

# MECHATRONIC SYSTEM MODELING

## *A Consistent Preliminary Design Process*

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Abstract: In order to describe a consistent and collaborative preliminary design process for mechatronic systems, this study deals with an automotive power lift gate scenario. First, a functional analysis is carried out with SysML from user requirements. This allows one to define suitable architectures and associated test cases. Each of them has to be analysed and optimized separately in order to select the best architecture and the best set of key parameters. The next step of the preliminary design is a modelling of its structure and its behaviour. In order to merge multi-physical and geometrical parameters, our generic method relies on a topological analysis of the system and generates a set of equations with physical and topological constraints previously defined. Finally, an interval analysis is implemented, allowing one to explore exhaustively the search space resulting from a declarative statement of constraints, in order to optimize the parameters under the constraint of the relevant test cases.

## 1 INTRODUCTION

Nowadays, system engineering problems are solved using a wide range of domain-specific modelling languages and tools. Standards such as ISO 15288 detail the large number of system aspects and various components of multi-domain systems (ISO/IEC 2001) (Turki, 2008). It is also not realistic to create an all-encompassing systems engineering language capable of modelling and simulating every aspect of a system. However, for multi-domain systems, a global approach is necessary. Indeed, each domain has its own methodologies and languages, thus impeding the consistency of the different modelling. Hence, a global optimization is difficult during the preliminary design process of these systems.

Mechatronic systems development involve considering the modelling of their components together with their interactions. Models can be used to formally represent all aspects of a systems engineering problem, including requirements, functional, structural, and behavioural modelling.

Additionally, simulations can be performed on these models in order to verify and validate the effectiveness of design decisions.

This study covers the preliminary design phase of a mechatronic system, in order to verify that the chosen design is in accordance with the system requirements and to verify that this chosen design minimizes risks in further design phases. Following the recent advances in Model Based System Engineering (Estefan, 2008), the preliminary design can be viewed as a model transformation process (Hartman and Kreische, 2005).

Based on the example of a power lift gate, our goal is to show how the engineering knowledge can be formalized and used all along the three following phases of the preliminary design process: requirements definition and functional analysis, geometrical and physical modelling, optimization.

Once the early design phases have been performed with SysML models, the physical modelling of the overall system has to be built, based on the topology of the system, in order to generate the equations required for the optimization

phase. This being done, the Design Space Exploration can be executed in order to discover the optimal design solution from all functional and architectural specifications and constraints. Indeed, the most efficient way to explore this design space is to reason about previous SysML models, thus proving in a mathematically rigorous way that all required properties and constraints are met.

## 2 A POWER LIFT GATE SCENARIO

An automotive power lift gate (Figure 1) includes a lift gate door hinged to a car body. This system moves the lift gate between its open and closed positions, thanks to electric cylinders (Figure 2) that replace the usual gas struts in a classic manual lift gate. It includes a motor and a gearbox that are fixed to the base tube and a jackscrew that drives the upper tube, helped by a spring, in order to sustain static forces. Both electric cylinders are identical and are fixed to the car body and the lift gate.

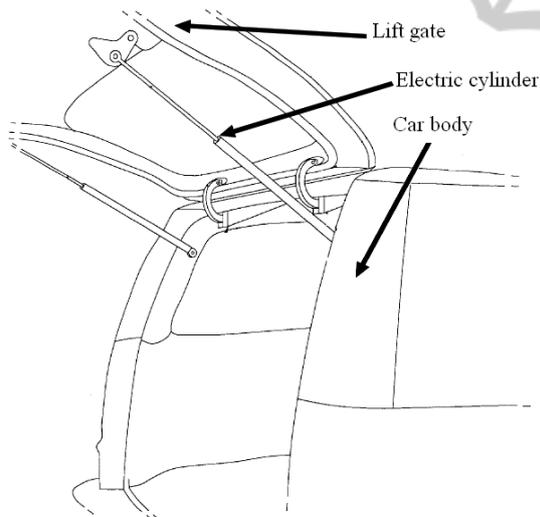


Figure 1: The Power Lift Gate Location on a Car Body.

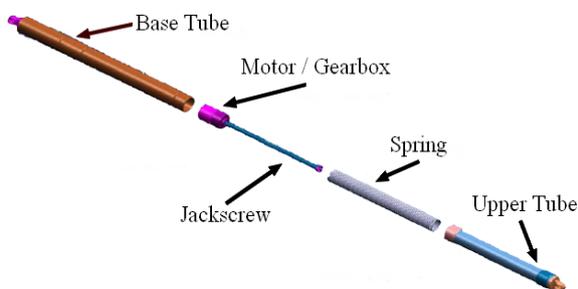


Figure 2: The Electric Cylinder Architecture.

## 3 THE PRELIMINARY DESIGN PROCESS

The proposed preliminary design process relies on a methodology that deals with different modelling, (SysML model, topological model) in order to provide consistent equations for an optimisation of the mechatronic system with interval analysis.

### 3.1 Modelling of the Power Lift Gate System with SysML

We propose a modelling of a power lift gate system by means of appropriate SysML models at the early stages of the technical engineering process. The different SysML diagrams make it possible for engineers from various disciplinary fields to share a common view about the system. First, we create an extended context diagram, in order to present the different interactions between the extended system (Lift gate + Electric cylinder) and its environment (Figure 3).

Then a Use Case Diagram is defined to describe the system services (Figure 4).

SysML Requirement Diagram can be used to clearly organize user and derived system requirements (Figure 5). By using a hierarchical representation of the requirements, clear gains can be made in the elaboration of requirements, in tradeoffs, as well as in the validation and the verification of requirements. Indeed, during design activities, verification activities need to be defined to satisfy system constraints and properties. Links between the Requirements Diagram and other models allow engineers to connect test criteria to test cases used throughout the development process. During the architecture analysis, system synthesis by assigning functions to identified physical architecture elements (subsystems, components) is carried out (Figure 6). Finally we create a kinematic joint diagram (Figure 7) with connectors regarding to application points and with links representing the field or the type of joints between two elements.

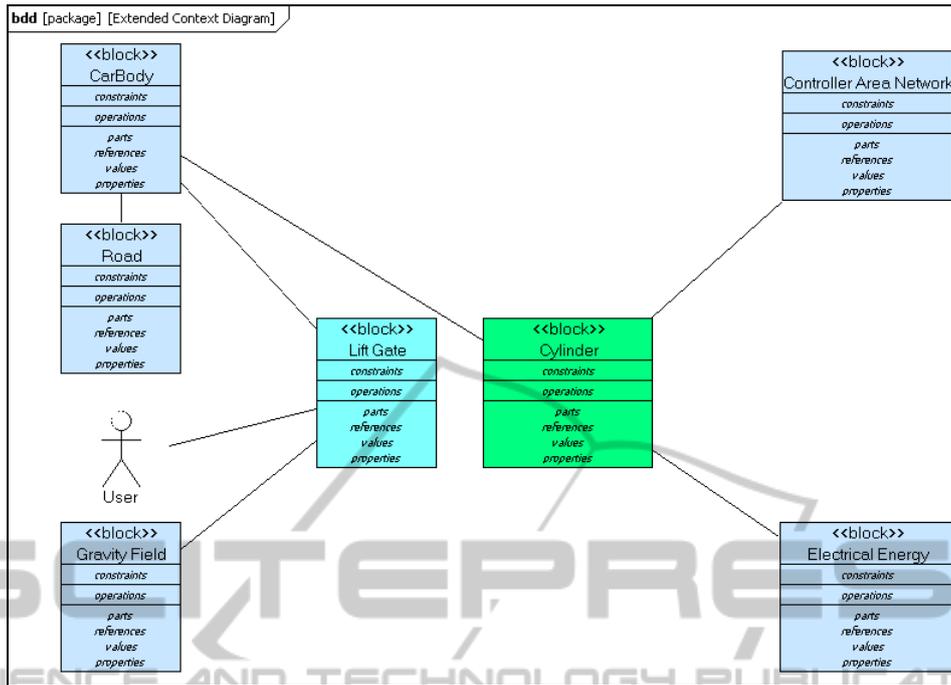


Figure 3: Extended Context Diagram of the Power Lift Gate System.

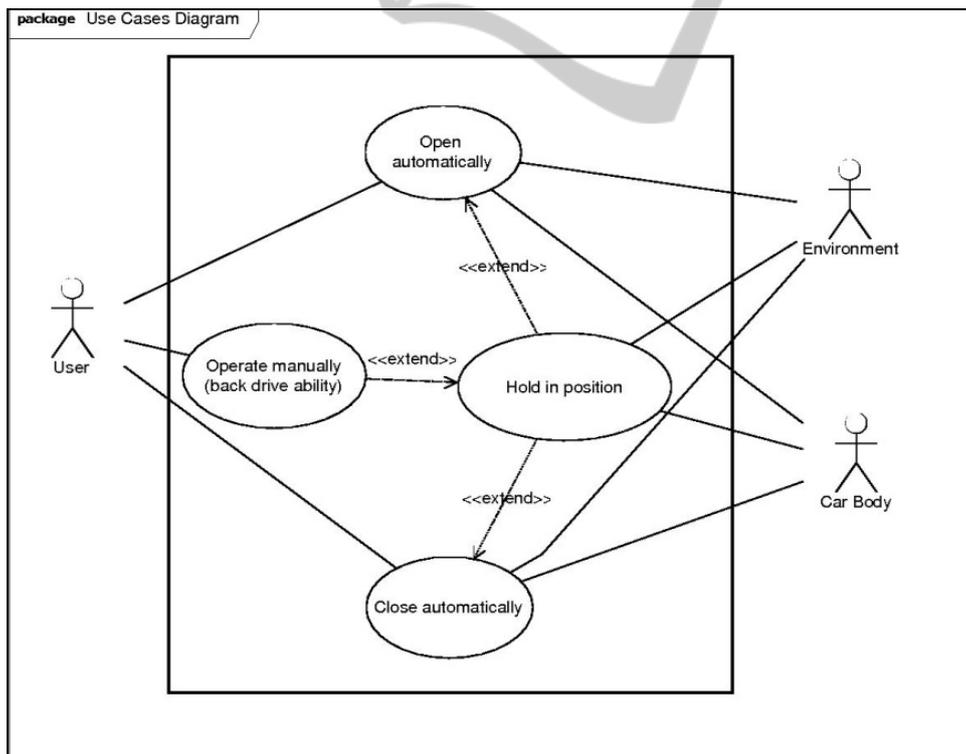


Figure 4: Use Cases Diagram of the Power Lift Gate System.



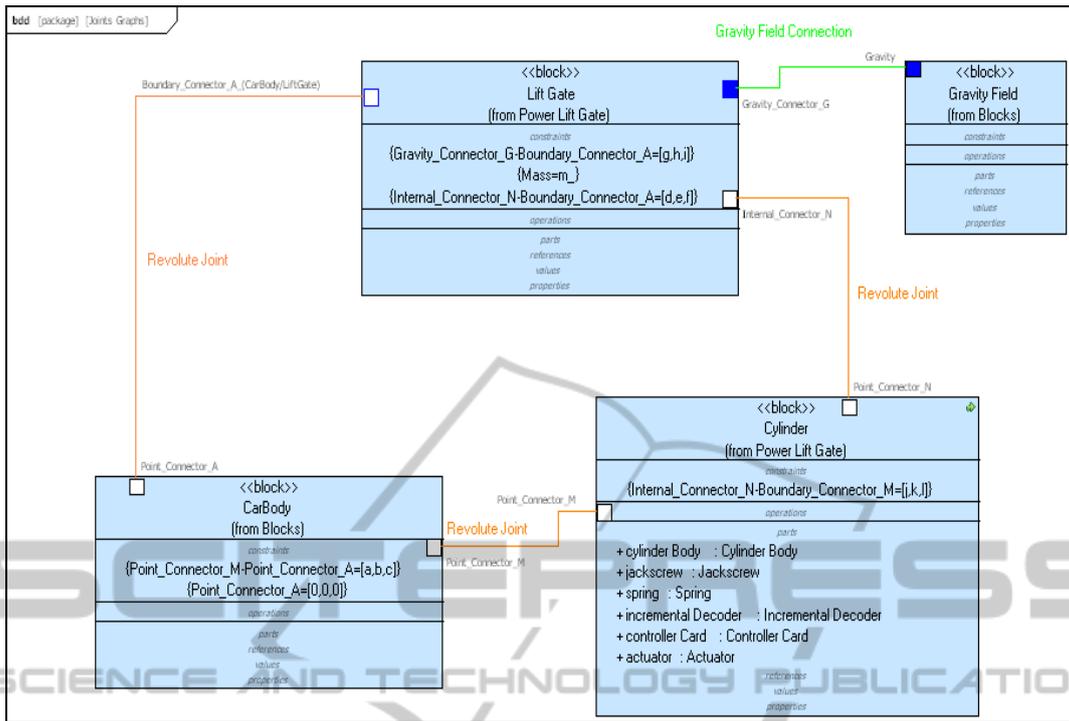


Figure 7: Kinematic Joints Relations Diagram in SysML.

### 3.2 Vector-based Mechanical Modelling Derived from System Topology

The previous SysML diagrams bring to light the key parameters and the topology of the power lift gate system. In order to optimize these key parameters, this mechanical problem has to be translated into equations. We propose to use a highly suitable method (Plateaux et al, 2008) for multi-domain systems such as automotive mechatronic components. Based on a topological analysis of the system, this generic method delivers equations that can be processed by a solver. It relies on the works of Kron (Kron, 1959), Branin (Branin, 1966) and Björke (Björke, 1995). Here, our method is restrained to the mechanical study of the static equilibrium of the lift gate but it may also be used to express the internal structure of the electric cylinder (screw and nut system, tubes, gearbox, spring, sensors, electrical engine and electronic components...).

The isolated system includes the lift gate with the electric cylinder between the points M and N, the car body being an external system. Let us assume that: the mechanical joints are perfect; points A, M and G belong to the system boundary; there is neither external mechanical force nor torque on internal

point N; P is the external force on the gravity centre G;  $F_C$  is the force created by the electrical cylinder, which corresponds to the internal force  $R_{MN}$ .

In order to model the architecture of the system, a topological graph has first to be defined from geometrical and mechanical definitions of the problem (figure 8).

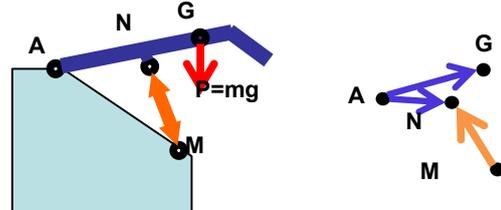


Figure 8: Power Lift Gate Topological Graph.

We use the kinematic joints diagram and the vectorial constraints between characteristic points of previous SysML diagrams to describe the topological structure. Indeed, each connector in the kinematic joints relations diagram represent a particular point, named “node” in the topological structure, and each link between two connectors give the nature of the kinematic screw, dual of its static screw. The automation of this process between SysML diagrams and our topological representation is made through the analysis of a xml/xmi generated

file from the SysML Kinematic Joints Relations Diagram (Figure 7). So, the boundary of the system is expressed by means of labels attached to each node (boundary) named (A, G, ...), like the SysML connectors, and to each branch (internal), all of them inherited from SysML diagrams.

Then, the topology has to be mathematically expressed (equation 1) using a connexion (or incidence) matrix named C and an algebraic graph that allows one to connect nodes and branches. The topological structure (graph) is overlaid with an algebraic structure. This global structure connects nodes and branches of the graph, and may include physical parameters which govern the behaviour of the system. This method has been thoroughly described in previous papers (Plateaux et al, 2007) (Plateaux et al, 2008).

$$\begin{array}{c} \text{Branches} \\ \text{(internal)} \end{array} \begin{array}{c} \left( \begin{array}{c} \text{AN} \\ \text{MN} \\ \text{AG} \end{array} \right) \end{array} \xrightarrow{C=(-1)} \begin{array}{c} \left[ \begin{array}{cccc} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{array} \right] \end{array} \begin{array}{c} \text{Nodes} \\ \text{(external)} \end{array} \begin{array}{c} \left( \begin{array}{c} A \\ M \\ N \\ G \end{array} \right) \end{array} \quad (1)$$

Thus, the transposed matrix  $C^T$  can be used to express (equation 2) the connection between internal and external mechanical forces and moments, defined with their associated static screws, with  $T_A$  standing for ‘‘screw of external mechanical action on point A’’ and  $T_{AN}$  standing for ‘‘screw of internal mechanical action on AN structure’’:

$$\begin{array}{c} \left( \begin{array}{c} T_{AN} \\ T_{MN} \\ T_{AG} \end{array} \right) \end{array} \xrightarrow{C^T=(-1)} \begin{array}{c} \left[ \begin{array}{ccc} -1 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \end{array} \begin{array}{c} \left( \begin{array}{c} T_A \\ T_M \\ T_N \\ T_G \end{array} \right) \end{array} = \begin{array}{c} \left( \begin{array}{c} T_{AN} + T_{AG} \\ T_{MN} \\ -T_{AN} - T_{MN} \\ -T_{AG} \end{array} \right) \end{array} \quad (2)$$

As a result, an equations system (3) is obtained, with the decomposition of screws in 4 force equations and in 4 moment equations expressed in the arbitrarily chosen point A:

$$\begin{array}{c} \left( \begin{array}{c} \{R_{AN}\}_A \\ \{M_{AN}\}_A \\ \{R_{MN}\}_A \\ \{M_{MN}\}_A \\ \{R_{AG}\}_A \\ \{M_{AG}\}_A \end{array} \right) \end{array} \xrightarrow{C^T=(-1)} \begin{array}{c} \left[ \begin{array}{ccc} -1 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \end{array} \begin{array}{c} \left( \begin{array}{c} R_A = R_{AN} + R_{AG} \\ R_M = R_{MN} \\ R_N = -R_{AN} - R_{MN} \\ R_G = -R_{AG} \\ M_A = M_{AN} + M_{AG} \\ M_M = M_{MN} \\ M_N = -M_{AN} - M_{MN} \\ M_G = -M_{AG} \end{array} \right) \end{array} \quad (3)$$

The equations system is solved and the static equilibrium of the lift gate system is expressed.

### 3.3 Computational Support for the Exploration of the Solution Space based on Constraint Programming and Interval Analysis

Special emphasis is also placed on interval-based computational methods (Jaulin et al, 2001) allowing one to explore exhaustively the search space resulting from a declarative statement of constraints (Yannou et al, 2003). Given the previous high level vector model linked to a given topology, formal calculus and causal ordering based on bipartite graphs theory (Duff, 1981) (Pothen and Chin-Ju, 1990) can be used to avoid part of the tedious work consisting in giving the mathematical expressions of some constraints as required to run dedicated solvers. The use of interval computations within a constraint programming paradigm (Blick et al, 2001) also provides a computational support to quantify uncertainties and to detect inconsistencies. From a methodological point of view, the refinement inherent to the design process is underlined.

A Constraint Satisfaction Problem (CSP) is usually defined by  $(X, D, C)$  where  $X = \{x_1, x_2, \dots, x_n\}$  is a set of variables,  $D = \{d_1, d_2, \dots, d_n\}$  is a set of domains such that  $\forall i \in \{1, \dots, n\}, x_i \in d_i$ , and  $C = \{C_1, \dots, C_m\}$  is a set of constraints depending on the variables in  $X$ . Each constraint includes information related to constraining the values for one or more variables. When continuous variables are considered, the use of interval analysis techniques naturally arises in order to represent the domains. Those methods make it possible to explicitly take uncertainties (in the sense of deterministic imprecision rather than probabilistic variability) into account in the preliminary design process. The use of an interval CSP solver (here, RealPaver) (Granvilliers, 2003) allows an exhaustive search within the search space  $D$  which is partitioned into three sets,  $D = D_0 \cup D_1 \cup D_2$ , the latter two being described by a box paving:  $D_0$  is a sub-domains of  $D$  where the constraints are never satisfied;  $D_1$  is a sub-domains of  $D$  where the constraints are always satisfied;  $D_2$  is a sub-domains of  $D$  where the satisfiability of the constraints has not been decided yet according to some stopping criterion (precision, for instance).

From an engineering design point of view, the variables in  $X$  can be a set of design parameters, the domains in  $D$  can be used to define the range of the search space of interest, and the constraints in  $C$  can be concurrently stated by several engineers in any order. Such a declarative modelling is a significant advantage of the CSP paradigm throughout the life

cycle of a Computer Aided Engineering (CAE) application (Raphael and Smith, 2003).

From a methodological point of view, the refinement inherent to the design process can be supported as follows: the poor initial knowledge results in a small number of constraints with few variables belonging to rather large intervals; then, the sequence of assumptions, trials and evaluations constituting the heart of an iteration within the design refinement loop allows the engineers to acquire knowledge, to organize it, and to gradually converge toward what will become the detailed solution (Aughenbaugh and Paredis, 2006).

In this paper, our case study is restricted to a few design parameters and focuses on the equilibrium requirement for the power lift gate. The design parameters are  $X = [x_{MB}, y_{MB}, x_{NL}, y_{NL}]$  i.e. the 2D coordinates of the electric cylinder fixation points  $M$  (on the car body) and  $N$  (on the lift gate). The equilibrium requirement is related to four constraints previously identified in the analysis based on SysML:

$C_{\Delta F}$ : “The additional force value  $\Delta F$  required to maintain the lift gate static equilibrium is inferior to some threshold level ( $\Delta F_{max}$ )”.  $\Delta F$  refers here to the force  $\Delta F$  defined as  $F_{cyl} = F_{spring} + \Delta F$ , where  $F_{cyl}$  is the cylinder force required to maintain the static equilibrium and where  $F_{spring}$  is the force of the spring within the power cylinder used to reduce the power of the electrical motor;

$C_L$ : “The electric cylinder length  $L$  is within the interval  $[L_{min}, L_{max}]$  related to the aperture angle of the lift gate”;

$C_M$ : “The car body fixation point  $M$  is within a specified area”;

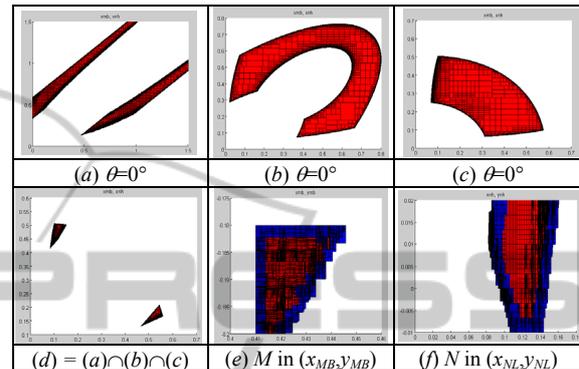
$C_N$ : “The lift gate fixation point  $N$  is within a specified area”.

Following formal computations guided using causal ordering techniques, all the constraints are expressed as functions of the design parameters, and the text file required as input of the interval CSP solver is so obtained. The preliminary design of the power lift gate then consists in using the interval solver outputs to understand the influence of the opening angle on the position of fixation points and to perform a (possibly iterative) refinement by selecting an area in the solution space.

Table 1 (a-d) illustrates the influence of the opening angle on the solution set. This corresponds to a preliminary study before an exhaustive search for all the opening angles. Focusing on an area in the search space corresponds to the refinement related to the preliminary design process. The reduced search

area allows a more precise exploration while preserving a reasonable computation time. The proposed refinement iteration aims at being reproduced all along the preliminary design process in order to converge toward the solution set, table 1 (e,f), that will be kept to initiate the detailed design of the power lift gate.

Table 1: Interval CSP Outputs.



## 4 CONCLUSIONS

We have presented a proven solution for a global multi-domain constraints-based preliminary design supported by a robust design methodology in conformance with System Engineering Standards. Based on three interactive design environments and illustrated by a mechatronic example, it demonstrates the power of collaborative engineering in model-based design. As a result, Model-Based System Engineering simplifies the development of mechatronic and other multi-domain systems by providing a common approach for design and communication across different engineering disciplines.

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