ACCUMULATING EVIDENCE BASED ON ESTIMATION THEORY AND HUMAN PSYCHOLOGY

Michael E. Farmer
University of Michigan-Flint, 302 East Kearsley St., Flint, MI, U.S.A.

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Abstract: The traditional D-S conditioning is based on a collection of 'experts' inputting their evidence and accumulating the beliefs. Researchers have often adopted this same mechanism for integrating evidence from single sources of evidence over time, such as seen in sensor networks. One issue with this approach is that the order of inputs does not matter. While this is sensible for a collection of experts we propose that it is not suitable for a single input providing streams of evidence. Likewise research in psychology show order of integration of evidence does matter, and depending on the application humans have a preference for recency or primacy. Estimation theory provides frameworks for analyzing data over time, and recently some researchers have proposed integrating evidence in an estimation-inspired manner. In light of this we propose a Kalman-filter based approach for integrating single sensor evidence over time where the evidence conflict plays the role of system noise in adapting the filter gain.

1 INTRODUCTION

There has been significant research in the artificial intelligence (AI) community with respect to evidential reasoning and updating beliefs, with the most common approaches being Bayesian and Dempster-Shafer. Likewise the field of cognitive psychology has also been heavily involved in research in belief updating. There are three key areas where the existing AI research in evidential reasoning has differed significantly from the findings in human cognition, (i) order effects, (ii) evidence impact reduction in long evidence streams, and (iii) evidence evaluation versus estimation.

As sensor networks become more prevalent, it is time to relook at the mechanisms of evidence accumulation over time to support belief revision and updating. In sensor networks inputs from multiple sensors over time are integrated resulting in both a lateral integration (sensor fusion across sensors) as well as a longitudinal integration (data integration over time for each sensor). This spatial-temporal combining of evidence leads to issues which have not been fully addressed to date and relate back to the three issues relating to human evidential reasoning.

The first issue, namely that of order independence, is one of the key tenets of the more common theories, and it is also a critical foundation of the AGM framework of logic and Dempster Shafer. Approaches such as Jeffrey's conditioning have been shown to have order dependence but it is an accident of the mathematics and not a conscious goal of the algorithm (Garber, 1980). This order independence is not however found to exist in human reasoning, and there are definite situations when order-effects are demonstrated in humans (Hogarth and Einhorn, 1992). as well as being desirable in sensor networks.

The second issue is that of how to integrate evidence over long streams of data. In human cognition 'as information accumulates, beliefs are expected to become less sensitive to the impact of new information because this represents an increasingly small proportion of evidence already processed" (Hogarth and Einhorn, 1992). This contrasts sharply with the common approaches in AI of Bayes and Dempster-Shafer which weight the entire history equally with the most recent input.

The third issue related to human cognition is based on what cognitive researchers call. In human evidential reasoning there are two approaches, evaluation and estimation. Evaluation is a process in which the reasoning tasks tend to be formulated into a true-false framework. Estimation tasks, however, are additive in nature and assess 'how

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much evidence is to be allocated to a belief' encoding (Hogarth and Einhorn, 1992). Cognitive researchers have found that 'estimation may be more frequently used than evaluation', whereas artificial reasoning systems tend to adopt evaluative approaches. In this paper we propose a framework for evidence accumulation from a single sensor over time devised from first-principles of Kalman filtering that resolves these differences between human subject behavior and artificial reasoning systems.

2 BELIEF REVISION IN HUMAN PSYCHOLOGY STUDIES

There is considerable research showing the importance of order in human reasoning. Hogart and Einhorn (Hogarth and Einhorn, 1992). suggest that the recency effect is more important when messages are inconsistent. This result was also verified by Wang et al. for an interesting Combat Information Center application (Wang et al., 1999). Baratgin and Politzer recently address the issue of updating (dynamic environment) in human decision making and confirm by reviewing numerous studies that "a message has greater contextual effects when it is learnt in the last position" (Baratgin and Politzer, 2007).

Likewise the repetition of evidence in human reasoning has also been shown to be important where repeated repetition of a message changes the opinions of the test subjects (Baratgin and Politzer, 2007). Hogarth and Einhorn modeled human behavior through anchoring and adjustment defined by (Hogarth and Einhorn, 1992):

$$S_k = S_{k-1} + \alpha S_{k-1}\left[s(x_k) - R\right] \quad \text{for } s(x_k) \le R, \tag{1}$$

and

$$S_k = S_{k-1} + \beta(1 - S_{k-1})\left[s(x_k) - R\right] \quad \text{for } s(x_k) > R, \tag{2}$$

where S_k is the current level of belief, S_{k-1} is the belief at the last update, $s(x_k)$ is the new evidence input into the system, and α and β are weights to enforce recency or primacy. This model also supports inertia through the mechanism where "as information accumulates and as people become more firmly committed to their beliefs, values of α and β would decline in a long series of evidence items" (Hogarth and Einhorn, 1992).

While the goal of a system which integrates evidence over extended time periods may not be to replicate these effects in human behavior, these results should provide cause for us to consider whether a robust evidential integration paradigm should have the flexibility to mathematically support them.

3 EXISTING APPROACHES TO EVIDENCE COMBINATION

There are numerous approaches in the literature for evidence combining and Hawthorne observes: "The issue regarding which kind of factor should be taken as primitive [for sequential evidence accumulation] is not a purely mathematical issue. It is an epistemological, or an empirical, or a pragmatic issue" (Hawthorne, 2004). We will briefly review some of them.

3.1 Basic Approaches for Evidence Combination

The most common approach for evidence combining is Bayes' conditioning when new evidence, E, becomes 'known' to be true (Shafer, 1976):

$$p_2(A) = p_1(A \mid E) \tag{3}$$

Another rule of belief revision based on Bayes conditioning include Jeffrey's which directly extends Bayes by allowing for uncertainty in the options for E (Garber, 1980):

$$p_2(A) = \sum_{e \in \Omega} p_1(A \mid E) p_{new}(E) \tag{4}$$

Dempster provides an alternative to conditioning through his rule of combination (Shafer, 1976):

$$m_1 \oplus m_2(Z) = \frac{\sum_{X \cap Y = Z} m_1(X).m_2(Y)}{1 - \sum_{X \cap Y = \phi} m_1(X).m_2(Y)} \tag{5}$$

where X, Y and Z are the elements of the power set.

Note in all of these approaches there are no mechanisms for specifically dealing with integrating temporal streams of data and in particular discounting information to support either recency/primacy or long term inertia.

3.2 Extensions of the Basic Theories to Temporal Evidence Streams

Dempster's Rule of Combination has been extended to process a temporal stream of sensor inputs by viewing *$m_2(a)$* "not as sensor S_j's observation, but instead as the previously combined observations

(Wu, Siegel, & Ablay, 2003):

$$m_1 \oplus m_2(Z) = \frac{\sum_{X \cap Y = Z} w_1 \cdot m_1(X) w_2 \cdot m_2(Y)}{1 - \sum_{X \cap Y = \phi} w_1 \cdot m_1(X) w_2 \cdot m_2(Y)}, \tag{6}$$

where the weights are computed according to:

$$w_i(t) = \sum_{n=0}^{\infty} c_i(t - n \cdot \Delta T) \cdot p^n, \tag{7}$$

and $c_i(t)$ is either 0 or 1 depending on whether the sensor estimate is correct or not, ΔT is the incoming data sampling rate, and p controls the decay rate of samples being considered.

Hawthorne discusses a collection of approaches to sequential updating based on the basic Jeffrey's updating model in Equation (4) (Hawthorne, 2004). He begins with a basic sequential update model that is amnestic in that it completely replaces any past evidence with the most recently gathered. He then discusses a likelihood-ratio update model which provides order independence, however this does not fit well with the objectives of providing a mathematical means for providing human-inspired evidential reasoning.

3.3 Alternative Approaches based on Estimation Theory

Integration of temporal streams of data sources typically employs estimation techniques. The basic processing requirements for any estimation system are highlighted in Figure 1 (Blackman, 1986). For the purpose of this paper we will focus predominantly on Filtering, Prediction and Gain Computation.

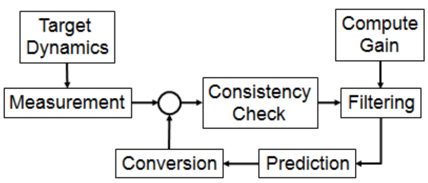

Figure 1: General Processing Flow for Estimation System.

For the Filtering task in Figure 1 the simplest filter is the moving average filter (Nakamura, Loureiro, & Frery, 2007):

$$\hat{x}(k) = \frac{1}{M} \sum_{i=0}^{M-1} z(k-i), \tag{8}$$

where z is the incoming belief stream and $x(k)$ is the output beliefs. This moving average filter can be extended to the Finite Impulse Response (FIR) filter where the sum is replaced by a *weighted* sum with coefficients chosen to control the frequency response of the filter. Dewasurendra, et al developed filters that integrate beliefs based on their frequency of occurrence, using (Dewasurendra et al., 2007):

$$Bel_k(B) = \sum_{i=1}^{N} \alpha_i \cdot Bel_{k-1}(B) + \sum_{i=1}^{N} \beta_i \cdot Bel_{k-i}(B \mid A) \tag{9}$$

and

$$\sum_{i=1}^{N} \alpha_i + \sum_{i=1}^{N} \beta_i = 1. \tag{10}$$

The α and β are weights which are defined to produce a desired transfer function for detecting the frequency behavior of the evidence being analyzed.

The simplest form of estimator which also incorporates the Prediction processing block in Figure 1 is the $\alpha - \beta$ tracker which is defined as (Blackman, 1986):

$$x_{est}(k) = x_{pred}(k) + \alpha \cdot \left[x_{obs}(k) - x_{pred}(k)\right] \tag{11}$$

$$v_{est}(k) = v_{est}(k-1) + \frac{\beta}{T} \cdot \left[x_{obs}(k) - x_{pred}(k)\right] \tag{12}$$

$$x_{pred}(k+1) = x_{est}(k) + T \cdot v_{est}(k), \tag{13}$$

where $x_{est}(k)$ is the estimate for the state x, $x_{pred}(k)$ is the prediction of the state at time k, $x_{obs}(k)$ is the current observation, $v_{est}(k)$ is the estimate for the velocity, and α and β are the fixed filter gains.

A framework for sequential belief updating that is based on fixed weights, analogous to the $\alpha - \beta$ tracker, has been developed by Premaratne, et al. (2007), using:

$$Bel_{k+1}(B_1) = \alpha_k \cdot Bel_k(B_1) + \beta_k \cdot Bel_k(B_1 \mid A), \tag{14}$$

where the weights are constrained by $\alpha_k + \beta_k = 1$ and $x_{pred}(k)$ corresponds to $Bel_k(B_1)$ and $x_{obs}(k)$ corresponds to $Bel_k(B_1 \mid A)$. Equation (14) exhibits some of the behavior we tend to expect in a signal processing framework, namely: "…when encountered with the same streaming information continuously, the belief converges to a value decided solely by this incoming information" (Premaratne, et al., 2007). The weight selection controls the relative importance of new versus historical evidence, thereby also providing support for primacy and recency.

The Kalman filter is the next step in complexity for estimation algorithms; however, to date no one has implemented a quantitative evidence accumulation system strictly based on the principles of Kalman filtering. Benferhat, et al. developed an analogy to the Kalman filter for *qualitative* belief revision within Possibility Theory that (Benferhet et al., 2000). In the following section we will develop a complete Kalman filter formalism for evidence accumulation.

4 PROPOSED STRUCTURE FOR A KALMAN FILTER FOR EVIDENCE FILTERING

Let us consider the estimate of a basic evidential state at some time increment *k*, to be $\hat{x}_E(k)$ and an incoming measure of such a state to be $x_{obs}(k)$. Assume an initial estimate of the uncertainty in that state is defined to be: σ_E^2 and the uncertainty in the measurements to be σ_M^2. For simplicity we will assume the state transition matrix and the measurement matrix are the identity matrices.

The basic estimate update equation would be (Gelb, 1974):

$$\hat{x}_E(k) = \hat{x}_E(k-1) + G(k) \cdot [x_{obs}(k) - \hat{x}_E(k-1)]. \quad (15)$$

Recall in the Hogarth and Einhorn model defined in Equations (1) and (2), there was a corresponding term in the brackets which accounted for the difference between the incoming evidence and a reference which served as the anchor. In a Kalman-based approach, the reference is the integrated belief state.

After each update of the filter we compute a new estimate for the estimate uncertainty to be (Gelb, 1974):

$$\hat{\sigma}_E^2(k) = (1 - G(k)) \cdot \hat{\sigma}_E^2(k-1), \quad (16)$$

where the gain term is simply (Gelb, 1974):

$$G(k) = \frac{\hat{\sigma}_E^2(k)}{\hat{\sigma}_E^2(k) + \sigma_M^2(k)}. \quad (17)$$

Equations (15) through (17) represent the simplest form possible for the Kalman filter. Note however, that the gain term defined in Equation (17) does not include the desired System Covariance which captures the uncertainty in the system model. The measurement and system noise uncertainty terms parallel the two fundamental types of uncertainty: Aleatory (relating to traditional issues of variability) and Epistemic which relates to ignorance and uncertainty in the state of knowledge) (Sentz, 2002.).

To add this term we need to note that if there is an added system error *h* introduced at time *k*, and then the state for that time would actually be (Gelb, 1974):

$$x_E(k) = x_E(k-1) + h(k). \quad (18)$$

This would then result in the covariance for the state estimate to be (Gelb, 1974):

$$\hat{\sigma}_E^2(k) = \hat{\sigma}_E^2(k) + \sigma_h^2(k). \quad (19)$$

The behavior of the gain and the resultant behavior of the filtering system relative to the *system* noise behavior, σ_h^2, is not obvious, but referring to Figure 2 we see that in the traditional Kalman formalism increases in the system noise result in increases in gain. This behavior is desirable from a traditional physical state filtering problem, since if there is a change in the target behavior that we did not anticipate in our model, we will need to increase the gain to be able to maintain a track on the object. There are two key questions we must address at this point when extending the Kalman filter to evidence filtering: (i) what behavior do we want from an evidential filtering viewpoint, and (ii) what does the term $\sigma_h^2(k)$ correspond to in the evidential reasoning domain.

To address these questions we will begin with the comment from Schubert: “A high degree of conflict is seen if there is a representation error in the frame of discernment; while a small conflict may be the result of measuring error” (Schubert, 2008). Recall the system noise represents the uncertainty in the representation. Therefore it is natural to consider the conflict between two sources of evidence as the parameter to serve as the system noise measure. The conflict in evidence is (Shafer, 1976):

$$\mathrm{K}_{12} = \sum_{X \cap Y = \phi} m_1(X).m_2(Y), \quad (20)$$

We propose there should be a decrease in the gain of the system in the face of conflict to allow us to evolve the system of beliefs to provide an estimation analog to traditional Demspter-Shafer where conflict is simply removed and the masses re-normalized. If we substitute $\sigma_h^2 = 1 - K_{12}$ in Equation (19) we get:

$$\hat{\sigma}_E^2(k) = \hat{\sigma}_E^2(k) + 1 - K_{12}(k). \quad (21)$$

The behavior will be as shown in Figure 3, increases in conflict result in a reduction in gain which means; evidence will be added to the system at a reduced level rather than discarded. As this conflicting evidence continues to come into the system (assuming it is a sustained change in environment), it will become less conflicting as the masses evolve, and the gain will increase as the belief system evolves.

Since one goal of the evidential filter is supporting desired time order preference we propose

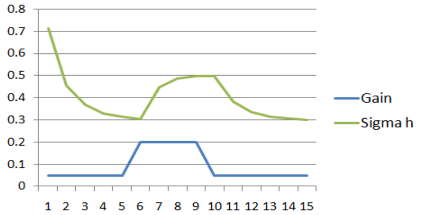

Figure 2: Traditional Kalman gain due to increase in σ_h^2.

introducing a term we call the recency factor, α_{rec}, to Equation (21) to bias the state estimate towards recency or primacy:

$$\hat{\sigma}_E^2(k) = \hat{\sigma}_E^2(k) + \alpha_{rec} \cdot (1 - K_{12}(k)). \qquad (22)$$

Our rationale for inserting the term into equation (21) is based on recalling it is derived from the addition of an offset into the state estimate (recall Equation (18)), and we are adding to this term with bias towards past or current measures based on the desired behavior of the filter for a given application. Figure 4 provides a set of curves for how the gain behaves over time for varying recency factors. Additionally Figure 5 shows the behavior of the gain for various recency factors when there is a sudden increase in conflict of the incoming measures with the existing state, which support our objectives: (i) higher gain for higher recency, and (ii) reduced gain during periods of conflict.

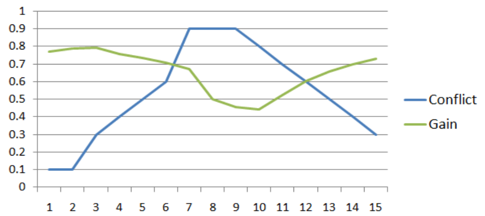

Figure 3: Effect on gain with increase in conflict.

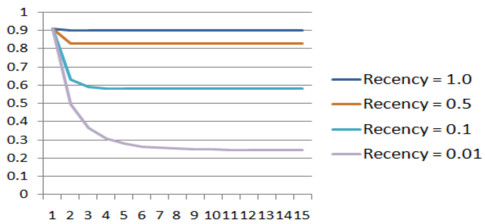

Figure 4: Gain versus recency factor, α_{rec}.

There are numerous candidates to serve the role of σ_M^2 in gain term defined in Equation (17). Recall in the traditional Kalman filter, the measurement variance is based on the signal-to-noise ratio of the

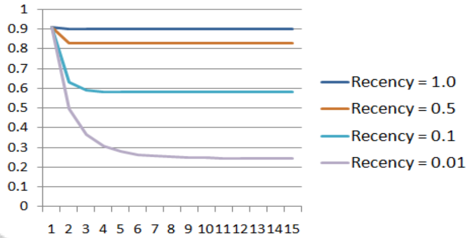

Figure 5: Gain versus recency factors, α_{rec} during sudden increase in conflict.

detected signals. For evidence filtering the most logical choice is the variance in the classification measure at each instance.

5 CONCLUSIONS AND FUTURE WORK

We have reviewed results from evidence accumulation in human subjects and discovered there are three key issues when compared to evidence accumulation in artificial intelligence, namely: (i) order effects, (ii) evidence impact reduction in long evidence streams, and (iii) evidence evaluation versus estimation. Based on this observation we proposed there is a need for an alternative estimation theoretic approach to accumulating evidence over long time streams from single sensors. We then reviewed various approaches for evidence accumulation, and showed the parallelisms between existing evidence filtering approaches to traditional FIR filters and $\alpha - \beta$ trackers and noted there are no systems strictly based on the Kalman filter. We then developed a formalism for evidence accumulation based on the principles of the Kalman filter. We then related the key Kalman noise terms, namely the measurement noise and the system noise to the ideas of aleatory and epistemic uncertainty in evidence accumulation.

As the concept of ignorance corresponds to the epistemic uncertainty, we propose that the conflict in the incoming evidential states can be used as a means of estimating the system noise. The measurement noise for systems which provide classification outputs can be estimated the classification system variance. Future work will be directed at further studying the ability to use conflict as a measure of system noise and to execute these algorithms on a typical evidence accumulation problem.

REFERENCES

Baratgin, J., & Politzer, G., 2007. The psychology of dynamic probability judgment: order effect, normative theories, and experimental methodology. *Mind & Society*. Springer.

Benferhet, S., Dubois, D., & Prade, H., 2000. *Kalman-like filtering in a qualitative setting.* INRIA.

Blackman, S. S., 1986. *Multiple-target Tracking with Radar Applications.* Artech House.

Dewasurendra, D., Bauer, P. H., & Premaratne, K., 2007. Evidence filtering. *IEEE Trans. On Signal Processing*. IEEE.

Dubois, D., & Prade, H., 1997. A synthetic view of belief revision with uncertain inputs in a framework of possibility theory. *International Journal of Approximate Reasoning*. Elsevier.

Garber, D., 1980. Discussion: Field and Jeffrey conditionalization. *Philosophy of Science*, Philosophy of Science Association.

Gelb, A., 1974. *Applied Optimal Estimation.* Cambridge: MIT Press.

Hawthorne, J., 2004. Three models of sequential belief updating on uncertain evidence. *Journal of Philosophical Logic*. Springer.

Hogarth, R. M., & Einhorn, H. J., 1992. Order effects in belief updating: The belief-adjustment model. *Cognitive Psychology*. Elsevier.

Liu, W., & Hong, J., 2000. Re-investigating Dempster's idea on evidence combination. *Knowledge and Information Systems*. Springer.

Nakamura, E. F., Loureiro, A. A., & Frery, A. C., 2007. Information fusion for wireless sensor networks: Methods, models and classifications. *ACM Computing Surveys*. ACM.

Premaratne, K., Dewasurendra, D. A., & Bauer, P. H., 2007. Evidence combination in an environment with heterogeneous sources. *IEEE. Trans on Systems, Man, and Cybernetics-Part A: Systems and Humans*. IEEE.

Schubert, J., 2008. Conflict management in Dempster-Shafer theory by sequential discounting using the degree of falsity. *Procs. of Intl. Conf. on Information Processing and Management of Uncertainty in Knowledge-based Systems*. Elsevier.

Sentz, K., 2002. *Combination of evidence in Dempster-Shafer theory - SAND 2002-0835*. SANDIA.

Shafer, G., 1976. *A Mathematical Theory of Evidence.* Princeton: Princeton University Press.

Wang, H., Zhang, J., & Johnson, T. R., 1999. Order effects in human belief revision. *Proceedings of the 1999 Cognitive Science Society Conference.* Cognitive Science Society.

Wu, H., Siegel, M., & Ablay, S., 2003. Sensor fusion using Dempster-Shafer theory II: Static weighting and Kalman filter-like dynamic weighting. *Procs. of Instrumentation and Measurement Technology Conference*. IEEE.

Zhao, J., & Osherson, D., 2010. Updating beliefs in light of uncertain evidence: Descriptive assessment of Jeffrey's rule. *Thinking & Reasoning*. Taylor & Francis Group.