MAGS
An Aco-based Model to Solve the Schedule Generation and Fleet Assignment Integrated Problem

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Abstract: Schedule Generation and Fleet Assignment problems usually are solved separately. The integrated solution for both problems, although desirable, leads to large scale models of the NP-Hard class. This article presents a mathematical formulation of this integrated problem along with a new heuristical approach, called MAGS, based on the ACO metaheuristic. Both the exact solution and the one provided by MAGS are obtained and compared for the case of a Brazilian airline. The results have shown the applicability of MAGS to real world cases.

1 INTRODUCTION

This paper presents a heuristic model that incorporates the Ant Colony Optimization metaheuristic to solve schedule generation and fleet assignment integrated problems, avoiding model simplifications that limit its application to real world problems (Caetano, 2011); (Caetano and Gualda, 2010); (Rabeanety et al., 2006).

Initially, a brief review of a linear programming model is presented, followed by the Multiple Ant Colony Group System heuristic (MAGS), based on the traditional Ant Colony Optimization (Dorigo and Stützle, 2004). Finally, a comparison between the results obtained through the metaheuristic and the optimal results obtained with the linear programming model is presented, followed by the conclusions of the study.

2 AIRLINE OPERATIONAL PLANNING

The traditional approach to solving the fleet assignment models are based on a space-time network, where arrival or departure airports are represented by nodes. There are two basic types of arcs on this representation: flight leg arcs – connecting nodes that represent different airports – or waiting time arcs – connecting nodes that represent different times at the same airport (Berge and Hoperstead, 1993 apud Sherali et al., 2006); (Hane et al. 1995).

These classical models assume that the flight schedule is previously defined, with every flight being covered. Traditionally, they do not include operational restrictions at airports. To overcome these limitations, it is necessary to define a more comprehensive model. The model presented in this paper is based on the concept of space-time modelling (Berge and Hoperstead, 1993 apud Sherali et al. 2006); (Hane et al. 1995), extended to cope with landing and departure slots by the addition of landing arcs that connect an arrival node to the first viable departure node, as shown in Figure 1.

The fleet assignment model can be integrated to schedule generation with the addition of a penalty for non served demand and relaxing the cover constraint so that not all flights must be assigned.

The following sets, parameters and decision variables are defined to describe the model:

- <u>Sets</u>

M: set of all markets, indexed by m; each market defines a demand and a time window which limits which flights can serve this demand.

N_f: set of all nodes for aircraft f, indexed by i, j, o, d or k, representing an airport at a specific time.

Nrd: set of nodes with departure restrictions.

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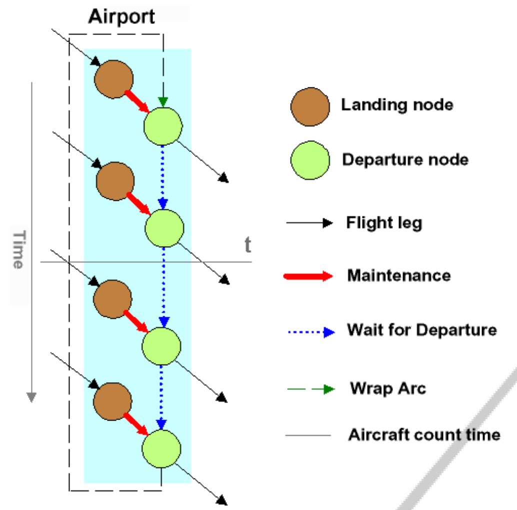

Figure 1: Space-Time Network.

Nra: set of nodes with landing restrictions.

F: set of all types of aircraft, indexed by *f*.

L: set of arcs that represent the movement of aircraft, maintenance, waiting on the ground or wrap, indexed by (*i*, *j*), being *i* the source node and *j* the destination node of the movement.

Lv: set of arcs that represent flight movements.

Lvd: set of arcs representing flights assigned to a market.

L_t: set of arcs whose origin time is equal to or less than t and destination time is after t. The time t is set to a valid time according to the problem.

L_m: set of arcs associated to market *m*.

- Parameters

D_m: unrestricted passenger demand on market *m*.

C^f: number of seats of aircraft of type *f*.

R_{ij}: unitary revenue for a passenger on the flight from node *i* to node *j*. Since (*i*,*j*) represent an specific flight – including day and time – each flight may be associated with a specific unitary revenue.

A_f: number of aircraft of type *f* available.

- <u>Decision Variables</u>

x^f_{ij}: number of aircraft of type *f* flowing through arc (*i*, *j*).

pa_{ij}: number of passengers associated to the flight from node *i* to node *j*.

d_{ij}: number of potential passengers (demand) associated to the flight from node *i* to node *j*.

The objective function (expression 1) seeks to minimize the sum of lost revenues. The first term represents the difference between maximum revenue for the assigned aircraft and the revenue received from assigned passengers. The second term is associated to the lost revenue due to lost demand.

$$[Min] \sum_{(i,j)\in Lv} \left\{ \left[\sum_{f\in F}\left(R_{ij}.C^f.x_{ij}^f\right) - R_{ij}.pa_{ij}\right] + \left[R_{ij}.\left(d_{ij}-pa_{ij}\right)\right] \right\} \quad (1)$$

$$\sum_{f\in F} x_{ij}^f \le 1 \quad \forall (i,j)\in Lvd \quad (2)$$

$$\sum_{o|(o,k)\in L} x_{ok}^f - \sum_{d|(k,d)\in L} x_{kd}^f = 0 \quad \forall k\in N_f, \forall f\in F \quad (3)$$

$$\sum_{(i,j)\in Lt} x_{ij}^f \le A_f \quad \forall f\in F \quad (4)$$

$$\sum_{f\in F}\sum_{d|(i,d)\in Lv} x_{id}^f \le 1 \quad \forall i\in Nrd \quad (5)$$

$$\sum_{f\in F}\sum_{o|(o,j)\in Lv} x_{oj}^f \le 1 \quad \forall j\in Nra \quad (6)$$

$$\sum_{f\in F} C^f.x_{ij}^f - pa_{ij} \ge 0 \quad \forall (i,j)\in Lv \quad (7)$$

$$d_{ij} - pa_{ij} \ge 0 \quad \forall (i,j)\in Lv \quad (8)$$

$$\sum_{(i,j)\in Lm} d_{ij} - D_m = 0 \quad \forall m\in M \quad (9)$$

Binaries:

$$x^f_{ij} \in \{0,1\} \text{ for } \forall(i,j) \in Lvd \quad (10)$$

Integers:

$$x^f_{ij} \ge 0 \text{ for } \forall(i,j) \in L\backslash Lvd \quad (11)$$

$$d_{ij} \ge 0 \text{ for } \forall(i,j) \in Lv \quad (12)$$

$$pa_{ij} \ge 0 \text{ for } \forall(i,j) \in Lv \quad (13)$$

Expressions 2 to 4 represent the traditional cover, balance and number of aircraft restrictions (Berge and Hoperstead, 1993 apud Sherali et al. 2006); (Hane et al. 1995).

Expressions 5 and 6 represents slot constraints, assuring that only one aircraft will depart or land on those nodes, respectively. Expressions 7 to 9 assure that each market demand will be associated to each flight and that the passengers of a flight will never be greater than the associated aircraft capacity.

The variables representing demanded flight arcs are binary, and are specified on expression 10. All other arc variables are integers greater than or equal to zero, as stated on expression 11, 12 and 13.

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3 ANT COLONY MODELING

Flight scheduling and fleet assignment are traditionally solved using integer linear programming techniques such as node clustering and constraint relaxation. However, practical instances, representing the operation of major airlines, remain a challenge, given the computational complexity involved. On the other hand, there are many heuristics that are capable of finding very good solutions to several types of combinatorial problems (Rayward-Smith et al., 1996 apud Abrahão, 2005), suggesting the search for heuristics that can provide appropriate solutions for the problem in lower processing times. The successful application to problems like Vehicle Routing Problem (VRP) and Aircraft Rotation Problem (ARP) draws attention to the metaheuristic known as Ant Colony Optimization (ACO), one of the many swarm intelligence metaheuristics (Teodorovic, 2008).

3.1 ACO Applied to the Schedule Generation and Fleet Assignment Integrated Problem

Although it was possible to adapt the basic ACO metaheuristic to solve the integrated flight schedule and fleet assignment problem, the results obtained through such approach were not satisfactory. Since the basic ACO leads to a single shortest path, it must be executed several times, assigning one aircraft at a time and removing the selected arcs from the list, leading to suboptimal solutions, with objective function values almost three times the optimal ones.

However, the problem has specific characteristics that can be used to improve the overall solution and thus an alternative heuristic is proposed, called Multiple Ant Group System (MAGS), incorporating elements of Multiple Ant Colony Optimization (MACO)(Vrancx and Nowé, 2006), Multiple Ant Colony System (MACS) and Elitist Ant System (EAS)(Dorigo and Stützle, 2004), as well as new elements not present on other ACO metaheuristic variants.

3.2 Multiple Ant Group System – MAGS Heuristic

MAGS is a multiple ant colony heuristic, such as MACS and MACO. As in MACO, a solution is represented by multiple ants; on the other hand, the number of ants that build a specific solution is previously known: there must be one ant per aircraft. The ants that compose a solution are called an *ant group*. A group may be composed of ants from different colonies and, similar to what is presented in MACS, each colony has a different objective function. This means that ants from each colony make decisions based on different criteria. In MACS, however, pheromones are identical for all colonies, which means it is substantially different from MAGS.

The proposed solution construction process is substantially different from the classical ACO, to reduce the number of invalid and unrealistic solutions. During the construction of a solution, the ants of a group will alternately choose graph arcs. The group's ant which will take the next step is randomly selected and whenever a flight arc is associated to an ant, this arc will be no longer available to other ants in the same group, ensuring the construction of solutions in which two or more aircraft do not share flights.

Additionally, when a flight is selected by an ant, part of the flight's market demand is also allocated, reducing the demand available for other flights that share the same market. This strategy avoids the association of ants to flight arcs for which the demand is no longer available in that solution. As the demand for each arc becomes dynamic during the construction of the solution, the problem presents similar characteristics to the dynamic routing in communication networks, as solved by the AntNet heuristic (Dorigo and Stützle, 2004). The exclusion of arcs and the demand allocation during the solution construction have relevant effects on the results, which are complementary to that provided by the repellent pheromones proposed on MACO, which continues to affect the selection probability of each arc.

Considering the described construction process, each ant group has the same role of a single ant in the basic ACO: the group of ants represents the complete objective function, each ant associated with a different term of it. The MAGS basic logic is presented in Figure 2.

As proposed by Dorigo and Stützle (2004), the nearest neighborhood solution can be adopted as an initial solution. On the addressed problem, the "nearest neighbor" was defined as the arc associated with the minimum revenue loss, avoiding waiting arcs whenever possible. The objective function value for this solution is used to determine the initial pheromone deposit on each arc.

Differently from the basic ACO, the initial pheromone deposit is not the same for all arcs. Arcs associated with smaller heuristic values must receive substantially more pheromones in the initial distri-

```
procedure MAGS
  Setup Parameters
  best=Nearest Neighborhood Solution
  Setup Pheromones
  for s Seasons
    for g Ant Groups
      Create Ant Group
      sol=Build Ant Group Solution
      sol=LocalSearch(sol)
      if( sol < 1,05*best OR
          sol > 2*best)
        sol=LocalSearch2(sol)
      if (sol < best) best=sol
    end for g Ant Groups
    Evaporate Pheromones
    Update Pheromome trails
    Update best AntGroup trail
  end for s Seasons
end Procedure
```

Figure 2: MAGS basic logic.

bution than those associated to higher heuristic values. Considering the basic ACO probability equation (expression 14), an increased pheromone deposit value on arcs that have low heuristic value will also increase their likelihood of being chosen, at least in the initial heuristic stage.

$$p_{ij} = \frac{\tau_{ij}^{\ \alpha} . \eta_{ij}^{\ \beta}}{\sum_{l|(i,l) \in N_i} \tau_{il}^{\ \alpha} . \eta_{il}^{\ \beta}} \tag{14}$$

The probability equation adopted (expression 15), however, presents some additional parameters. The first one is ϕ_{ij}, which represents the amount of pheromone of other colonies, as in MACO, with their respective coefficient γ. Additionally, the parameter ρ_{ij} reduces the probability of selecting a sequence of several unprofitable flights. The ρ_{ij} value is always 1.0 for profitable flight, maintenance, and waiting arcs. For unprofitable flight arcs, its value starts as 1.0 but upon the addition of an unprofitable arc to the solution, the value of ρ_{ij} is reduced by 50% for the next unprofitable flight arc. This value is only reset to 1.0 when a profitable flight is selected to compose the solution.

$$p_{ij} = \frac{\tau_{ij}^{\ \alpha} . \eta_{ij}^{\ \beta} . \left(\frac{1}{\phi_{ij}}\right)^{\gamma} . \rho_{ij}}{\sum_{l|(i,l) \in N_i} \tau_{il}^{\ \alpha} . \eta_{il}^{\ \beta} . \left(\frac{1}{\phi_{il}}\right)^{\gamma} . \rho_{il}} \tag{15}$$

The η_{ij} value is proportional to the flight profit, given that there is demand available in the market. The η_{ij} is made equal to τ_{ij} for repositioning flight arcs – which have no markets associated to them –, waiting arcs and maintenance arcs, since the heuristic value based on lost revenue on these arcs would be always non positive. The objective of this measure is also to reduce the myopic heuristic behavior, adding more emphasis on historical quality of the solutions containing a specific arc, which is represented by the pheromone deposit value.

After the initial pheromone is distributed, **g** ant groups are generated, but no changes are made on pheromones, as in the basic ACO. As the generation of the ant groups is completed, pheromone evaporation takes place, at a fixed rate, and then all **g** ant groups will update their pheromone trails. As in the EAS, the best solution will reinforce its own pheromone trail, leading to convergence toward that solution.

The pheromone deposit for each ant group is proportional to the objective function value, as in the basic ACO, but each ant of that group shall deposit only part of the group total pheromone: the amount of pheromone each ant of a group deposits is proportional to the ant's contribution to the quality of the solution represented by that group. The proposed distribution rule is defined by the expression 16, where τ_f is the deposit of each ant, with $\tau_g = 1/C^g$, where C^g is the cost of the solution represented by the group, calculated through the objective function. R_f is the revenue generated by that ant and MR_g is the maximum revenue that could be generated by that group of ants.

$$\tau_f = \tau_g . \{0.5 + [R_f / 2.MR_g]\} \tag{16}$$

This formulation guarantees that each ant will deposit a value not smaller than 50% of the deposit calculated for the group and also ensures that it will increase when the ant has a large contribution to the group total revenue.

It is important to notice that when each group finishes its solution construction, a local search is performed to improve that solution. This local search is divided into two steps: LS, which is quicker and handles all the solutions, and LS2, which is slower and processes only the solutions that have a value close to the optimum one or are too far from it. The LS is a procedure that removes sequences of two unprofitable flight arcs. This procedure also includes a corrective heuristic, which adjusts the solution so that each ant's terminal and initial airports are the same. The LS2 supplements LS, looking for profitable flights that could replace waiting arcs on each ant's path.

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3.3 Application and Results

The mathematical model and MAGS were applied to instances based on a domestic regional airline case that carries 104 weekly flights and uses three ATR-42/300 aircraft (for 50 passengers each). Alternative flight networks were generated with different fleet configurations, involving Embraer 120 (for 30 passengers each), and Embraer 170 (for 70 passengers each). Some of those instances also include alternative flights for a new destination, expanding the base network to 164 weekly flights, plus thousands of potential repositioning flights with no markets associated to them.

The demand distribution adopted on each instance can be of three different types:

- Fixed: the demand associated to each flight is fixed at 50 passengers.
- Flight: the demand is associated to each flight and is the average demand per flight, based on values provided by the Brazilian Civil Aviation regulatory agency – ANAC (2007).
- Period: the demand between two airports associated to a period of day – morning or evening – is the average demand by day period, based on values provided by ANAC (2007).

The instances were solved by integer linear programming techniques through Gurobi Optimizing software version 3, on an Intel Core2 Quad computer with 2GB of memory, using four processing cores and 200GB of available virtual memory. MAGS was implemented using Java SE version 6, running on the same equipment while using only one of the cores, since MAGS was not implemented using parallel programming.

Table 1 shows the results obtained by both the exact mathematical model and MAGS for some of the instances, given a weekly schedule. The values of the objective function represent the total lost revenue and, thus, the lower the value, the better the solution.

Analyzing the results, it is possible to notice that MAGS leads to results very close to those obtained by the exact model, with small standard deviations and much smaller processing times – none of the instances took more than an hour on each run. The minimum values obtained, shown in Table 2, are even closer to the optimal ones: while the average values are distant by up to 6% of the optimum, the minima are no more than 3% greater than the optimum value for each case.

The average processing time per arc was of 0.04 seconds. The processing times, though not directly linear to the number of arcs, are very low compared to processing times of the mathematical model solved with the Gurobi Optimizer, even without a parallel implementation for MAGS.

Table 1: Exact model and MAGS average results.

Instance	Demand Type	Fleets / Aircraft	Arcs	Flight Arcs / Repositioning Arcs (Potential)	Exact Model		MAGS (10 Runs Average)		
					O.F. Value	Time(s)	O.F. Value***	Std.Dev.	Avg. Time (s)
1	Fixed	3 / 3	35.367	312 / 15.636	0	3	0	0	0
2	Fixed	4 / 4	35.367	312 / 15.636	92.200	3	92.200	0	430
3	Fixed	3 / 3	44.907	492 / 20.316	175.000	7	177.500	0	803
4	Flight	3 / 3	44.907	492 / 20.316	897.345	14	916.475	0	828
5	Flight	5 / 5	74.845	820 / 33.860	809.365	172.800*	855.192	23.398	1.386
6	Flight	3 / 5	44.907	492 / 20.316	809.365	14.249	843.317	8.468	1.108
7	Period	1 / 3	14.969	164 / 6.772	870.120	68	878.324	3.615	2.236
8	Period	2 / 3	29.938	328 / 13.544	814.150	78.133**	831.716	3.220	2.792
9	Period	3 / 5	29.938	328 / 13.544	788.550	172.800*	807.112	9.452	3.372

(*) Processing was interrupted after the 2-day time limit (172.800 seconds).
(**) Processing was interrupted due to insufficient memory (2GiB RAM + 200GiB harddisk virtual memory).
(***) Average values do not include the construtive heuristic results.

Table 2: MAGS minimum results.

Instance	MAGS (10 Runs Minimum)		
	Value*	MAGS / Optimum	Avg. Time (s)
1	0	100.0%	0
2	92.200	100.0%	430
3	177.500	101.4%	803
4	916.475	102.1%	828
5	832.135	102.8%	1.386
6	833.865	103.0%	1.108
7	875.470	100.6%	2.236
8	829.170	101.8%	2.792
9	801.030	101.6%	3.372

(*) Minimum values do not include the constructive heuristic results.

4 CONCLUSIONS

This study presented and compared results of two types of models to solve the flight schedule and the fleet assignment problems in an integrated way. Both models incorporate the same objective function and constraints, including real world operational restrictions such as slots at airports.

One of the models relies on Linear Programming. The other, addressed to solve large scale problems, a heuristic approach called MAGS – Multiple Ant Group System – is based on the ACO metaheurisic. MAGS presents a distinctive way to determine the initial pheromone level on each arc, as well as alternative rules for the construction of solutions, each one being represented by multiple ants. In addition, the multiple ant solution representation required a new rule for pheromone updating. The exact model could reach optimal solutions for relatively small instances. MAGS has reached very close results to the optimal ones in smaller processing times, addressing the possibility to utilize it to solve larger real world problems.

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