HANDLING IMPRECISE LABELS IN FEATURE SELECTION WITH GRAPH LAPLACIAN

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Keywords: Feature selection, Imprecise labels, Graph laplacian.

Abstract: Feature selection is a preprocessing step of great importance for a lot of pattern recognition and machine learning applications, including classification. Even if feature selection has been extensively studied for classical problems, very little work has been done to take into account a possible imprecision or uncertainty in the assignment of the class labels. However, such a situation can be encountered frequently in practice, especially when the labels are given by a human expert having some doubts on the exact class value. In this paper, the problem where each possible class for a given sample is associated with a probability is considered. A feature selection criterion based on the theory of graph Laplacian is proposed and its interest is experimentally demonstrated when compared with basic approaches to handle such imprecise labels.

1 INTRODUCTION

The feature selection step is known to be of fundamental importance for classification problems. Its objective is to get rid of irrelevant and/or redundant features and to identify those being really useful for the problem. The benefits of feature selection for classification can be numerous and include: better prediction performances of the classification models, easier interpretation of these models, deeper understanding of the problems and/or reduced feature acquisition and storage cost (Guyon and Elisseeff, 2003).

Due to its importance, feature selection has been extensively studied in the literature, especially for standard classification problems, i.e. for problems where each sample point is associated without ambiguity to exactly one class label (see e.g. (Dash and Liu, 1997; Kwak and Choi, 2002)). It has revealed extremely useful in many fields among which one can cite for example text classification (Yang and Pedersen, 1997) or gene expression based classification (Ding and Peng, 2003).

Quite surprisingly, to the best of our knowledge, very few attempts have been made up to now to achieve feature selection in problems where the class labels are uncertainly or imprecisely specified and can even be erroneous. However, such a situation can be frequently encountered in practice and thus deserves to be investigated. Indeed, it is frequent that a human supervision is required to assign labels to sample points. This is for example the case in the medical domain where a diagnostic has to be made based on a micro-array or a radiography. Such a task can be hard to perform and experts sometimes hesitate between different classes or propose a single class label but associate it with a measure of the confidence they have in this label.

In this work, we address the problem of feature selection in the case where an expert gives, for each sample x_i ($i = 1 \ldots n$), a probability value p_{ij} to each possible class c_j ($j = 1 \ldots l$) such that $\sum_{j=1}^{l} p_{ij} = 1 \, \forall i$. Such a supervision has been called possibilistic labels in the literature; it can also be obtained by combining the opinion of several experts about the membership of a point to a given class. As a concrete problem, (Denoeux and Zouhal, 2001) evokes the recognition of certain transient phenomena in EEG data, whose shapes can be very hard to distinguish from EEG background activity, even for trained physicians; experts can rarely be sure about the presence of such phenomena, but can be able, however, to express a possibility about this presence. Such labels are also encountered in fuzzy classification problems. Indeed when classes are not well defined, they can sometimes be better represented by fuzzy labels, measuring the degree of membership of the samples to each of the classes. This is for example the case when the labels are obtained through the fuzzy c-means clustering al-

Doquire G. and Verleysen M. (2012).
HANDLING IMPRECISE LABELS IN FEATURE SELECTION WITH GRAPH LAPLACIAN.
In *Proceedings of the 1st International Conference on Pattern Recognition Applications and Methods*, pages 162-169
DOI: 10.5220/0003712101620169

gorithm (Bezdez and Pal, 1992).

The proposed feature selection algorithm is a ranking technique based on the Laplacian Score (He et al., 2006), originally introduced for feature selection in unsupervised problems. The basic idea is to select features according to their locality preserving power, i.e. according to how well they respect a proximity measure defined between samples in an arbitrary way. In this paper, it is shown how this criterion can be extended to the uncertain label framework.

The rest of the paper is organized as follows. In Section 2, related work on feature selection and uncertain label analysis is presented. The original Laplacian Score is described in Section 3 and the proposed new criterion, called weighted Laplacian Score (WLS), is introduced in Section 4. Section 5 presents experimental results assessing the interest of WLS before Section 6 concludes the work.

2 RELATED WORK

As explained above, feature selection has been widely investigated in the literature; traditional approaches include wrapper or filter strategies. Wrappers make use of the prediction (classification or regression) model to select features in order to maximize the performances of this model. Such methods thus require building a huge number of prediction models (with potential hyperparameters to tune) and are typically slow. However, the performances of the prediction models with the selected features are expected to be high (Kohavi and John, 1997).

On the other hand, filters are independent of any prediction model; they are rather based on a relevance criterion. The most popular of those criteria include the correlation coefficient (Hall, 1999), the mutual information (Peng et al., 2005) and other information-theoretic quantities (Meyer et al., 2008) among many more. Filters are in practice faster and much more general than wrappers since they can be used prior to any prediction model; they are considered in this work. More recently, embedded methods, performing simultaneously feature selection and prediction have also raised a huge interest (Yuan and Lin, 2006) since the publication of the original LASSO paper (Tibshirani, 1996).

Classifiers for problems with uncertain label have also been proposed in the literature; more specifically the Dempster-Shafer theory of belief functions has proven to be very successfull in this context (Denoeux and Zouhal, 2001; Jenhani et al., 2008; Côme et al., 2009). Indeed, it offers a convenient and very general way to model one's belief about the class label of a sample. This belief can be expressed as in this paper but can also take more general forms as a set of possible class labels or the probability of a class with no additional information. Moreover, the Transferable Belief Model (Smets et al., 1991) permits to combine elegantly the different pieces of belief concerning a sample class membership.

Despite its importance, the problem of feature selection with imprecise class labels has received few attention in the literature. In (Semani et al., 2004), the authors consider a fully supervised classification problem before relabelling automatically all the data points in order to take into account the classification ambiguity. In (Wang et al., 2009), the Hilbert-Schmidt independence criterion is used to achieve feature selection with uncertain labels. However, the possibility of label noise is not studied in this paper which is rather focused on multi-label like problems.

It is worth noting that more specific weakly supervised problems have already been considered in the literature. This is the case for semi-supervised problems in which a small fraction of the samples are associated with an exact class label while no label at all is associated with the other samples (Chapelle et al., 2006). An extension of the Laplacian Score has been proposed for feature selection in this context (Zhao et al., 2008) while many other methods also exist in the literature, e.g. (Zhao and Liu, 2007).

A closely related but different problem is the one where only pairwise constraints between samples are given. This means that the exact class labels are unavailable but that, for some couples of points, it is known whether or not they belong to the same class. Here again, feature selection for this paradigm has been achieved successfully with an extension of the Laplacian Score (Zhang et al., 2008).

3 THE LAPLACIAN SCORE

This section briefly presents the Laplacian Score, introduced in (He et al., 2006) for unsupervised feature selection. As already discussed, the method aims at selecting the features preserving at most the local structure of the data, or, in other words, the neighborood relationships between samples.

Let X be a given data set containing m samples $\mathbf{x_i}$ ($i = 1 \dots m$) and f features $\mathbf{f_r}$ ($r = 1 \dots f$). Denote by f_{ri} the r^{th} feature of the i^{th} sample of X. It is possible to build a proximity graph representing the local structure of X. This graph consists in m nodes, each representing a sample of X. An edge is present between node i and node j if the corresponding points $\mathbf{x_i}$ and $\mathbf{x_j}$ in X are close, i.e. if $\mathbf{x_i}$ is among the k nearest neighbors of $\mathbf{x_j}$ or conversely, where k is a parameter of the method. Traditionally, the proximity measure used to determine the nearest neighbors of a point is the Euclidean distance, denoted as $d(.,.)$.

Based on this proximity graph, a matrix S can be built in the following way:

$$S_{i,j} = \begin{cases} e^{-\frac{d(\mathbf{x_i},\mathbf{x_j})}{t}} & \text{if } \mathbf{x_i} \text{ and } \mathbf{x_j} \text{ are close} \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

with t being a suitable positive constant. We also define $D = diag(S\mathbf{1})$, with $\mathbf{1} = [1\dots1]^T$, as well as the graph Laplacian $L = D - S$ (Chung, 1997).

The mean of each feature $\mathbf{f_r}$ (weighted by the local density of the data points) is then removed; the new features are defined by

$$\tilde{\mathbf{f_r}} = \mathbf{f_r} - \frac{\mathbf{f_r^T}D\mathbf{1}}{\mathbf{1^T}D\mathbf{1}}\mathbf{1}.$$

The objective of this normalisation is to prevent a non-zero constant vector such as $\mathbf{1}$ to be assigned a zero Laplacian score (and thus to be considered as highly relevant) as such a feature obviously does not contain any information. It is then possible to compute the Laplacian score of each feature $\mathbf{f_r}$ as

$$L_r = \frac{\tilde{\mathbf{f_r}}^T L \tilde{\mathbf{f_r}}}{\tilde{\mathbf{f_r}}^T D \tilde{\mathbf{f_r}}} \qquad (2)$$

and features are ranked according to this score, in increasing order.

More details can be found in the original paper (He et al., 2006), where the authors also derive a connection between (2) and the canonical Fisher score.

4 THE WEIGHTED LAPLACIAN SCORE FOR POSSIBILISTIC LABELS

In this section, the proposed feature selection criterion for possibilistic labels is introduced. As already stated, all the developments presented here could as well be applied to the case where an expert only provides one class label for each point of the training set, but associates this label with a coefficient indicating the certainty he has on his prediction. The section ends with a theoretical justification of the WLS.

4.1 The Proposed Algorithm

Consider again the data set $X \in \Re^{m\times f}$. Each sample point $\mathbf{x_i}$ has *a single true* class label c_j belonging to the set of the l possible class labels $c_1 \dots c_l$. In practice, however, this label is not precisely known. Instead, each point $\mathbf{x_i}$ is rather associated with a probability value p_{ij} for each possible class label such that $\sum_{j=1}^{l} p_{ij} = 1\,\forall i$. These values can be directly obtained from an expert or come from the combination of several experts opinions. Obviously, the probability that two points in X have the same class label can be computed as follows:

$$S_{ij}^{sim} = \sum_{k=1}^{l} p_{i,k}p_{j,k}. \qquad (3)$$

Thus, $S_{i,j}^{sim}$ is simply the scalar product between the vectors of the labels probability for $\mathbf{x_i}$ and $\mathbf{x_j}$.

The algorithm starts by building a graph G^{sim} with m nodes, each corresponding to a point in X. An edge exists between node i and node j if the probability that the two samples $\mathbf{x_i}$ and $\mathbf{x_j}$ have the same class label is greater than 0.

From the matrix S^{sim}, it is possible to define $D^{sim} = diag(S^{sim}\mathbf{1})$ and the graph Laplacian of G^{sim}, $L^{sim} = D^{sim} - S^{sim}$.

Let us then construct a matrix $S^{dis} \in \Re^{m\times m}$, corresponding to the probabilities that two samples belong to different classes:

$$S_{ij}^{dis} = 1 - \sum_{k=1}^{c} p_{i,k}p_{j,k} = 1 - S_{ij}^{sim} \qquad (4)$$

and let us define $D^{dis} = diag(S^{dis}\mathbf{1})$ as well as $L^{dis} = D^{dis} - S^{dis}$.

The importance of each feature $\mathbf{f_r}$ is eventually computed by the weighted Laplacian score that we define to be

$$WLS_r = \frac{\mathbf{f_r^T}L^{sim}\mathbf{f_r}}{\mathbf{f_r^T}L^{dis}\mathbf{f_r}}; \qquad (5)$$

the features can be ranked according to this score, in increasing order. The number of features to keep to build the model has to be determined in advance.

4.2 Justification

In the above developements, the graph G^{sim} defines a structure between the points by connecting those possibly sharing the same clas label. Based on this structure, the matrix S^{sim} (3) weights the importance of the similarity between the samples, i.e. $S_{i,j}^{sim}$ will be high if the probability that $\mathbf{x_i}$ and $\mathbf{x_j}$ belong to the same class is high and will be low otherwise.

Following these considerations, a good feature $\mathbf{f_r}$ can be seen as a feature respecting the structure defined by G^{sim}. Indeed, intuitively, if $\mathbf{x_i}$ and $\mathbf{x_j}$ have the same label with a great probability (or equivalently if $S_{i,j}^{sim}$ is large), then it is expected that f_{ri} and f_{rj} are close (or at least closer than points having a very low probability of belonging to the same class). Distant features for close points in the sense of S^{sim} have thus to be penalized and not considered as relevant.

In the same way, two samples belonging to different classes with a high probability are intuitively expected to be far from each other. Therefore, it makes sense to penalize close features for sample points which are distant according to S^{dis}.

A sound criterion to assess the quality of the features is consequently to prefer those minimizing the following objective:

$$\frac{\sum_i \sum_j (f_{ri} - f_{rj})^2 S_{ij}^{sim}}{\sum_i \sum_j (f_{ri} - f_{rj})^2 S_{ij}^{dis}}. \tag{6}$$

This way, if $S_{i,j}^{sim}$ grows, $(f_{ri} - f_{rj})^2$ has to be small for the feature r to be good and the structure defined by G^{sim} is respected. Similarly, $(f_{ri} - f_{rj})^2$ and S_{ij}^{dis} have also to be high simultaneously.

In the following, we show the equivalence between expressions (5) and (6) of the WLS feature selection criterion.

As already explained, the diagonal matrix $D^{sim} = diag(S^{sim}\mathbf{1})$ thus $D_{ii}^{sim} = \sum_j S_{ij}^{sim}$. Using $L^{sim} = D^{sim} - S^{sim}$, the graph Laplacian of G^{sim}, some simple calculations give:

$$\begin{aligned}\sum_i \sum_j (f_{ri} - f_{rj})^2 S_{ij}^{sim} &= \sum_i \sum_j (f_{ri}^2 + f_{rj}^2 - 2 f_{ri} f_{rj}) S_{ij}^{sim} \\ &= \sum_i \sum_j f_{ri}^2 S_{ij}^{sim} + \sum_i \sum_j f_{rj}^2 S_{ij}^{sim} \\ &\quad - 2 \sum_i \sum_j f_{ri} f_{rj} S_{ij}^{sim} \\ &= 2\mathbf{f_r^T} D^{sim} \mathbf{f_r} - 2\mathbf{f_r^T} S^{sim} \mathbf{f_r} \\ &= 2\mathbf{f_r^T} (D^{sim} - S^{sim}) \mathbf{f_r} \\ &= 2\mathbf{f_r^T} L^{sim} \mathbf{f_r}.\end{aligned} \tag{7}$$

In the same way, it is easy to prove that

$$\sum_i \sum_j (f_{ri} - f_{rj})^2 S_{ij}^{dis} = 2\mathbf{f_r^T} L^{dis} \mathbf{f_r}.$$

It then appears clearly that minimizing (6) is equivalent to minimizing (5):

$$\min_{\mathbf{f_r} \in X} \frac{\mathbf{f_r^T} L^{sim} \mathbf{f_r}}{\mathbf{f_r^T} L^{dis} \mathbf{f_r}} = \min_{\mathbf{f_r} \in X} \frac{\sum_i \sum_j (f_{ri} - f_{rj})^2 S_{ij}^{sim}}{\sum_i \sum_j (f_{ri} - f_{rj})^2 S_{ij}^{dis}}. \tag{8}$$

Even if the scores produced by criteria (5) and (6) are equal, it is interesting to consider both approaches. Indeed, even if formulation (6) is more intuitive, the use of spectral graph theory allows us to establish connections between WLS and other feature selection criteria such as the Fisher score (He et al., 2006).

5 EXPERIMENTAL RESULTS

This section is devoted to the presentation of experimental results showing the interest of the proposed feature selection approach, taking the uncertainty of the class labels into account. The tests have been performed on both artificial and real-world data sets for binary and multi-class problems.

5.1 Methodology

To simulate the uncertainty of an expert and the possible errors he or she makes when labelling data points, the following methodology has been adopted. If the number of samples in the data set is m, m values β_i are drawn from a Beta distribution of mean μ and variance $\sigma = 0.1$. With probability β_i, the true label l_j of sample $\mathbf{x_i}$ is switched to one of the other possible labels $l_{s \neq j}$, with the same probability for each label. The probability value associated with the true class label is set to $p_{ij} = 1 - \beta_i$ and the probability of the possibly switched label is set to $p_{is} = \beta_i$.

The performances of the WLS are compared to those obtained with two other similar strategies, both neglecting the uncertainty of the class labels. The first one consists in considering as true the labels $y_i^{error} (i = 1 \ldots m)$ obtained after the switch procedure described above. The second one uses the labels $y_i^{max} (i = 1 \ldots m)$ associated with the highest probability for each sample point.

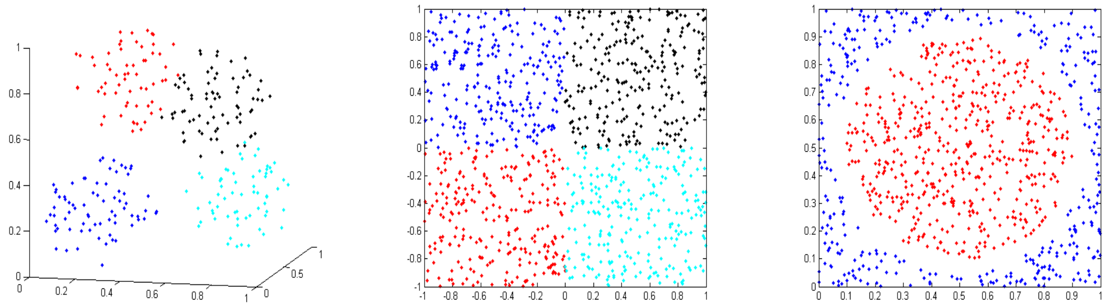
Based on these labels, two matrices $S^{sim,error}$ (resp. $S^{sim,max}$) and $S^{dis,error}$ (resp. $S^{dis,max}$) are built by setting

$$S_{i,j}^{sim,error(max)} = \begin{cases} 1 & \text{if } y_i^{error(max)} = y_j^{error(max)} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

and $S_{ij}^{dis,error(max)} = 1 - S_{ij}^{sim,error(max)}$. We again define $D^{sim,error(max)} = diag(S^{sim,error(max)}\mathbf{1})$ as well as $L^{sim,error(max)} = D^{sim,error(max)} - S^{sim,error(max)}$ (and similarly $L^{dis,error(max)}$) and the score of each feature is then computed as:

$$WLS_r^{error(max)} = \frac{\mathbf{f_r^T} L^{sim,error(max)} \mathbf{f_r}}{\mathbf{f_r^T} L^{dis,error(max)} \mathbf{f_r}}. \tag{10}$$

Equation (10) is thus the counterpart of Equation (5) for fixed labels. It is similar to the criterion proposed for semi-supervised feature selection (Zhao et al., 2008) and for pairwise constraints (Zhang et al., 2008).



(a) (b) (c)

Figure 1: Illustration of the three first artificial problems; (a) Spheres, (b) Squares, (c) Circle.

5.2 Artificial Problems

Table 1: Percentage of relevant features obtained by the three feature selection techniques on the *spheres* data set.

μ	WLS	y^{max}	y^{error}
0.3	100	99.33	96.67
0.35	98	93.33	92
0.4	97.33	90	88.67
0.45	91.33	80	80

Table 2: Percentage of relevant features obtained by the three feature selection techniques on the *squares* data set.

μ	WLS	y^{max}	y^{error}
0.35	100	98	96
0.4	99	94	96
0.45	99	93	90
0.5	96	81	78

The three strategies described above are first compared on five artificial data sets, for which the relevant features are known.

The first one is called the *spheres* problem. It consists in dividing the data points into four classes corresponding to four spheres of radius 0.25 and centered respectively in $(0.25, 0.25, 0.25), (0.25, 0.75, 0.75), (0.75, 0.75, 0.25)$ and $(0.75, 0.25, 0.75)$. A data set composed of 6 features uniformly distributed between 0 and 1 is built; only the three first ones, used as coordinates in a three-dimensional space, are useful to define the class labels. The sample size is 50.

The second problem, called *squares*, consists in four classes defined according to four contiguous squares whose size length is 1. Again, 6 features uniformly distributed between 0 and 1 are built while only the first two serve as coordinates in a two-dimensional space and are relevant to discriminate between the classes. The sample size is 100.

The third artificial problem, denoted *circle*, is a binary classification one. A circle of radius 0.4 is set at the center of a square of size length 1 and two classes are defined according to whether or not a point lies inside the circle. A ring of width 0.05 separates the two classes. Only the first two features are thus relevant in this case; the sample size is 500. Figure 1 represents the three first articial problems.

The fourth and fifth problems both have 10 features $f_1 \dots f_{10}$ and a sample size of 300. The class labels are generated by discretizing the following outputs:

$$Y_4 = \cos(2f_1)\cos(f_3)\exp(2f_3)\exp(2f_4) \qquad (11)$$

and

$$Y_5 = 10\sin(f_1 f_2) + 20(f_3 - 0.5)^2 + 10f_4 + 5f_5, \quad (12)$$

this last problem being derived from (Friedman, 1991). More precisely, the sample points are first ranked in increasing value of Y_4 or Y_5. They are then respectively divided into three and two classes containing the same number of consecutive points.

To compare the different feature selection strategies, the criterion is the percentage of relevant features among the n_r best ranked features, with n_r being the total number of relevant features for a given problem. All the artificial data sets have been randomly generated 50 times to repeat the experiments. [!ht]

Tables 1 to 5 present the results obtained with var-

Table 3: Percentage of relevant features obtained by the three feature selection techniques on the *circles* data set.

μ	WLS	y^{max}	y^{error}
0.25	100	100	92
0.30	97	97	87
0.35	89	85	74
0.40	80	72	64

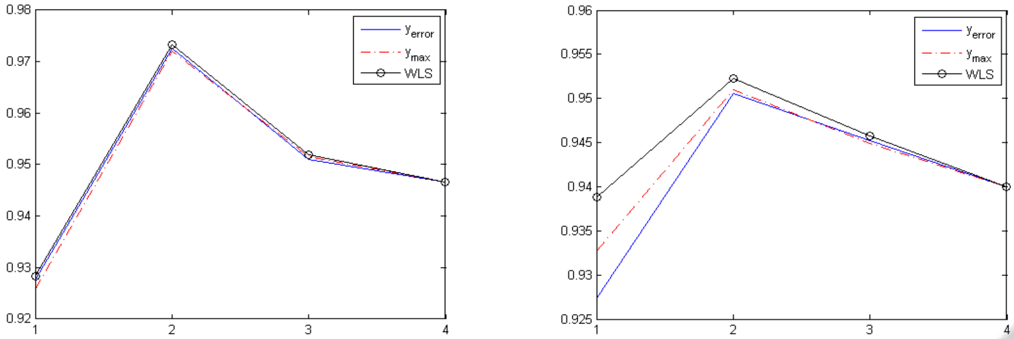


Figure 2: Accuracy of a 1NN classifier as a function of the number of selected features for the *Iris* data set with $\mu = 0.2$ (left) and $\mu = 0.3$ (right).

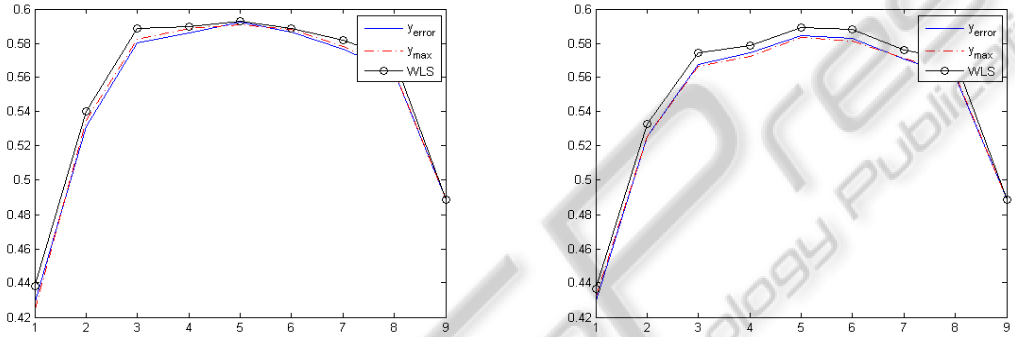


Figure 3: Accuracy of a 1NN classifier as a function of the number of selected features for the *Breast Tissue* data set with $\mu = 0.2$ (left) and $\mu = 0.3$ (right).

Table 4: Percentage of relevant features obtained by the three feature selection techniques on the Y_4 data set.

μ	WLS	y^{max}	y^{error}
0.25	95.5	94.5	88.5
0.30	95	89	87
0.35	89.5	82.5	82
0.40	84.5	75	74

Table 5: Percentage of relevant features obtained by the three feature selection techniques on the Y_5 data set.

μ	WLS	y^{max}	y^{error}
0.25	96.8	93.6	91.6
0.30	94	90	84.8
0.35	84.8	79.2	74.4
0.40	76.4	72.4	59.6

ious values of μ, depending on the complexity of the problem. The results obtained on the five considered artificial problems lead to very similar conclusions. Indeed, for each problem and each value of μ, the proposed WLS always outperforms its two competitors by more accurately detecting the relevant features. Moreover, the WLS performances are very satisfactory since, for example, it selects on average more than 90% of relevant features when $\mu = 0.3$. This indicates the adequateness of the proposed feature selection strategy for problems with uncertain labels.

As could be expected, the differences in performance between the methods are slightly smaller when μ remains low. However, when μ is raised to more than 25% or 30%, the advantage of considering the uncertainty for feature selection appears clearly. As an example, one can notice that for the *spheres* data set, WLS selects 7.33% and 11.33% more relevant features than the y^{max} based strategy for respectively $\mu = 35\%$ and $\mu = 45\%$. This difference in performances increases to 15% on the *squares* data set when $\mu = 50\%$. Generally, the method based on the observed class labels y^{error} performs the worse.

5.3 Real-world Data Sets

To further assess the interest of the proposed feature scoring criterion, experiments are also carried out on three real-world data sets. The first one is the well known *Iris* data set, whose objective is to assign each sample to one of three iris types, based on four fea-

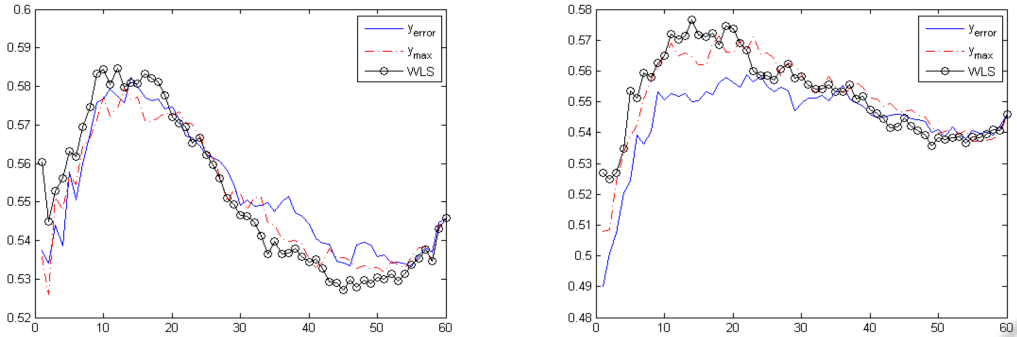


Figure 4: Accuracy of a 1NN classifier as a function of the number of selected features for the *Mines* data set with $\mu = 0.2$ (left) and $\mu = 0.3$ (right).

tures. The sample size is 150. The second data set is called *Breast Tissue*; it contains 106 samples and ten features. The goal is to classify breast tissues into four possible classes. The third data set is called *Mines vs. Rocks*. The objective is to decide whether a sonar signal was bounced off by a metal cylinder (a mine) or by a rock based on 60 features corresponding to the energy within a particular frequency band. The sample size is 208. These three data sets can be obtained from the UCI Machine Learning Repository website (Asuncion and Newman, 2007).

Since the most relevant features are not known in advance for these three data sets, the comparison criterion will be the accuracy of a classifier using the features selected by the three methods. More precisely, a 1-nearest neighbors (1NN) classifier will be used, as it is kwown to suffer dramatically from the presence of irrelevant features. The exact class labels will be used for the classification step while, as has been done before, the feature selection will be achieved with the possibly permuted labels and the expert information. This way the ability of the methods to select relevant features for the true original problem can be compared.

Figures 2, 3 and 4 present the accuracy of the 1NN classifier as a function of the number of selected features for $\mu = 20\%$ and $\mu = 30\%$. For each data set and each μ, the label permutation phase is randomly repeated 50 times and the results are obtained as an average over a five-fold cross validation procedure.

The results confirm the interest of the proposed WLS. Indeed, for the *Iris* and the Breast Tissue data sets, the WLS leads to better or equal classification performances than its two competitors for any number of features and both contamination rates. The differences in performance are of course larger when $\mu = 30\%$. The *Mines* data set also confirms that the WLS is able to detect relevant features more quickly than the methods which do not take label uncertainty into account. As can be seen in Figure 4, WLS outperfoms the other two approaches for the first 12 and the first 16 features when μ equals 20% and 30% respectively. In both cases, it leads to a global best classification accuracy.

6 CONCLUSIONS

This paper proposes a way to achieve feature selection for classification problems with imprecise labels. More precisely, problems for which each class label is associated with a probability value for each sample are considered. Such problems can result from the hesitation of an expert anotating the samples or from the combination of several experts' opinion; they are likely to be encountered when a human supervision is required to assign a class label to the points of a data set and is thus important to consider in practice. Indeed, such situations are frequently encountered for medical or text categorisation problems (among others) where errors are also possible.

The suggested methodology is based on the theory of graph Laplacian, which received a great amount of interest for feature selection the last few years. The idea is to rank the features according to their ability to preserve a neighborhood relationship defined between samples. In this paper, this relationship is defined by computing the probabilities that two points share the same class label. Obviously, the exact same methodology could as well be applied for problems where only one possible label is given with a measure of the confidence about the accuracy of this label.

Experiments on both artificial and real-world data sets have clearly demonstrated the interest of the proposed approach when compared with methods also based on Graph laplacian that do not take the label uncertainty into account.

ACKNOWLEDGEMENTS

Gauthier Doquire is funded by a Belgian F.R.I.A. grant.

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