal{F}_{t_R}\right]\right). \tag{3}$$

  All introduced quantities are conditional on the information (i.e. the corresponding filtration $\mathcal{F}_{t_R}$) given at the reference time t_R, which means that quantiles or expectations calculated at time t_R only include information available up to time t_R.

  It has to be noted that for a one-to-one relationship of PFVaR and the potential future exposure concept in counterparty credit risk, the maximum peak VaR would have to be defined by
$$\begin{aligned}\mathrm{MPVaR}_{\alpha,\beta}(T) &= \max_{t\in[t_R,T]}\left[\mathrm{PVaR}_{\alpha,\beta}(t)\right]\\ &= \max_{t\in[t_R,T]}\left[q_\beta\left(\mathrm{VaR}_\alpha(t) \mid \mathcal{F}_{t_R}\right)\right].\end{aligned}$$

  In the above definition (3), the order of maximization and percentile has been switched as it is more meaningful for practical applications: the maximum peak VaR is exceeded by $\mathrm{VaR}_\alpha(t)$ in the period $[t_R, T]$ with a probability of $1-\beta$.

The concept of potential future VaR can be easily generalized to *potential future market risk* when replacing VaR by any arbitrary market risk measure. For the calculation of the PFVaR figures an appropriate model needs to be specified to project risk figures forward in time. Analogously to risk modelling, the choice of the appropriate methodology, i.e. the choice between historical bootstrapping or Monte Carlo simulation (e.g. GARCH models or discretized SDEs), has a strong impact on the resulting figures. Depending on the purpose and on the time horizon, Spangler and Werner suggested to either use a pure and simplistic historical bootstrapping method similarly to (Christoffersen and Goncalves, 2005) or, alternatively to use a more sophisticated integrated economic scenario generator, see (Davidson, 2008), which is especially designed for projections of the joint evolution of risk factors over longer time horizons. For the specific calculation of the PFVaR figures and the concrete choice of the model, let us refer to (Spangler and Werner, 2010).

3 APPLICATIONS OF PFVAR

PFVaR as such represents an add-on to existing market risk frameworks without the need to amend a bank's current market risk systems. For instance,

[2] As the VaR time horizon h is fixed throughout the following, we skip it for notational convenience.

based on the algorithm presented by Spangler and Werner, a bank's market risk system can be re-used to calculate the corresponding PFVaR figures without much additional effort. The requirements on the computational capabilitites of the market risk (and front office) system are actually the same as for CVA (counterparty value adjustment) or counterparty credit risk calculations, which are nowadays already in place for most banks.

From a management perspective, PFVaR provides a consistent framework to measure and handle long term model risk within a bank's planning and management processes:

- Taking the *expected VaR* at certain time horizons $T_1, \ldots, T_n$ yields precise information on how fast risk will decay on average, especially compared to traditional time-to-maturity or duration concepts. The expected VaR not only covers the ageing effect of the portfolio, but can also account for expected increases in volatility or correlation. Thus, the sustainability of hedging activities over longer time horizons can easily be analyzed based on the expected VaR.

  It is furthermore well-suited for the planning process, and especially suitable for planning of new business. In general, new business volumes should be chosen in such a way that the expected future risk fits to the overall planning figures for future time horizons. In such a context, time horizons from one to three years are usually considered by banks. These longer time horizons therefore require the same models for the evolution of the risk factors as in long term counterparty credit risk modelling. However, in contrast to scenarios calibrated for counterparty credit risk purposes based on historical data, the focus on planning usually requires that the expected risk factor evolution coincides with some *best estimate planning scenario* provided by a bank's strategic planning department.

- The future VaR distribution, and especially the *peak VaR*, provides additional valuable insights for the limit management process, as for a fixed portfolio it shows by how much future market risk figures may fluctuate. To avoid limit breaches due to changes in volatilities and correlation, these fluctuations should be taken into account when setting limits based on reasonable levels of β. Whereas any new trade should be considered against limits derived from the expected future VaR, the size of capital buffers in risk capital planning should be based on the excess of the peak VaR against the expected VaR. Then, assuming a limit breach of the expected VaR limit is not due to new business or trading activities, the excess of the risk against the limit can be absorbed by the additional buffer. Only when this buffer is exhausted, a limit breach is reported.

- Further, economic capital models under going-concern assumptions do not only need to model trading losses at some future point in time, but also need to forecast the future VaR consumption.[3] Given the obvious ambiguity about the future market risk exposure, the peak VaR provides guidance on how (a maybe downscaled version of) the current portfolio would behave in a stressed environment as assumed by economic capital models. Thus, in going-concern economic capital models, market risk capital should be sufficient to cover both severe losses (i.e. decreases in market value) plus a stressed market risk level.

  Taking the stress test point of view, peak VaR also represents a good choice for VaR figures within stress scenarios used in macro-economic integrated stress tests. The peak VaR therefore can act as a supplement to usual risk figures, if β is chosen in accordance to the severity of the applied stress test.

- Finally, the *maximum peak VaR* represents the maximum VaR figure which may be observed in a stress scenario where markets become completely illiquid and risks cannot be reduced by hedging activities. Given the maximum peak VaR, both the likelihood as well as the amount of a potential limit breach can be derived, which can, for example, be used in contingency planning.

4 EXAMPLE: TWO-FACTOR DELTA-NORMAL VAR FRAMEWORK

Let us consider a hypothetical portfolio consisting of Greek and Spanish floating rate government bonds with different maturities. Since the portfolio consists of floating rate bonds only, interest rate risk plays a minor role and will therefore be neglected in the following examinations. The remaining details of the setup are the same as in Spangler and Werner: for credit spread risk measurement, we use a two-factor delta-normal VaR approach; credit spread market movements are explained by two credit spread risk factors (zero spreads) which are assumed to have a flat

[3]Under a going-concern point of view, it cannot be reasonably assumed that all market risks are completely hedged.

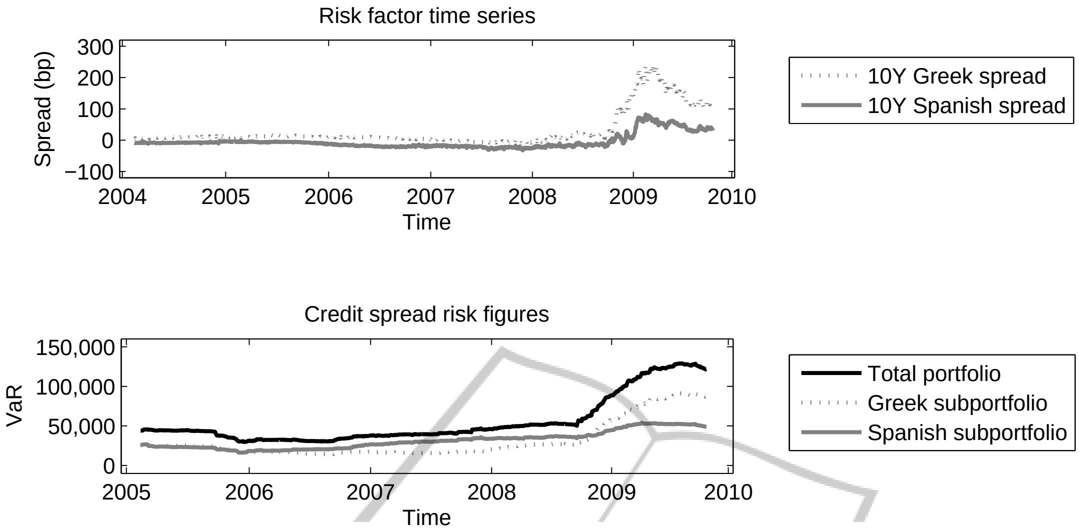

Figure 1: Risk factors and corresponding portfolio credit spread risk figures over time.

term structure that can change over time[4]. As risk factor proxies we have chosen 10-year asset swap spread time series for Greek and Spanish government bonds, cf. Figure 1, upper part. The lower part of Figure 1 shows the historical 1-day 99% credit spread VaR of the assumed portfolio.

4.1 Portfolio VaR Decomposition

In order to identify the impact of changing volatilities and correlations, changes in risk factor levels and ageing effects on VaR figures let us consider Figure 2 which shows the decomposition of logarithmic VaR changes from initial time t_I to time T according to (4), decomposed into changes caused by shifts in the covariance matrix C_t and changes in the sensitivity S_t.

$$\underbrace{\ln\left(\frac{\mathrm{VaR}_\alpha(T)}{\mathrm{VaR}_\alpha(t_I)}\right)}_{\text{Total impact}} = \underbrace{\ln\left(\frac{\sqrt{S_T^\top \cdot C_T \cdot S_T}}{\sqrt{S_T^\top \cdot C_{t_I} \cdot S_T}}\right)}_{\text{Impact of covariance}} \quad (4)$$

$$+\underbrace{\ln\left(\frac{\sqrt{S_T^\top \cdot C_{t_I} \cdot S_T}}{\sqrt{S_{t_I}^\top \cdot C_{t_I} \cdot S_{t_I}}}\right)}_{\text{Impact of sensitivity}} \quad (5)$$

Although the influence of sensitivities on VaR figures is considerable – VaR is reduced by up to $\approx 50\%$ of the original value – the impact of the covariance matrix is even stronger: it accounts for a factor of $\approx 570\%$. Although the effects tend to cancel out to a certain extent, a once feasible portfolio (i.e. within market risk limits) can easily exceed its limit over time by a large or actually economic capital threatening amount. For a further analysis, Figure 2 also details the effects of the sensitivities and the covariance matrix, which shows that for the given portfolio, ageing dominates level effects. Further, changes caused by covariance shifts are mainly due to (Greece) volatilities, not correlation.

Looking at a rolling one-year horizon instead of the complete VaR history, Figure 3 displays a rather constant influence of ageing, leading to an annual risk reduction of about 15% per annum ($\mathrm{VaR}_\alpha(t) \approx 0.85 \cdot \mathrm{VaR}_\alpha(t-1Y)$) compared to a maximum increase of 270% over one year caused by rising volatilities. From Figure 3 it can be seen that VaR increases steadily from 2007 to 2009 and then explodes from the beginning of 2009 onwards.

4.2 PFVaR Analysis

To illustrate the concept of PFVaR, Figure 4 shows a sample of 100 bootstrapped scenarios of length 250 days against the actual historical evolution of the Spanish credit spread risk factor.

As can be seen from Figure 5 (upper part), the concept of potential future works well before September 2008.

- The expected VaR shows that VaR reducing ageing effects are expected to be compensated by an increase in volatility (due to the moving estima-

[4]As rather similar results are obtained if a more general framework (i.e. varying term structures) is considered, we focus on this simplified setting for the brevity and clarity of presentation.

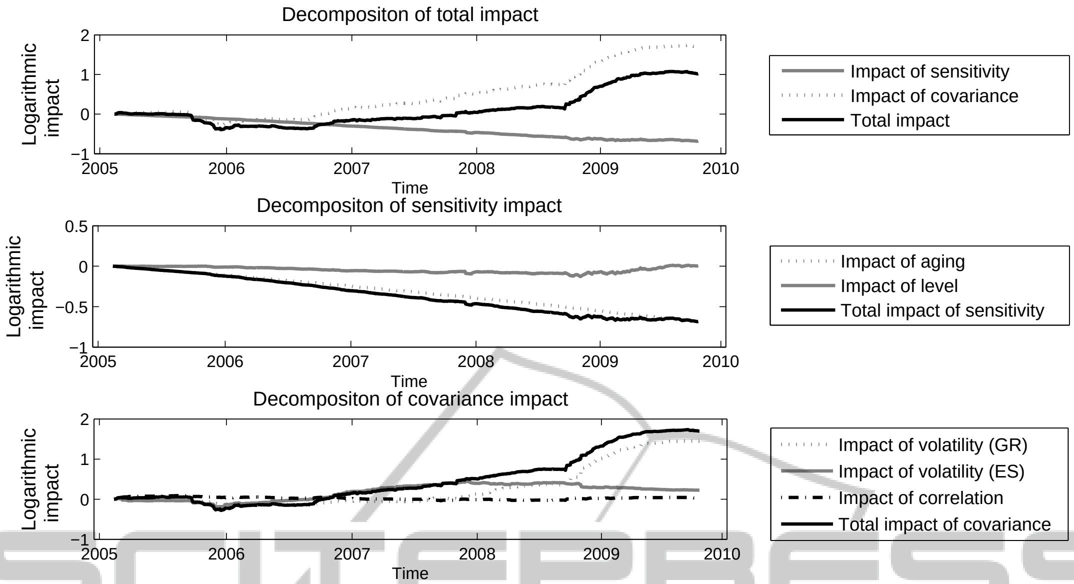

Figure 2: Impact of sensitivity and covariance parameters on portfolio VaR.

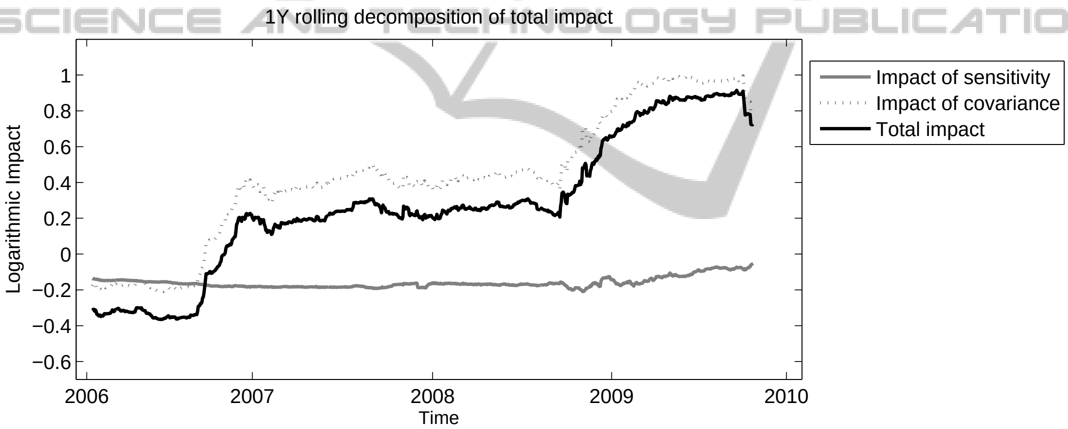

Figure 3: Impact of sensitivity and covariance parameters on portfolio VaR over a rolling one-year time horizon.

tion window), resulting in relatively stable VaR figures on average. The (maximum) peak VaR gives a reasonable upper bound on VaR, as long as there is no regime switch in the market.

- Without additional capital, no new business can be planned for, as the expected VaR remains on the same level as the initial VaR of January 2008. This is in contrast to the maturity or duration profile which would have indicated a potential for new business around 5% to 15%. The gap is mainly due to increasing volatilities as a very calm period at the beginning of 2007 is dropped from the rolling 250-day time window for the historical data used for VaR calculations.
- While the VaR limit can be fixed at the current level and no new business is possible, an additional limit or capital buffer for market risk of around 30% is indicated by the model. The peak VaR figure signals that such a buffer might become necessary due to rising volatilities.

However, as can also be clearly seen in the upper part of Figure 5, the whole approach would have only worked until autumn 2008; the regime switch due to the financial crisis in September 2008 could not have been predicted by pure historical bootstrapping. Now, taking the possibility of regime shifts into account (for instance, a 1% probability to switch to an unstable economy with tripled volatilities), the picture dramatically changes, cf. the lower part of Figure 5.

- The MPVaR ratio increases from 130% to almost 200%, which means that the capital buffer should have been set around 100% of the initial market

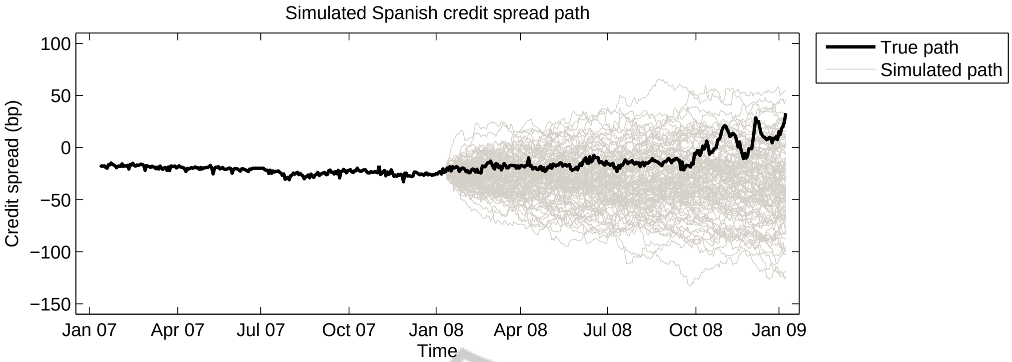

Figure 4: Spanish credit spread paths obtained by historical bootstrapping.

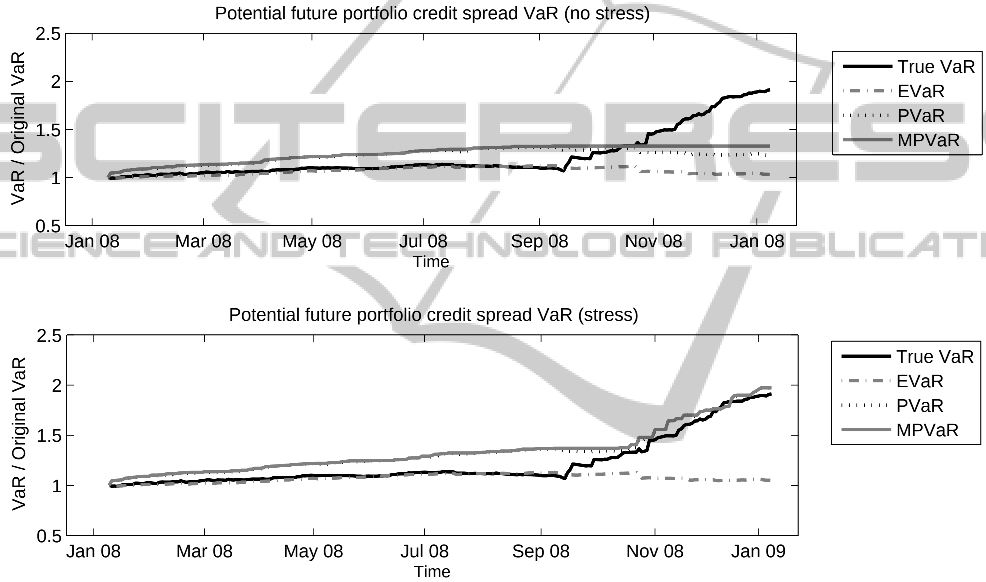

Figure 5: Potential future credit spread VaR ($\alpha = \beta = 99\%$) and actual credit spread VaR evolution under no stress and in a stressed scenario.

risk capital instead of only 30%. Interestingly, due to the low probability of moving to such a stressed regime such a capital buffer is not necessary for the near future, however, becomes much more likely for longer time horizons.

- As an alternative to such a large capital buffer for a volatility increase, alternative measures could be considered in contingency planning, for example so called *crash puts*, i.e. far out-of-the-money CDS options or redemption options on the original bonds.

This analysis demonstrates that an over-reliance on historical data might lead to an underestimation of future risk. Instead, several regimes should be taken into account, and, preferably be linked to stress test concepts. For example, one could use assumptions from stress test concepts (which are nowadays mandatory due to Basel II and Basel III) and incorporate these into potential stress regimes in the simulation of future market conditions. In addition, counteracting measures identified as potential remedies in stress situations should be included in the simulation of the potential future market risk. In such a way, current stress testing efforts can be accompanied by an additional quantitative analysis based on the concept of PFVaR.

This analysis furthers shows that the correct estimation of the future evolution has a significant impact on the resulting risk figures. Although the whole approach is applicable in a rather general context, i.e. it is not restricted to multivariate normal distribution or linear dependence of instrument prices on risk factors, any difference of the simulated future from the true future distribution results in a deviation of the estimated figures from realized figures later on.

4.3 Application in Portfolio Optimization

In principle, the concept of PFVaR can also be used in a portfolio optimization context. Technically, it is quite simple to generalize traditional risk-return optimization approaches to the newly suggested PFVaR measures. However, although the replacement of ordinary risk measures like VaR by PFVaR measures looks quite simple on first glance, it has to be noted that the interpretation changes quite significantly: In traditional approaches, the expected return from time t_R to $t_R + h$, i.e. $\mathbf{E}[V_{t_R+h} - V_{t_R}]$, is compared against a risk measure applied to the profit-and-loss distribution of the net asset values $V_{t_R+h} - V_{t_R}$ (here, h denotes the time horizon of the risk measure). If the risk measure is now replaced by a potential future risk measure, then the risk of the profit-and-loss distribution of $V_{T+h} - V_T$ for some future time T is compared against the return from time t_R to T (or $T+h$), which does not represent a meaningful setup.

Instead, it is much more meaningful to introduce additional constraints based on PFVaR measures in traditional portfolio optimization. For example, one might consider the situation that the expected return from time t_R to T should be maximized (usually $T - t_R$ represents one year), while risk capital – represented by the VaR of the profit-and-loss distribution of $V_T - V_{t_R}$ – is limited by the available capital of the bank. In such a case, it would be meaningful to introduce additional PFVaR constraints for smaller time horizons, which would guarantee that a short term VaR on a time horizon of h does not exceed a certain much smaller limit throughout the whole time period from t_R to T.

5 RESUMEE

We have shown that VaR figures cannot be expected to be constant, but may vary over time due to several reasons, such as changes in covariance parameters and sensitivities. We have further observed that all effects might be similarly important and need to be taken into account. As a remedy against model risk arising from these changes, the new concept of potential future market risk has been introduced and motivated and it has been argued how this can be incorporated into a bank's planning cycle. Although PFVaR numbers themselves depend on the simulated risk factors, i.e. a certain dependence on models and assumptions is always inherent, we have illustrated that this concept has several powerful applications, especially in new business planning and economic capital buffer planning. Still, a careful selection of the simulation assumptions is key to a successful application of the PFVaR concept. Eventually, we believe that the PFVaR concept can also be successfully applied in a portfolio optimization context, but we leave this for future research, as this needs much more detailed technical considerations.

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