

# A FUZZY SCHEME FOR IMAGE NOISE REDUCTION

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**Abstract:** The improvement of acquisition devices increases the need for processing of multicomponent images. In this context, the noise reduction is a preliminary preprocessing step affecting the results of the other image operations. This paper proposes a framework explaining usual noise reduction methods by the means of two fuzzy logic techniques: first a pixel fuzzification and second a defuzzification for estimating the filtered values. A new density-based filter is built for removing both impulse noise and Gaussian noise. The filter we propose is robust against outliers and it improves the classical bilateral approach for noise reduction of multicomponent images.

## 1 INTRODUCTION

In the framework of image processing, one of the first tasks consists in removing or reducing noise from the images (Gonzales and Woods, 1992). The improvement of acquisition devices increases the need for processing multicomponent images obtained from different channels (Kotropoulos and Pitas, 2001; Bovik, 2000). The independent processing of image components turns out to be inappropriate and leads to strong artifacts (Lukac et al., 2006). Thus the noise reduction of multicomponent images is an active field of research in satellite remote sensing, robot guidance, electron microscopy, medical imaging, color processing and real-time applications (Lin and Hsueh, 2000; Wong et al., 2004; Gallegos-Funes and Ponomaryov, 2004). This paper focuses on this preprocessing step for reducing both additive Gaussian noise and impulse noise. Additive Gaussian noise corrupts images because of the imprecision of acquisition devices. Impulse noise is generally produced by the transmission devices (Bovik, 2000).

The noise reduction consists in filtering the image, classically by computing a barycenter within a window. The selection of barycentric coordinates is the main key of noise reduction methods. The fuzzy techniques also addresses this issue of noise reduction (Ville et al., 2003; Morillas et al., 2009; Camarena et al., 2010). In this paper, we consider that the filtering window is a fuzzy set. First we determine these fuzzy sets associated to each pixel. This step

corresponds to a fuzzification of the pixels. Second the estimation of the filtered value corresponds to a defuzzification (Leekwijck and Kerre, 1999). Moreover the pixels of a multi-component image have both 2-dimensional spatial coordinates and n-dimensional photometric coordinates associated with the n components of the image. The bilateral filtering is a classical way taking into account both the spatial aspect and the photometric aspect of images in image processing. Bilateral filter of Tomasi and Manduchi (Tomasi and Manduchi, 1998) is the archetype of such bilateral approach. Thanks to the aggregation operators (Detyniecki, 2001), the fuzzy logic enables us to generalize the bilateral approach of filtering. Unfortunately Bilateral filter is not robust against outliers. Thus this paper proposes a new bilateral filter based on density estimation that provides robustness against outliers.

The paper is organized as follows: Section 2 presents the general framework selecting fuzzy neighborhood of each pixel for image filtering. Section 3 is devoted to the defuzzification step for estimating the filtered value of a pixel. In Section 4 we study the combination of fuzzy neighborhood improving the classical bilateral filtering (Tomasi and Manduchi, 1998). This approach is applied to reduce Gaussian noise and impulse noise in color images. The last Section proposes a discussion and concludes this paper.

## 2 FUZZY NEIGHBORHOOD OF A PIXEL

Let  $p$  be a pixel of a multicomponent image  $I$  with  $d$  components. Let  $I(p)$  be its photometric vector. Reducing the noise,  $I(p)$  is replaced by the filtered value  $I^*(p)$  which is estimated within a window  $W_p$  centered on  $p$ . Let  $p_1, p_2, \dots, p_N$  be the  $N$  pixels of  $W_p$  ( $N = n \times n$ ).  $I^*(p)$  is usually a barycenter of  $I(p_1), I(p_2), \dots, I(p_N)$  defined by:

$$I^*(p) = \frac{1}{\sum_{1 \leq i \leq N} \mu(i)} \sum_{1 \leq i \leq N} \mu(i) I(p_i) \quad (1)$$

where  $\mu(i)$  are the barycentric coordinates of  $I^*(p)$ .

In the fuzzy logic frame,  $\mu(i)$  becomes the membership value of the pixel  $p_i$  to a fuzzy set  $\tilde{p}$ . This fuzzy set has its support in  $W_p$ . Then the first step of the filtering procedure consists in selecting this fuzzy neighborhood of  $p$ . This fuzzification step is detailed in the following subsections.

### 2.1 Fuzzy Spatial Neighborhood

When the membership values  $\mu(i)$  depend only on the spatial locations of the pixels  $p_i$ , then a fuzzy spatial neighborhood  $\tilde{p}_{spat}$  is defined for filtering. Gaussian filter is the archetype of these spatial filters. The membership values  $\mu_{spat}(i)$  are defined by:

$$\mu_{spat}(i) = \exp\left(-\frac{dist_{spat}^2(p, p_i)}{2\sigma_{spat}^2}\right) \quad (2)$$

where  $dist_{spat}$  is the Euclidean distance and  $\sigma_{spat}$  is the standard deviation of the Gaussian filter. Note that these fuzzy neighborhoods are normalized fuzzy sets (Bouchon-Meunier, 1995) and their largest membership values are equal to 1.

### 2.2 Fuzzy Photometric Neighborhood

When the membership values depend only on the closeness between the photometric values  $I(p_i)$  and  $I(p)$ , then the fuzzy neighborhood of  $p$  is designed in the photometric space. Rank filter or vector median filters (Astola et al., 1990) give examples of such photometric filters. They are obtained by ordering the vectors  $I(p_1), I(p_2), \dots, I(p_N)$ . The estimation of  $I^*(p)$  is based on the ranks of  $I(p_i)$  vectors. In such cases, the membership values  $\mu_{phot}(i)$  of the fuzzy photometric neighborhood  $\tilde{p}_{phot}$  ignore the spatial location of the pixels  $p_i$ .

By analogy to the fuzzy spatial neighborhood, the Gaussian distribution also permits to give another definition of  $\tilde{p}_{phot}$ . The support of the fuzzy set remains

$W_p$ . But the distance  $dist_{phot}$  is computed in the photometric space (e.g. Euclidean distance). Then the membership function is defined by:

$$\mu_{phot}(i) = \exp\left(-\frac{dist_{phot}^2(I(p), I(p_i))}{2\sigma_{phot}^2}\right) \quad (3)$$

where  $\sigma_{phot}$  is the standard deviation of the Gaussian distribution in the photometric domain.

Because of the noise,  $I(p)$  could be inappropriate as the center of a photometric neighborhood. Therefore we propose another approach for defining a fuzzy photometric neighborhood of  $p$ .

### 2.3 Fuzzy Neighborhood based on Density

For each pixel  $q$  in  $W_p$   $\tilde{p}_{phot}^q$  is a fuzzy neighborhood of  $p$  centered on  $I(q)$ . The membership functions of  $\tilde{p}_{phot}^q$  are defined by:

$$\mu_{phot}^q(i) = \exp\left(-\frac{dist_{phot}^2(I(q), I(p_i))}{2\sigma_{phot}^2}\right) \quad (4)$$

where  $q \in W_p$ . These  $N$  fuzzy sets are aggregated using the arithmetic mean of their membership functions. Then the function  $\mu_{dens}$  we obtain corresponds to a local estimation of a probability density function (PDF). Improving PDF estimation we preserve against outliers and noise by ruling out  $\tilde{p}_{phot}^p$  (i.e.  $\tilde{p}_{phot}$ ) when estimating the density (Herbin and Bonnet, 2002). The membership function of this new fuzzy set based on density is defined by:

$$\mu_{dens}(i) = \frac{1}{C} \sum_{q \in W_p, q \neq p} \mu_{phot}^q(i) \quad (5)$$

where  $C$  is a normalization coefficient. This paper proposes this approach through robust density estimation to define a new fuzzy photometric neighborhood.

### 2.4 Bilateral Approach of Fuzzy Neighborhood

To keep the advantage of both spatial and photometric approaches, the t-norms (Bouchon-Meunier, 1995) (i.e. a conjunction operator) permit to combine the fuzzy spatial neighborhood and the fuzzy photometric neighborhood. Tomasi and Manduchi (Tomasi and Manduchi, 1998) use the algebraic t-norm for computing their bilateral filter. Then the membership values of  $\tilde{p}_{bilat}$  is defined by:

$$\mu_{bilat}(i) = \mu_{spat}(i) \times \mu_{phot}(i) \quad (6)$$

In this paper, we use the classical minimum operator as t-norm combining both spatial and photometric density-based neighborhoods. The fuzzy bilateral neighborhood  $\tilde{p}_{bidens}$  we propose is the conjunction of these two fuzzy sets. Therefore the membership function  $\mu_{bidens}$  is defined by:

$$\mu_{bidens}(i) = \min\left(\mu_{spat}(i), \mu_{dens}(i)\right). \quad (7)$$

### 3 DEFUZZIFICATION

The goal of this section is to estimate the filtered value  $I^*(p)$  from the fuzzy neighborhoods of  $p$ . This step corresponds to a defuzzification process (see a review of the defuzzification methods in (Leekwijck and Kerre, 1999)). The defuzzification is obtained using two stages: the first one operates in the spatial domain and the second one operates in the photometric domain.

The most classical defuzzification method is based on the maximum of membership values. In the context of multicomponent images, the maxima method in the spatial domain consists in selecting the pixel  $p_i$  for which the membership value  $\mu(i)$  is maximal. Let  $\bar{p}$  be this pixel defined by:

$$\bar{p} = \arg \max_{p_i \in W_p} \left( \mu(i) \right) \quad (8)$$

In this paper, the membership function  $\mu_{bidens}$  is used to determine  $\bar{p}$ . Therefore  $\bar{p}$  corresponds to the mode of our density estimation.

Another usual defuzzification method consists in computing the center of gravity of a fuzzy set where the weights are the membership values. This method is used in the photometric domain.  $I(\bar{p})$  is considered as the center of the fuzzy photometric neighborhood of  $p$ . Then the filtered value  $I^*(p)$  is defined by:

$$I^*(p) = \frac{1}{\sum_{1 \leq i \leq N} \mu_{phot}^{\bar{p}}(i)} \sum_{1 \leq i \leq N} \mu_{phot}^{\bar{p}}(i) I(p_i) \quad (9)$$

Indeed this barycenter inside the window  $W_p$  is the filtered value we propose to reduce noise in multicomponent images.

### 4 APPLICATION TO COLOR IMAGES

To assess our method, we use color images with three components: Red, Green and Blue. Images are



(a) Reference Image (Parrots)



(b) Noised Image



(c) Vector Median Filter



(d) Bilateral Filter



(e) Density-based Filter

Figure 1: Comparison of noise reduction filters: (a) Reference image (Parrots), (b) Part of Noised image (Noised), (c) Vector Median filtered image (VM), (d) Bilateral filtered image (BILAT), (e) Fuzzy Density-based filtered image (DENS).

corrupted with two kinds of independent and identically distributed noise. A low level noise is designed through additive Gaussian noise, and high level noise is modeled by impulse noise. The goal is to reduce both low level noise and high level noise by filtering corrupted images.

The classical mean squared error (MSE) evaluates the results by averaging the squared differences of filtered and reference images. In this context  $MSE$  is defined by:

$$MSE(I^*) = \frac{1}{\#I} \sum_{p \in I} dist_{phot}(I^*(p), I(p))^2 \quad (10)$$

where  $\#I$  is the number of pixels of the images. We

separate MSE into two parts  $MSE^-$  and  $MSE^+$ .  $MSE^-$  is defined by:

$$MSE^-(I^*) = \frac{1}{\#I^-} \sum_{\delta(p) \leq T} \delta^2(p) \quad (11)$$

where  $I^- = \{p : \delta(p) \leq T\}$ , and  $MSE^+$  is defined by:

$$MSE^+(I^*) = \frac{1}{\#I^+} \sum_{\delta(p) > T} \delta^2(p) \quad (12)$$

where  $I^+ = \{p : \delta(p) > T\}$ . In this paper, the threshold  $T = 10$  is used to separate low level noise and high level noise.

Table 1: Assessments of noised image (Noised), vector median filtered image (VM), bilateral filtered image (BILAT) and fuzzy density-based filtered image (DENS) using  $MSE$  with 388,112 pixels,  $MSE^-$  with  $N^-$  pixels, and  $MSE^+$  with  $N^+$  pixels ( $N^- + N^+ = 388,112$ ).

Image	$MSE$	$MSE^-$	$N^-$
Noised	2873.6	256,921	46.1
VM	98.2	338,773	32.9
BILAT	2803.0	336,865	28.6
DENS	69.0	370,588	21.3
Image	$MSE$	$MSE^+$	$N^+$
Noised	2873.6	131,191	8410.9
VM	98.2	49,339	545.6
BILAT	2803.0	51,247	21040.3
DENS	69.0	17,524	1077.5

In this paper, the filtering windows has  $5 \times 5$  pixels. Estimating the density in the photometric space, a large value of  $\sigma_{phot}$  is preferred for smoothing PDF estimation. Then we use  $\sigma_{phot} = 30.0$ . In the spatial domain,  $\sigma_{spat}$  is empirically determined ( $\sigma_{spat} = 0.5$ ). In the defuzzification process,  $\sigma_{phot}$  value controls the smoothing when filtering. The value which gives the best results is  $\sigma_{phot} = 10$ .

Evaluating the results, we compare a corrupted image (Noised), a classical bilateral filtered image (BILAT), a vector median filtered image (VM) and our fuzzy density-based filtered image (DENS). Table 1 gives the mean square errors obtained when assessing the noise reduction. These results show that the bilateral filter is inappropriate in the case of high level noise (i.e. outliers) and vector median filter cannot smooth enough the image for reducing low level noise when preserving the edges. Figure 1 confirm these results.

The defuzzification process uses a weighted mean of the photometric vectors which permits to smooth the image. The higher  $\sigma_{phot}$  value, the smoother the image. If  $\sigma_{phot}$  is too small, then the filter does not smooth the filtered image. Therefore it does not

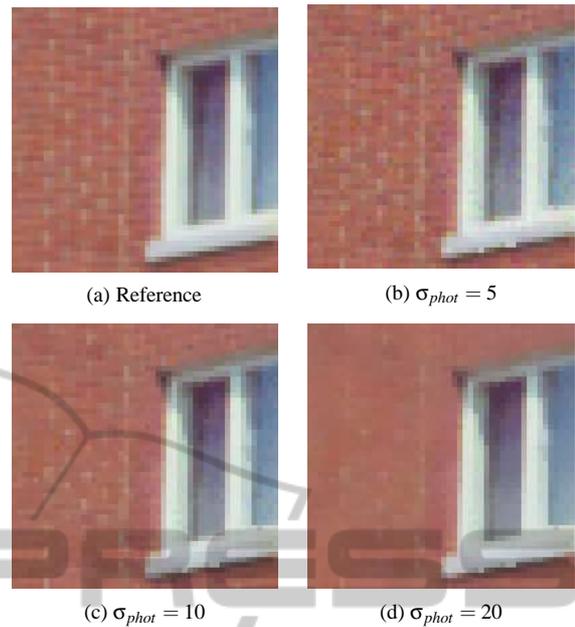


Figure 2: Reducing noise and level of texture; (a) part of a reference image and fuzzy density-baser filtered images with (b)  $\sigma_{phot} = 5$ , (c)  $\sigma_{phot} = 10$ , (d)  $\sigma_{phot} = 20$ .

enough reduce low level noise. If  $\sigma_{phot}$  is too large, the fine details could disappear when filtering because of a too large smoothing. Figure 2 displays the results obtained with  $\sigma_{phot} = 5$ ,  $\sigma_{phot} = 10$ , and  $\sigma_{phot} = 20$ . The value  $\sigma_{phot} = 10$  gives convenient results between smoothing for reducing low level noise and preserving details.

## 5 CONCLUSIONS

This paper adapts the classical fuzzy scheme for data analysis in the framework of noise reduction for multicomponent images. This scheme consists in a data fuzzification following by a defuzzification allowing the decision. The approach we propose is based on first the selection of adaptive fuzzy neighbourhoods of the pixels (i.e. the fuzzification) and second a defuzzification taking into account both spatial and photometric aspects of images. This fuzzy logic approach allows us to model the most classical filters used in the framework of image processing. Therefore this fuzzy scheme offers new angles for noise reduction of multicomponent images. The new density-based filter we propose reduces both high level noise (impulse noise) and low level noise (Gaussian noise). Like Bilateral filter, our filter reduces low level noise preserving details because its anisotropic nature. But it is also as robust against outliers (i.e. high level noise) as the

vector median based filters are. Therefore the fuzzy scheme permits us to design a new filter taking into account the advantages of two classic filters for reducing both high and low level noise.

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