COMPLEXITY ANALYSIS OF A HYBRID THRESHOLD-BASED ZF-MMSE EQUALIZER FOR SIC-BASED MIMO-OFDM RECEIVERS

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Abstract: In this paper, we propose a new reception scheme for MIMO-OFDM systems, which can switch between two different equalization methodologies, accordingly, on a subcarrier basis. In fact, the proposed receiver is based on the ordered successive interference cancellation (SIC) technique and uses jointly zero forcing SIC (ZF-SIC) and minimum mean squared error SIC (MMSE-SIC) according to a defined threshold value. The modulation scheme considered in this study is QPSK. Our main objective, here, is the provision of the computational complexity efficiency. Hence, a complexity analysis is provided and upper and lower complexity bounds for the proposed scheme are also derived.

1 INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is proposed as one of the key technologies for modulation and signal propagation. On the other hand, multiple-input multiple-output (MIMO) technologies hold the premise of achieving significant performance improvement and capacity enhancement in such systems. Moreover, MIMO fading channels can be explored to provide either spatial diversity gain (SD) in order to enhance the system robustness, or spatial multiplexing gain (SM) to the scope of the system capacity increase and the transmission gain. Due to the complementary benefits of MIMO and OFDM, the realization of MIMO-OFDM systems is, therefore, of a great importance (Lee et al, 2006) to ensure both the effectiveness and the reliability of future service demands in modern wireless ad hoc networks.

The interference effect represents a major efficiency inhibitor while induces a typical upper bound to the performance of MIMO-OFDM systems. Successive interference cancellation (SIC) represents quite an effective methodology, which tends to counteract the later limitation. The equalization methodology used in conjunction with the SIC-based reception, determines both the reliability of SIC, in terms of the bit-error-rate (BER) performance, and the overall computational complexity of the process. The maximum likelihood (ML) criterion is the most optimal yet most demanding equalization technique, in terms of the BER performance and the computational complexity, respectively. Particularly, the computational burden of an ML equalizer is found to be overwhelmed for most practical applications (Verdú, 1998). Hence, most of the research community has rather focused on suboptimal equalization techniques, based on either zero forcing (ZF) or minimum mean squared error (MMSE) detection. Generally, in a noisy environment, MMSE-SIC is more error resilient than ZF-SIC at the cost of a higher computational complexity and vice versa (Kim et al, 2009).

We focus, in this paper, on the performance of SIC for MIMO-OFDM systems and, more specifically, we propose a joint detection methodology based on ZF and MMSE equalization. Upon a signal decoding, the decision statistic, which determines whether a symbol will be carried out by a ZF-SIC or by an MMSE-SIC, exclusively, depends on a threshold. The value of the threshold is defined on an OFDM frame (or block) basis by taking into account the statistics extracted from the received information on each subcarrier. In this study we focus on multiuser MIMO-OFDM systems where all the transmissions established using QPSK modulation schemes. For the best of our knowledge,

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a hybrid ZF-MMSE-SIC reception methodology has never been proposed for MIMO-OFDM implementations.

2 SYSTEM MODEL

We consider a MIMO-OFDM system consisting of N_C subcarriers with N_T and N_R ($\geq N_T$) transmit and receive antennas, respectively.

After the CP removal, the receiver conventionally performs fast Fourier transform (FFT) at each subcarrier, which yields to the actual received signal expressed as

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{w}_n \tag{1}$$

where $\mathbf{y}_n = \left[y_0^n + y_1^n + \cdots + y_{N_R-1}^n\right]$ and $\mathbf{x}_n = \left[x_0^n + x_1^n + \cdots + x_{N_T-1}^n\right]$ represent the received and the transmit signal vector, respectively, at the n-th subcarrier. Also, the $\mathbf{w}_n = \left[w_0^n + w_1^n + \cdots + w_{N_R-1}^n\right]$ is a zero-mean complex AWGN vector with covariance matrix given as

$$\mathbf{\Phi}_{\mathrm{w}} = \mathbb{E}\{\mathbf{w}_{(n)}\mathbf{w}_{(n)}^H\} = \sigma_w^2 \mathbf{I}_{N_R}, \tag{2}$$

where σ_w^2 represents the noise variance introduced by the communication channel and $\mathbf{I}_{N_R}$ is the $N_R \times N_R$ identity matrix. We also provide a compact input-output relation between all the OFDM subcarriers at all the transmit and receive antennas of all the network transmitting nodes, which is expressed as

$$\underline{\mathbf{Y}} = \sum_{n=0}^{N_C-1} \mathbf{H}_n \mathbf{P}_n \mathbf{x}_n + \mathbf{w}_n = \underline{\mathbf{H}}\underline{\mathbf{X}}\underline{\mathbf{P}} + \underline{\mathbf{W}} \tag{3}$$

where

$$\underline{\mathbf{Y}} = [\mathbf{y}_{(0)}{}^T, \mathbf{y}_{(1)}{}^T, \ldots, \mathbf{y}_{(N_C-1)}{}^T]^T,$$

$$\underline{\mathbf{X}} = [\mathbf{x}_{(0)}{}^T, \mathbf{x}_{(1)}{}^T, \ldots, \mathbf{x}_{(N_C-1)}{}^T]^T,$$

$$\underline{\mathbf{P}} = [\mathbf{P}_{(0)}{}^T, \mathbf{P}_{(1)}{}^T, \ldots, \mathbf{P}_{(N_C-1)}{}^T]^T,$$

$$\underline{\mathbf{W}} = [\mathbf{w}_{(0)}{}^T, \mathbf{w}_{(1)}{}^T, \ldots, \mathbf{w}_{(N_C-1)}{}^T]^T$$

$$\underline{\mathbf{H}} = [\mathbf{H}_{(0)}, \mathbf{H}_{(1)}, \ldots, \mathbf{H}_{(N_C-1)}]^T.$$

Note that $\underline{\mathbf{H}}$ is a multi-block diagonal matrix, assuming that both ISI and ICI which are caused mainly due to time or frequency offsets, are perfectly compensated by an appropriate CP length.

3 PROPOSED SCHEME

On MIMO-OFDM infrastructures, the ZF equalization leads to the noise enhancement because the pseudoinverse channel matrix is not always added destructively and, hence, it could result to the potentially colored additive noise at the receiver. Moreover, the diversity gain provided by the multiple receive antenna array for the interference suppression is eliminated along with the channel matrix coefficients by exploiting the ZF equalizer in MIMO channels, which results in a lower overall diversity order (Zhang et al, 2009). Particularly, the selected stream for detection at the first SIC step has a diversity gain of N_R, while the stream at the last SIC step has a diversity gain of $N_R - N_T + 1$, which is a rather undesirable condition.

On the other hand, in case of MMSE-SIC, the interference is not totally removed. However, the imperfect interference cancellation is compensated by providing a higher diversity performance in the decoding process. Moreover, MMSE does not enhance the noise coefficients in comparison to the respective ZF equalization, whereas the higher diversity order tenet is found to be beneficial, especially in low SINR regions (Zijian et al, 2008). It is straightforward that a conventional MMSE equalizer results to a lower BER probability in comparison to a respective ZF one. Nevertheless, the higher computational cost is a fundamental prerequisite for the enhanced BER performance. Hence, the selection of the appropriate equalizer is still debatable, depending mostly on the users' QoS requirements or the network manufacturer.

Taking into account the benefits and the drawbacks of the abovementioned equalizers, we propose a novel framework for MIMO-OFDM systems, which is based on a hybrid detection-switching technique, associated with the conventional SIC method. Since we are focusing on a QPSK modulation, the signal is demodulated as two independent BPSK signals in quadrature, thereby only the real or the imaginary symbol part is captured in each branch. The detection criterion is tightly predetermined on an OFDM frame basis according to a threshold value, gained by the statistics collected upon the reception of an OFDM frame. In this content and while the transmitting symbols are independent of each other, we introduce a threshold S, representing the mean amplitude of the overall received signal (Marabissi et al, 2006), which is expressed as

$$S = \beta\left(\frac{1}{N_C N_R}\left(\Re\left(\left|\underline{\mathbf{Y}}\right|\right)+\Im\left(\left|\underline{\mathbf{Y}}\right|\right)\right)\right)$$
$$= \beta\left(\frac{1}{N_C N_R}\sum_{n=0}^{N_C-1}\left(\Re\left(\left|\mathbf{y}_n\right|\right)+\Im\left(\left|\mathbf{y}_n\right|\right)\right)\right) \quad (4)$$
$$= \beta\left(\frac{1}{N_C N_R}\sum_{n=0}^{N_C-1}\sum_{j=0}^{N_R-1}\left(\Re\left(\left|y_n^j\right|\right)+\Im\left(\left|y_n^j\right|\right)\right)\right)$$

where β is a constant which plays the role of a tuning parameter.

Therefore, upon a signal reception y_n^j, the decision statistic is determined by using ZF detection if both $\Re|y_n^j| \geq S$ and $\Im|y_n^j| \geq S$ or by using MMSE detection if $\Re|y_n^j| < S$ and $\Im|y_n^j| < S$. Moreover, switching the more reliable signals to ZF-SIC and the less reliable ones to MMSE-SIC, we also balance the overall complexity of the process. Under this regime, we set the accuracy of the reception process upon a *reliability* classification basis. Furthermore, an appropriate balance on the tradeoff between the performance efficiency and the complexity reduction, depending always on the S value, is accomplished.

4 COMPLEXITY ANALYSIS

We provide computational complexity upper and lower bounds of the proposed scheme and we also present a cross-analysis demonstration between the proposed scheme and the quite complex MMSE-SIC, in a MIMO-OFDM environment. The computational complexity has been evaluated with respect to the number of the expected floating point operations (flops). Since the signal coefficients as well as the channel matrix coefficients are complex-valued, all the appropriate operations including multiplications, additions and divisions, are conducted upon complex values. In the rest of the paper, when we discuss computational complexity we refer to the complex operations (COs) including only complex multiplications (CMs) and complex additions (CAs). We count each complex CA as two flops and each complex CM as six flops (Luo et al, 2007), (Gan et al, 2009), in order to evaluate the overall computational complexity of the proposed scheme.

From the property of Hermitian matrices, only the half of the complexity can be obtained in comparison to the complexity of computing a general matrix of the same size. Typically, the main computational burden is obtained by the number of CMs. Computing $\mathbf{H}^H\,\mathbf{H}$ requires $N_R N_T^2 - 1/2\, N_T^2 + 1/2\, N_T$ CMs, while $3/2\, N_T^3 + (N_R + 1)N_T^2 + 1/2\, N_T$ CMs are needed in order to calculate $(\mathbf{H}^H\,\mathbf{H}+\sigma^2\,\mathbf{I}_{N_T})^{-1}\mathbf{H}^H$, assuming that the real-valued $\sigma^2\,\mathbf{I}_{N_T}$ is *priori* estimated at the receiver. However, the exact number of COs for a matrix inversion event may vary, depending mostly on how the channel coefficients are handled. Several elimination approaches (e.g. Gauss-Jordan elimination) or advanced signal processing techniques based on matrix decomposition methods (e.g. QR or LDL^H decomposition), result to potentially different computational burden at the cost of either BER performance or hardware gain. We, hence, approximate the complexity of the channel inversion, with respect to the CAs, CMs and COs, as $\mathcal{O}(N_T^3)$. In case of a SIC reception, the channel inversion procedure is suppressed due to the channel nulling operation at each SIC step. Thus, the computational burden at the i-th SIC step is obtained as $\mathcal{O}((N_T - i)^3)$.

The lower complexity bound of the proposed scheme can be obtained as $\beta \rightarrow 0$, where all transmitting symbols at all the OFDM subcarriers are carried out by a ZF-SIC reception. Therefore, the total number of COs in the case where $N_T = N_R$ is obtained as

$$CO_{\text{Lower Bound}} = \sum_{i=0}^{N_C-1}\left(2N_T^3 + 2N_T^2(1 - 2i) + 2N_T(i^2 - i + 1) + \mathcal{O}((N_T - i)^3)\right) \quad (5)$$

Similarly, the upper complexity bound of the proposed scheme can be obtained as $\beta \rightarrow +\infty$, where all transmitting symbols at all the OFDM subcarriers are carried out by an MMSE-SIC reception, exclusively. Hence, the total number of COs considering the case of $N_T = N_R$, is obtained as

$$CO_{\text{Upper Bound}} = \sum_{i=0}^{N_C-1}\left(3N_T^3 + N_T^2(2 - 6i) + N_T(3i^2 + 2 - i) + \mathcal{O}((N_T - i)^3)\right) \quad (6)$$

In order to show the fraction of saved complexity of the proposed scheme, we introduce the quotient

$$\xi = \frac{Flops_{proposed}}{Flops_{MMSE}} \quad (7)$$

where $Flops_{proposed}$ denotes the total number of flops for the proposed hybrid scheme and $Flops_{MMSE}$ denotes the respective number of flops for the conventional MMSE-SIC. In fig. 1, the performance of ξ is depicted with respect to β for varying number of $N_T = N_R$ antennas, considering a MIMO-OFDM network with 128 subcarriers. As an illustrative example, in case of $N_T = N_R = 6$ and when $\beta = 3.5$, the proposed scheme requires $0.78 \times Flops_{MMSE}$ number of flops in order to complete an OFDM frame reception. Consequently, this reflects to a strong reduction in the overall computational complexity, since the proposed methodology does not produce any additional overhead to the hardware gear or to the system latency. Furthermore, it is worth mentioning that the complexity efficiency of the proposed scheme is a result of the diversity of the detection approach, while the computational cost for the decision of the most appropriate equalizer depends on the calculation of S, which is negligibly small.

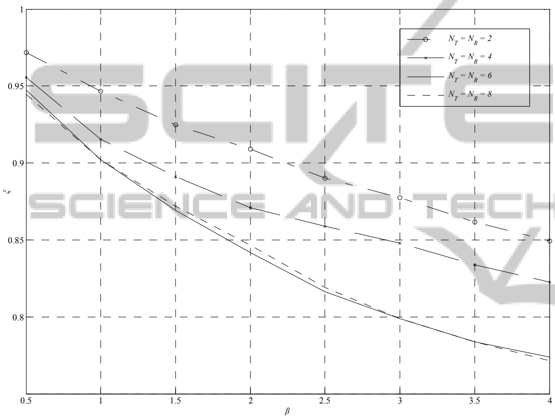

Figure 1: Fraction of saved complexity of the proposed scheme in comparison to the conventional MMSE-SIC for different β values, in a MIMO-OFDM system with N_C = 128. A small number of transmit and receive antennas is considered in order to approach realistic scenarios, suitable for practical implementations.

5 CONCLUSIONS

In this paper, we presented a new detection-switching approach for SIC-based receivers in MIMO-OFDM systems. The proposed scheme is implemented by two well-known equalizers jointly. More specifically, it switches between ZF and MMSE equalization according to a certain threshold, which is determined by the mean received amplitude of the overall signal within an OFDM frame. All the included transmissions have been implemented under QPSK modulation alphabets. Upper and lower asymptotic complexity bounds have derived through a computational complexity analysis. We showed that by applying the proposed detection switching approach, up to 22% complexity savings can be obtained. Some of our most important future aspects are the study of the proposed hybrid SIC under MIMO-OFCDM systems and a definition of an appropriate threshold under QAM modulation schemes.

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REFERENCES

H. Lee, B. Lee and I. Lee, "Iterative Detection and Decoding With an Improved V-BLAST for MIMO-OFDM Systems," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, 2006, pp. 504-513.

S. Verdú, *Multiuser Detection*, Cambridge University Press, 1998.

E. C. Kim, J. S. Park and J. Y. Kim, "Co-channel interference cancellation based on ZF/MMSE SIC with optimal ordering for cooperative communication systems," *IEEE 9th Int. Symp. Commun. Inf. Technol., Incheon, Korea*, Sept. 2009, pp. 404-409.

H. Zhang, H. Dai and B. Hughes, "Analysis on the diversity-multiplexing tradeoff for ordered MIMO SIC receivers," *IEEE Trans. Commun.*, vol. 57, no. 1, 2009, pp. 125-133.

A. Zijian, J. Berkmann, C. Spiegel, T. Scholand, G. H. Bruck, C. Drewes, B. Gunzelmann, P. Jung, "On MIMO With Successive Interference Cancellation Applied to UTRA LTE," *IEEE 3rd Int. Symp. Commun., Control and Signal Process.*, St. Julians, Malta, March 2008, pp. 1009-1013.

D. Marabissi, R. Fantacci and S. Papini, "Robust Multiuser Interference Cancellation for OFDM Systems With Frequency Offset," *IEEE Trans. Wireless Commun.*, vol. 5, no. 11, 2006, pp. 3068-3076.

Z. Luo, S. Liu, M. Zhao and Y. Liu, "A Novel Fast Recursive MMSE-SIC Detection Algorithm for V-BLAST Systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, 2007, pp. 2022-2025.

Y. H. Gan, C. Ling and W. H. Mow, "Complex Lattice Reduction Algorithm for Low-Complexity Full-Diversity MIMO Detection," *IEEE Trans. Signal Process., vol. 57, no. 7, 2009, pp. 2701-2710.*