ENGINE CALIBRATION PROCESS OPTIMIZATION

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Abstract: Before an engine can be scheduled in the Product Development cycle for inclusion in a vehicle, it must be calibrated in such a way that it satisfies a variety of regulatory tests over a range of conditions. The current engine calibration process involves conducting a design of experiments at a representative number of steady state points in order to satisfy all required regulatory tests: test engineers use a standard 16×16 grid with standard grid spacing and then conduct a design of experiments on a subset of those points - about 120 of them. This work explores how to reduce the engine calibration process time by finding the best 16×16 grid choice (i.e. the best spacing on both the engine speed and torque axes) and the minimum number of points on the grid to test in order to satisfy regulatory constraints around NO_X, particulate matter, noise, and fuel consumption. Our proposed method models the problem as a Binary Integer Program that simultaneously selects the best grid spacing and optimized number of points to test, while guaranteeing that all specified constraints hold. We present an example that demonstrates how we can reduce the number of necessary test points by approximately 56%.

1 INTRODUCTION

As vehicle emission and fuel economy standards continue to tighten, manufacturers respond by developing increasingly more complex engine systems with advanced control strategies. The process of calibrating such an engine (i.e. assigning the desired values to control parameters) quickly becomes a daunting task for calibration engineers. In the case of a modern internal combustion engine that may have six or more inputs (e.g. injection timings, injection quantities, intake manifold pressure, and exhaust gas recirculation rate), generating data for the calibration task is a time consuming and costly endeavor. If we consider the *simple* case where the response of the engine could be reasonably modeled with a quadratic function (i.e. each control factor can be understood by using three settings), and the engine speed and load regime (i.e. the range of engine rotational speed and available output torque) are each segmented by 16 grid quadrants, then the calibration engineer would be need to run $16^2 \times 6^3 = 256 \times 4,096 = 55,296$ test points: this is derived from the (number of quadrants)$^{\text{engine speed} \times \text{torque}}$ $\times$ (number of inputs)$^{\text{number of settings}}$. At roughly 5 minutes per test point, data collection alone would take over *six months!* Confound this with the fact that calibrations must be developed for different operating conditions and engine operation modes, and the product development timeline quickly becomes uncompetitive.

There has been significant work using design of experiment (DoE) and mathematical optimization techniques to minimize the amount of input data needed for every given speed and load combination (e.g. (Yoshida et al., 2011), (Maloney, 2009), (Castagné et al., 2008), and (Langouët et al., 2008)): the goal is to reduce the number of input combinations to some fraction of the possible combination of inputs and settings (e.g. $6^3 = 4,096$ combinations when there are six inputs and three settings). However, this work does not address on which of the $16^2 = 256$ speed and load combinations (i.e. test points) a calibration engineer should focus their efforts, as it is not feasible to consider every combination. This selection of test points needs to be determined in such a way to satisfy testing of typical transient drive cycles needed to pass certification (i.e. the Environmental Protection Agency (EPA) Federal Test Procedure (FTP) 75 test cycle (EPA, 1977)).

Steady state (SS) engine development consists of maintaining constant speed and load for prolonged periods of time (e.g. five minutes or more). This is not, however, typical of how most vehicle owners operate their vehicles. Vehicles are usually driven in

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a transient manner with engine speed and load constantly changing with pedal position. The transient test in Figure 1 is a discretized version of an FTP test, but note that the time spent at any of these points is less than one second.

Figure 1 illustrates a conventional 16×16 grid (calibration table points), a discretized representation of a transient drive cycle (FTP74 Test Points), typical engine calibration test points, and an engine's full load curve (FLC). The transient drive cycle, usually mandated by a regulatory agency such as the EPA, is intended to represent a "typical" vehicle's driving pattern. The engine calibration test points are usually a subset of the 256 possible speed and load combinations discussed above. The FLC essentially represents the output capability of the engine. The typical engine test points denoted by circles in Figure 1 include all points directly above and below the FLC, all points on the zero axis, and every other grid point. The current process involves performing some level of calibration work: in-vehicle idle validation for zero load points for idle and off-idle performance, set point mapping at the FLC to ensure hardware limits are maintained, and a DoE at selected points to ensure emissions compliance and minimize fuel consumption. The work included in this paper is an attempt to minimize the number of speed/load points to test and to find the best grid location for these points instead of always considering a fixed subset of points as demonstrated by the calibration test points in Figure 1. The goal is to expedite the product development life-cycle and significantly reduce testing costs.

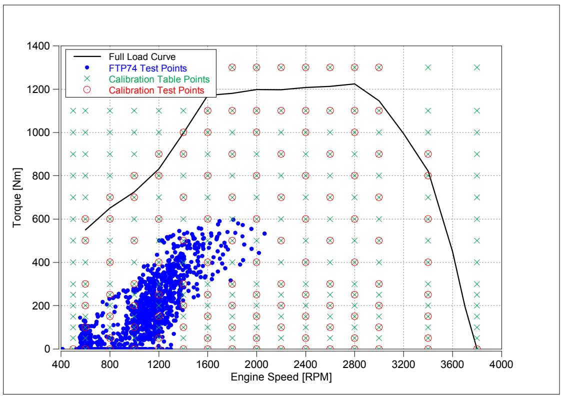

Figure 1: Typical engine operating regime.

2 PROBLEM FORMULATION

We model this problem as an Binary Integer Program (BIP), where all functions are linear, and all decision variables are binary. This problem is related to set-covering problems (Balas and Padberg, 1972), (Wolsey, 1998), where we find the minimum-cost cover. However, in the traditional set covering problems, the points from which to determine the cover are pre-defined. In our problem, we have to simultaneously determine both the points from which to select the cover (i.e. which SS points will be in the grid) and the best cover (i.e. which SS points we will test). We consider that a transient point or SS point not selected in the grid is covered by a selected SS point if it is within a certain distance to any selected SS point.

2.1 Inputs

The inputs below describe the necessary information to run the model: several are illustrated in Figure 2.

E^s = engine speed starting point = $\min_k\{FLE_k\}$.
E^e = engine speed ending point = $\max_k\{FLE_k\}$.
T^s = torque starting point.
T^e = torque ending point = $\max_k\{FLTU_k\}$.
p = length/width dimensions of final grid.
GX^{min} = minimum allowed for the grid to be spaced in the engine speed direction.
GY^{min} = minimum allowed for the grid to be spaced in the torque direction.
q = number of possible grid points on x-axis
r = number of possible grid points on y-axis
n = $q \times r$ number of possible SS points.
m = number of transient points.
K $= \{1,\ldots,q\}$: set of grid point indices on x-axis.
R $= \{1,\ldots,r\}$: set of grid point indices on y-axis.
S $= \{1,\ldots,n\}$: set of possible SS points.
I $= \{1,\ldots,m\}$: set of transient points.
dt_{ij} = distance between transient point $i \in I$ and possible SS point $j \in S$.
$ds_{j_1 j_2}$ = distance between two SS points $j_1, j_2 \in S$.
DT^{max} = max distance allowed between a transient point and its closest selected SS point.
DS^{max} = max distance allowed between a SS point and its closest selected SS point.
DX^{max} = max grid spacing in x direction.
DY^{max} = max grid spacing in y direction.
E_j = engine speed $j \in S$.
T_j = torque $j \in S$.
FE^{max} = max engine speed value of FLC.
FT^{max} = max torque value of FLC.
FLE_k = FLC engine speed value $\forall\ k \in K$.
FLT_k = FLC torque value $\forall\ k \in K$.
$FLTU_k$ = FLC torque upper value $\forall\ k \in K$.
$FLTD_k$ = FLC torque lower value $\forall\ k \in K$.

We define the $m \times n$ incidence matrix A and $n \times n$ incidence matrix B as follows

$$a_{ij} = \begin{cases} 1 & \text{if } dt_{ij} \leq DT^{max} \\ 0 & \text{otherwise} \end{cases} \quad \forall\ i \in I \text{ and } j \in S$$

$$b_{j_1 j_2} = \begin{cases} 1 & \text{if } ds_{j_1 j_2} \leq DS^{max} \\ 0 & \text{otherwise} \end{cases} \quad \forall\ i, j \in S$$

Figure 2 is illustrative of many of the input parameters. It shows the FLC on the grid of all possible SS points, which are denoted by circles. The transient points are superimposed on this figure and are denoted by stars. The torque value, $T^e = 1300$ TRQ, corresponds to the SS point that is directly above the highest point on the FLC. In this figure, $FT^{max} = 1255$ TRQ is just slightly above the possible SS point, x_{1897}, which has a torque value of 1250; hence, T^e is given the next highest torque value, 1300 TRQ. There are a total of $n = 1755$ possible SS points. In our implementation, the indexing scheme for each possible SS point starts from the bottom left corner of the grid and continues up from left to right (see Figure 3). Each possible SS point has an associated engine speed and torque, but is not indexed over engine speed and torque. For example, in Figure 2 x_{161} is a SS point with an engine speed, E_{161}, and a torque, T_{161}, where $(E_{161}, T_{161}) = (2100, 100)$. Also shown in the Figure are $FE^{max} = 3800$ RPM, $E^e = 3800$ RPM, $(FLE_{10}, FLT_{10}) = (1050, 818.25)$, $FLTD_{10} = 800$ TRQ, and $FLTU_{10} = 850$ TRQ.

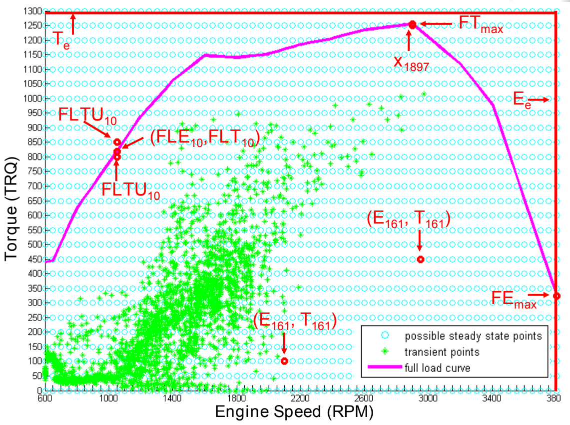

Figure 2: This figure shows all possible grid points in the engine speed and torque space labeled with example input parameters.

2.2 Variables

In this section, we introduce the decision variables for the model. The first binary variable will determine if a possible SS point will be included in the 16×16 grid.

$$x_j = \begin{cases} 1 & \text{if } j \in S \text{ is selected to be a SS} \\ & \text{point in the } 16 \times 16 \text{ grid} \\ 0 & \text{otherwise} \end{cases}$$

The second binary variable will determine which of the SS points that are selected to be included in the 16×16 grid will have a DoE run at that point. These selected points will have to cover the other grid points for which a DoE will not be run and cover all transient points:

$$y_j = \begin{cases} 1 & \text{if } j \in S \text{ is a SS point selected for} \\ & \text{testing} \\ 0 & \text{otherwise} \end{cases}$$

2.3 Objective Function

Our objective function is currently to minimize the size of the cover. Since we consider all points in the cover of equal value, we are just minimizing the number of SS points needed to cover the transient points and all other possible SS points. Our current objective is to minimize the cover points, which is reflected in

$$\min \sum_{j \in S} y_j. \tag{1}$$

The objective function could easily account for different *costs* associated with each selected point to include things such as the actual distance between the cover of SS points and the points it is covering. This would capture the case where the greater the distance between the point in the cover and the points it is covering, the worse the cover. For example, this could accommodate if the error rate for interpolating a SS value was worse for a greater distance between the interpolated point and its cover point. However, this could increase the number of tested points.

2.4 Constraints

This section describes the cover constraints, as well as additional constraints that capture certain points that must be included in the cover, grid spacing requirements, and constraints to ensure the resulting grid is 16×16. To illustrate these constraints, we introduce a small example, where $p = 6$, $E^s = 500$, $E^e = 1200$, $T^s = 0$, $T^e = 450$, $GX^{min} = 50$, $GY^{min} = 50$, $DX^{max} = 150$, and $DY^{max} = 150$. There are $q = 12$ possible engine speeds from which to choose for the grid in the x-direction and $r = 10$ possible torque values from which to choose for the grid in the y direction. Note that in this example we need to choose a 6×6 grid (i.e. 36 points) from a total of 120 points, whereas in the typical problem we need to choose 256 points to form the grid from a total of 1755 points. Figure 3 shows the indexing scheme for this size problem: the highlighted indices are the grid points directly above and below the FLC (i.e. $FLTU_k$ and $FLTD_k\ \forall\ k \in K$).

R	Torque												
10	450	109	110	111	112	113	**114**	**115**	116	117	118	119	120
9	400	97	98	99	100	**101**	**102**	**103**	**104**	105	106	107	108
8	350	85	86	87	**88**	**89**	90	91	**92**	93	94	95	96
7	300	73	74	75	**76**	77	78	79	80	**81**	82	83	84
6	250	61	62	**63**	64	65	66	67	68	**69**	70	71	72
5	200	49	50	**51**	52	53	54	55	56	57	**58**	59	60
4	150	37	**38**	39	40	41	42	43	44	45	**46**	**47**	**48**
3	100	**25**	**26**	27	28	29	30	31	32	33	34	**35**	**36**
2	50	**13**	14	15	16	17	18	19	20	21	22	23	24
1	0	1	2	3	4	5	6	7	8	9	10	11	12
Engine Speed		500	650	700	800	850	900	950	1000	1050	1100	1150	1200
K		1	2	3	4	5	6	7	8	9	10	11	12

Figure 3: This figure shows all possible grid points and corresponding index from which a 6×6 grid must be chosen.

2.4.1 Choosing a Grid

The first constraint guarantees that we have exactly $p\times p$ SS points in the grid:

$$\sum_{j\in S} x_j = p\times p \tag{2}$$

Note that this constraint does not guarantee that all of the points are chosen to be in the same rows and columns. Figure 4 shows an example selection of $6\times 6 = 36$ highlighted points that were chosen based on this constraint, but they do not form a 6×6 grid.

10	450	**109**	**110**	111	112	113	**114**	**115**	116	**117**	118	119	**120**
9	400	97	98	**99**	**100**	101	**102**	**103**	104	105	106	107	**108**
8	350	85	86	87	88	89	90	91	92	93	**94**	95	96
7	300	**73**	**74**	75	76	77	**78**	79	80	**81**	82	83	84
6	250	61	62	63	64	**65**	66	67	68	69	70	**71**	72
5	200	49	50	51	52	53	54	55	56	57	58	59	60
4	150	**37**	**38**	39	40	41	**42**	**43**	44	**45**	46	47	**48**
3	100	25	26	27	**28**	29	30	31	32	33	**34**	35	36
2	50	**13**	**14**	15	16	17	**18**	**19**	20	**21**	22	23	24
1	0	1	**2**	3	4	**5**	6	**7**	8	**9**	10	11	**12**
	0	500	650	700	800	850	900	950	1000	1050	1100	1150	1200

Figure 4: This figure shows highlighted in blue the $6\times 6 = 36$ points that were chosen to satisfy constraint (2).

The next two constraints address this problem and guarantee that we have exactly p columns and p rows chosen to compose the grid. Note that the two constraints (3) and (4) make constraint (2) redundant, but having it improves the solve time by around 5% based on our testing, as it gives a better representation of the convex hull (typically, additional constraints in BIPs help improve solve time (Geoffrion, 1976)). The constraint to ensure we choose exactly p columns is

$$\sum_{j_2\in S:E_{j_1}=E_{j_2}} x_{j_2} = p x_{j_1} \ \ \forall\ j_1\in K. \tag{3}$$

The following guarantees we choose exactly p rows:

$$\sum_{j_2\in S:T_{j_1}=T_{j_2}} x_{j_2} = p x_{j_1} \ \ \forall\ k\in\{0,\dots,r-1\} \tag{4}$$

$$\forall\ j_1\in S: j_1 = qk+1.$$

Figure 5 shows an example grid selection, choosing columns 1, 2, 4, 7, 8, and 12 and rows 2, 4, 5, 7, 9, and 10. The next few constraints enforce maximum spacing between the grid points in both the engine speed and torque directions. We ensure that the first (i.e., left-most) column in the grid must be at most DX^{max} away from the starting value of E^s:

$$\sum_{j\in S:E_j\le E_1+DX^{max}} x_j \ge p. \tag{5}$$

10	450	**109**	**110**	111	**112**	113	114	**115**	**116**	117	118	119	**120**
9	400	**97**	**98**	99	**100**	101	102	**103**	**104**	105	106	107	**108**
8	350	85	86	87	88	89	90	91	92	93	94	95	96
7	300	**73**	**74**	75	**76**	77	78	**79**	**80**	81	82	83	**84**
6	250	61	62	63	64	65	66	67	68	69	70	71	72
5	200	**49**	**50**	51	**52**	53	54	**55**	**56**	57	58	59	**60**
4	150	**37**	**38**	39	**40**	41	42	**43**	**44**	45	46	47	**48**
3	100	25	26	27	28	29	30	31	32	33	34	35	36
2	50	**13**	**14**	15	**16**	17	18	**19**	**20**	21	22	23	**24**
1	0	1	2	3	4	5	6	7	8	9	10	11	12
	0	500	650	700	800	850	900	950	1000	1050	1100	1150	1200

Figure 5: This figure an example grid with exactly 6 rows and 6 columns.

Note that in Figure 5, column 1 is selected, and since $E^s = 500$, this constraint holds.

Next, we require the grid to be spaced at most DY^{max} in the torque direction:

$$x_{j_1} + \sum_{j_2\in S: j_2>j_1\wedge|T_{j_1}-T_{j_2}|\le DY^{max}} x_{j_2} \ge p + \sum_{c\in S:T_c=T_{j_1}} x_c \tag{6}$$

$$\forall\ k\in\{0,\dots,((T^{end}-DY^{max})/GY^{min})\}$$

$$\forall\ j_1\in S: j_1 = qk+1.$$

Following Figure 5, we can see that the maximum space between any two rows is 100, and since $DY^{max} = 100$, this constraint holds.

Finally, we guarantee that the grid can be spaced at most DX^{max} in the engine speed direction:

$$x_{j_1} + \sum_{j_2\in S: j_2>j_1\wedge|E_{j_1}-E_{j_2}|\le DX^{max}} x_{j_2} \ge p + \sum_{c\in S:E_c=E_{j_1}} x_c \tag{7}$$

$$\forall\ j_1\in\{1,\dots,((E^{end}-DX^{max}-E^s)/GX^{min}+1)\}.$$

Following Figure 5, we can see that the maximum space between any two columns is 200 (e.g. columns 8 and 12), and since $DX^{max} = 150$, this constraint is violated. Figure 6 shows a grid selection that satisfies all constraints up to this point, with a maximum space between any two columns equal to 150.

10	450	**109**	**110**	111	**112**	113	114	**115**	116	117	**118**	119	**120**
9	400	**97**	**98**	99	**100**	101	102	**103**	104	105	**106**	107	**108**
8	350	85	86	87	88	89	90	91	92	93	94	95	96
7	300	**73**	**74**	75	**76**	77	78	**79**	80	81	**82**	83	**84**
6	250	61	62	63	64	65	66	67	68	69	70	71	72
5	200	**49**	**50**	51	**52**	53	54	**55**	56	57	**58**	59	**60**
4	150	**37**	**38**	39	**40**	41	42	**43**	44	45	**46**	47	**48**
3	100	25	26	27	28	29	30	31	32	33	34	35	36
2	50	**13**	**14**	15	**16**	17	18	**19**	20	21	**22**	23	**24**
1	0	1	2	3	4	5	6	7	8	9	10	11	12
	0	500	650	700	800	850	900	950	1000	1050	1100	1150	1200

Figure 6: This figure is an example grid selection that satisfies all constraints in Section 2.4.1.

2.4.2 Required Grid Points

There are a couple of types of SS points that are required to be a part of the grid. The first requirement forces the grid to contain points that have values greater than or equal to the max torque on the FLC. This is equivalent to forcing the maximum value in the grid to be selected.

$$\sum_{j\in S:T_j=T^e} x_j = p. \tag{8}$$

Similarly, the grid has to contain points that have values greater than or equal to the max engine speed on the FLC.

$$\sum_{j \in S : E_j = E^e} x_j = p. \tag{9}$$

We can see that the grid of chosen SS points in Figure 6 satisfies these two constraints because row 10 and column 12 are selected.

The next requirement is that points on the zero axis must be selected:

$$\sum_{j \in K} x_j = p. \tag{10}$$

Note that the grid of chosen SS points in Figure 6 does not satisfy this constraint because row 1 is not selected. Figure 7 shows a grid selection that satisfies this constraint, as well as all of the constraints in Section 2.4.1. Note that selected rows have been changed: row 1 instead of row 2 and row 3 instead of row 4 to maintain $DY^{max} = 100$.

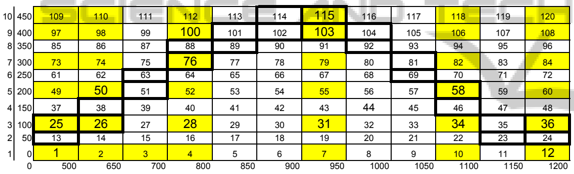

Figure 7: This figure is an example grid selection that satisfies constraints (2) - (15) with the cells containing the indices above and below the FLC outlined and the cover points in large, bold text.

2.4.3 Cover Constraints

This section describes the standard cover constraints and also additional constraints needed to require certain selected grid points to be in the cover. The first standard type of cover constraints guarantee that all transient points are covered:

$$\sum_{j \in S} a_{ij} y_j \geq 1 \;\; \forall \; i \in I. \tag{11}$$

Next, we ensure that all SS points that are not in the cover are covered:

$$\sum_{j_2 \in S} b_{j_1 j_2} y_{j_2} \geq 1 \;\; \forall \; k \in K : E_k \leq E^{end} \;\; \forall \; j_1 \in S : \; T_{j_1} \leq FLTU_k \wedge E_{j_1} = FLE_k. \tag{12}$$

The following constraints are not standard covering constraints: they guarantee that the points directly above and below the FLC are not only included in the selected grid but are also selected as part of the cover. First, though, we must ensure that if a point is chosen to be part of the cover, then it must have been selected to be a part of the grid:

$$y_j \leq x_j \;\; \forall \;\; j \in S. \tag{13}$$

Next, we guarantee that p SS points directly above the FLC are selected to test:

$$\sum_{k \in K : E_k \leq E^{end}} \;\; \sum_{j \in S : T_j = FLTU_k \wedge E_j = FLE_k} y_j = p. \tag{14}$$

Additionally, we guarantee that p SS points directly below the FLC are selected to be in the cover:

$$\sum_{k \in K : E_k \leq E^{end}} \;\; \sum_{j \in S : T_j = FLTD_k \wedge E_j = FLE_k} y_j = p. \tag{15}$$

Finally, we must ensure that all the SS points in the selected grid that are above the point directly chosen above the FLC are not selected as a part of the cover:

$$\sum_{k \in K} \;\; \sum_{j \in S : T_j > FLTU_k \wedge E_j = FLE_k} y_j = 0. \tag{16}$$

Figure 7 shows an example selected grid that satisfies all of the constraints: the indices in the highlighted cells represent the selected grid, the indices directly above and below the FLC are outlined in black, and the cover points are denoted by large, bold text.

We used IBM ILOG CPLEX Optimization Studio 12.2 (IBM, 2010) to solve the BIP. CPLEX takes on average 90 minutes to obtain an optimal solution.

3 EXAMPLE

The following example is of the grid and SS test point selection for a typical internal combustion engine. The target of this model application is to reduce the number of speed/load point combinations necessary to develop an engine calibration that delivers equivalent performance in terms of NO_X, particulate, noise, and fuel consumption.

3.1 Inputs

We provide the values for the inputs introduced in Section 2.1. Refer to Figure 2 for a pictorial of the entire grid space, the FLC, and the transient points for the internal combustion engine in this example.

In Table 1, we provide the values for the engine start and end speed, the torque start and end speed, the grid dimensions, minimum grid spacing in the engine and torque grid directions, maximum engine speed and torque for the FLC, and cover requirements. Other parameters that are calculated from these values are the number of possible grid points along the x-axis, the y-axis, and the number of possible SS points: $q = \frac{E^e - E^s + GX^{min}}{GX^{min}} = \frac{3800-600+50}{50} = 65$, $r = \frac{T^e - T^s + GY^{min}}{GY^{min}} = \frac{1300-0+50}{50} = 27$, and $n = q \times r = 65 \times 27 = 1755$.

Table 1: Input Parameters: scalar numbers.

Param.	Value	Unit	Param.	Value	Unit
E^s	600	RPM	n	1755	
E^e	3800	RPM	m	2618	
T^s	0	TRQ	DT^{max}	300	
T^e	1300	TRQ	DS^{max}	300	
p	16		DX^{max}	550	RPM
GX^{min}	50	RPM	DY^{max}	150	TRQ
GY^{min}	50	TRQ	FE^{max}	3800	RPM
q	65		FT^{max}	1255	TRQ
r	27				

We define the sets for the engine speed and torque grid point indices, the possible SS points, and the transient points as follows: $K = \{1,\ldots,65\}$, $R = \{1,\ldots,27\}$, $S = \{1,\ldots,1755\}$, and $I = \{1,\ldots,2618\}$. The engine speed, E_j, starts with 600 RPM and ends at 3800 RPM with an increment of 50 RPM. Torque, T_j, has a starting point of 0 TRQ and increments by 50 TRQ with a maximum of 1300 TRQ. The distance between possible SS point j_1 and j_2 for every $j_1, j_2 \in S$ is calculated using the Euclidean Formula $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, where x_1 and x_2 are the engine speed of j_1 and j_2, and y_1 and y_2 are the corresponding torque values. Similarly, we use the Euclidean distance formula to calculate the distance between all transient points $i \in I$ and SS points $j \in S$. As there are 2,619 transient points, we do not include them in this paper. The following points on the FLC are in terms of engine speed (RPM) and torque (TRQ): (0,373), (650, 445), (800, 623), (1000, 778), (1200, 939), (1400, 1061), (1600, 1147), (1800, 1140), (2000, 1152), (2200, 1185), (2400, 1205), (2600, 1234), (2900, 1255), (3000, 1209), (3200, 1117), (3400, 979), and (3800, 325).

3.2 Results

In this section, we present the optimization grid spacing and test point results for the internal combustion engine used in our study. We will also demonstrate how the selected grid and test points satisfy all constraints.

First, we can see in Figure 8, the selected grid points (defined by the 256 selected SS points) for the 16×16 grid. This grid selection satisfies the constraints specified in (2), (3), and (4) to have exactly 256 points chosen, with exactly 16 rows and 16 columns. In addition, the maximum spacing between the grid points in this example in the engine speed direction is 550 RPM and 150 TRQ in the torque direction, which satisfies constraints (5), (6), and (7).

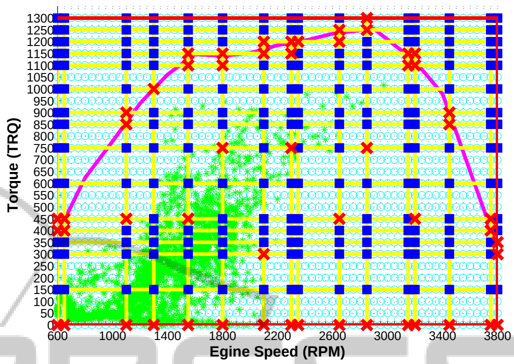

Figure 8: 54 Selected SS Points.

Furthermore, we confirm that the selected grid points satisfy *i*) constraints (8) and (9) that require the grid to contain points that have values greater than or equal to the max engine speed and max torque on the FLC; and *ii*) constraint (10) that forces the points on the zero axis to be selected. Figure 8 shows three lines that represent these grid constraints and demonstrate that our solution satisfies them.

The second part of our solution identifies which of the grid points are selected as SS points to test. Our solution yields a minimum number of 54 out of 256 SS points to test, which are identified by an X in Figure 8. We can see that test points are selected only from points that have been chosen to be in the grid: this satisfies constraint (13). Also note that all 16 SS points directly above and below the FLC are selected to be tested, and none of the points beyond the 16 points above the SS points are chosen to be tested: this satisfies constraints (14), (15), and (16).

Finally, we demonstrate how all of the transient points and SS points that were not selected for testing are covered by the SS points selected for testing (constraints (11) and (12)). Figure 9 shows which points are covered by the selected test points. Each selected test point, denoted by an X, has a radius of 300 that is marked by a circle. Any transient point or SS point that is in the circle is covered by that SS point at the center of the circle: note that due to different scaling in the torque and engine axes, the circles appear as ellipses. We can see that all of the transient and SS points that were not selected for testing are covered by at least one of the 54 SS points selected for testing.

For this example, the current testing process for the internal combustion engine would have required

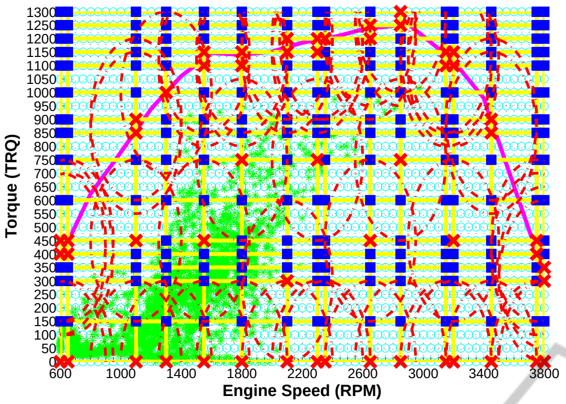

Figure 9: This figure shows the coverage of non-selected transient and SS points.

testing 122 points with the standard grid spacing. Our method selects a different grid spacing and chooses only 54 points to test, giving us around a 56% reduction in the number of points needed for testing.

4 SUMMARY

Completing a DoE at all possible steady-state engine operating points is a time consuming process. This project focused not on reducing the time to complete the DoE, but on reducing the number of experiments that needed to be performed. We captured the constraints to ensure that by minimizing the number of tests to perform, we could still satisfy regulatory requirements and internal testing constraints.

We introduced a BIP formulation that is based on a set-covering approach to select the best grid dimensions and to minimize the number of SS points. This formulation yields an optimal solution to simultaneously solving the grid selection and covering problem. We demonstrated the optimization methodology for a typical internal combustion engine and provided an optimal grid selection that resulted in an approximately 56% reduction in the points for which to perform a DoE.

While we applied this approach to the area of engine calibration, this method could also be applied to other automotive related areas. One example would be to select the minimum number of points on a surface on which to weld in order to satisfy certain material properties. Another example would be to minimize the number of stamping facilities and to determine the best locations to add stamping facilities in order to guarantee that every assembly plant would have at least one stamping facility within a certain distance. These are just a couple of examples that illustrate the diverse types of problems for which this approach can be applied.

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