2-DOF PI-FUZZY CONTROLLERS FOR A MAGNETIC LEVITATION SYSTEM

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Abstract: This paper treats aspects concerning the design of two-degree-of-freedom (2-DOF) PI-fuzzy controllers dedicated to the position control of magnetic levitation system. 2-DOF Mamdani and Takagi-Sugeno PI-fuzzy controller structures based on the fuzzification of some linear blocks in the 2-DOF linear controller structures are discussed. A design approach of three new cascade control system structures is offered. The design approach carries out first the pole placement design of the inner state feedback control system. The 2-DOF PI-fuzzy controllers in the outer loops are next designed to merge separately designed linear PI controllers accounting for the linearization of the process model at certain operating points. Samples of real-time experimental results related to a laboratory equipment are given to validate the new fuzzy control system structures and the design approach.

1 INTRODUCTION

The two-degree-of-freedom (2-DOF) controllers are successful with respect to the one-degree-of-freedom ones as they ensure very good control system performance indices (overshoot, settling time, etc.) defined in the performance specifications related to reference input tracking and disturbance input regulation (Åström, 1995; Araki, 2003; Bascetta, 2008; Precup et al., 2009). But the main drawback of the control systems (CS) with 2-DOF controllers is that the overshoot reduction is paid by slower responses for reference input variations.

The systematic design and stability of fuzzy CS have received much attention recently and many significant results have been reported recently (Gusikhin et al., 2007; Lam, 2009; Chohra et al., 2010; Linda and Manic, 2011; Liu et al., 2011). The fuzzy logic is inserted in 2-DOF CS structures to ensure the further performance improvement. A 2-DOF controller which involves a one-step-ahead fuzzy prefilter in the feed-forward loop and a PI-fuzzy controller in the feedback loop meant for the foot trajectory tracking control of a hydraulically actuated hexapod robot is discussed in (Barai and Nonami, 2007). A 2-DOF CS that consists of a conventional foreword internal model controller and a feedback fuzzy controller for an electro-hydraulic servo system is suggested in (Li and Xiong, 2008). A simulated 2-DOF Mamdani fuzzy controller for automotive semi-active suspension control is presented in (Bei, 2009). Different structures of 2-DOF Mamdani and Takagi-Sugeno (TS) PI(D)-

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fuzzy controllers applied to speed and position control of servo systems are given in (Preitl et al., 2010). Several nonlinear control approaches including fuzzy control have been proposed recently to deal with magnetic levitation systems. They include the Lyapunov-based guaranteed stability (Shameli et al., 2007), adaptive robust nonlinear control (Wu and Hu, 2009) and fuzzy control (Dragoş et al., 2010).

This paper suggests twofold contributions. First, 2-DOF PI-fuzzy controllers applicable as both Mamdani and TS PI-fuzzy controller structures are offered. Second, three cascade CS structures for a magnetic levitation system laboratory equipment (MLSLE) are proposed. The new CS structures employ an inner state feedback CS and 2-DOF PI-fuzzy controllers in the outer loops. The design approach for these structures starts with the pole placement design of the inner state feedback CS. The 2-DOF PI-fuzzy controllers in the outer loops are next designed to merge separately designed linear PI controllers accounting for the linearization of the process model at certain operating points.

Our new contributions are important and advantageous with respect to other recent solutions analyzed in the literature because they ensure very good CS performance with respect to reference and disturbance inputs. In addition, our fuzzy control solutions belong to the class of low cost solutions as the design approaches are relatively simple and the structures are easily to implement.

This paper is organized as follows. The 2-DOF PI-fuzzy controller structures are presented in Section 2. The process models related to the MLSLE are discussed in Section 3. The design of the new CSs and samples of real-time experimental results are given in Section 4. The concluding remarks are highlighted in Section 5.

2 2-DOF PI-Fuzzy CONTROLLER STRUCTURES

Three frequently used 2-DOF linear CS structures focused on the linear PI(D) controller with the transfer function (t.f.) $C(s)$ are presented in Figure 1 as the set-point filter structure (a), the feedforward structure (b) and the feedback structure (c) (Araki and Taguchi, 2003; Precup et al., 2009). The main variables in Figure 1 are r – the reference input, r_1 – the filtered set-point, y – the controlled output, $e=r-y$ or $e=r_1-y$ – the control error, u – the control signal, and d_1, d_2 and d_3 – the three types of disturbance inputs.

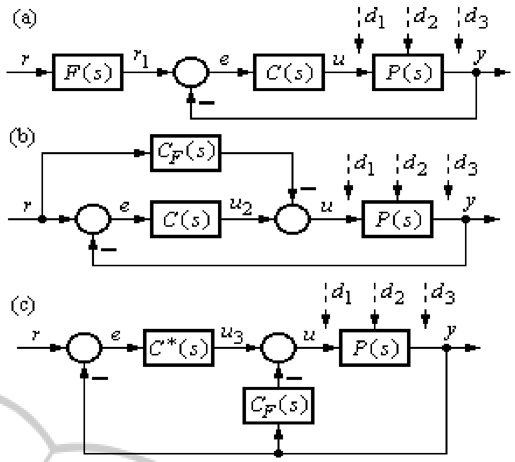

Figure 1: 2-DOF linear CS structures.

In Figure 1, $P(s)$ indicates the t.f. of the plant which is linear in this case but generally nonlinear in 2-DOF PI(D)-fuzzy CS structures. The t.f $C(s)$ of PI controller is

$$C(s) = k_c(1 + T_c s)/s\,, \tag{1}$$

where k_c is the controller gain and T_c is the integral time constant.

The expressions of the t.f. of the reference input filter, referred to also as set-point filter, in Figure 1 (a) and of the t.f. of the rest of two blocks in Figure 1 (b) and (c) are

$$\begin{aligned} &F(s) = (1+T_F s)/(1+T_c s), T_F \geq 0 \\ &C_F(s) = k_c(T_c - T_F), \\ &C^*(s) = k_c(1+T_F s)/s. \end{aligned} \quad , \tag{2}$$

where T_F is the filter time constant.

The 2-DOF CS structures presented in Figure 1 (a), (b) and (c) are equivalent as they are characterized by the same controller t.f. in the linear case:

$$\begin{aligned} &G_{u,y}(s) = u(s)/y(s) = -k_c(1+T_c s)/s, \\ &G_{u,r}(s) = u(s)/r(s) = k_c(1+T_F s)/s. \end{aligned} \tag{3}$$

They will be fuzzified as follows aiming the CS performance enhancement.

An alternative to the operational representation of the t.f.s of the three 2-DOF controllers is based on in the definition of the generic PI block with the t.f.

$$G^{\tau}(s) = k_c(1+\tau s)/s, \tau \geq 0\,. \tag{4}$$

The t.f. defined in (4) is used in different particular forms to express the components with dynamics in Figure 1 to be fuzzified. The fuzzification of the block with the t.f. $C(s)$ in the set-point filter structure and in the feedforward structure is based on the following relationship that results from (1) and (4):

$$C(s) = G^{T_c}(s)\,. \tag{5}$$

The fuzzification of the block with the t.f. $C^*(s)$ is supported by (2) and (5) that lead to

$$C^*(s) = G^{T_F}(s)\,. \tag{6}$$

The fuzzification of the generic PI block with the t.f. defined in (4) starts with its discretization. The fuzzification results in the fuzzy block FB-τ presented in Figure 2 (a), where FB is the Mamdani or the TS fuzzy block without dynamics (Preitl et al., 2010). The block FB is based on the input membership functions with the shapes and parameters defined in Figure 2 (b). These input membership functions are used in the Mamdani fuzzy block and in the TS fuzzy block as well; the output membership functions are defined only for the Mamdani fuzzy block. The low cost aim was accounted for to set three input membership functions for each FB input.

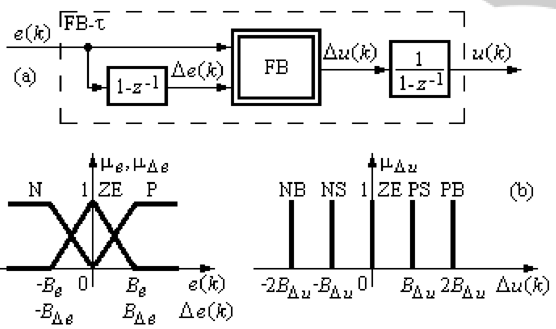

Figure 2: Structure (a) and membership functions (b) of block FB-τ.

The tuning parameters of the block FB-τ (shown in Figure 2 (b)) are B_e, $B_{\Delta e}$ and $B_{\Delta u}$ for the Mamdani fuzzy block FB-τ, and B_e and $B_{\Delta e}$ for the TS fuzzy block FB-τ. The discretization involves setting the sampling period T_s in accordance with the requirements of quasi-continuous digital control. Tustin's method is next applied to discretize the continuous-time linear PI block with the t.f. $G^{\tau}(s)$ resulting in the recurrent equation of the incremental digital generic PI block and on its parameters given in (7) and (8), respectively:

$$\Delta u(k) = K_P[\Delta e(k) + \mu\, e(k)]\,, \tag{7}$$

$$K_P = k_c\,(\tau - T_s/2),\, \mu = 2T_s/(2\tau - T_s)\,. \tag{8}$$

Mamdani's MAX-MIN composition is used in the inference engine of the Mamdani fuzzy block FB-τ, and the centre of gravity method is used in the defuzzification module of FB-τ. The SUM and PROD operators are used in the inference engine of the TS fuzzy block FB-τ, and the weighted average method is used in the defuzzification module of FB-τ. The rule base of the TS fuzzy block FB-τ is (Preitl et al., 2010):

$$\begin{aligned}
&\text{Rule 1: IF } e(k) \text{ IS N AND } \Delta e(k) \text{ IS P}\\
&\qquad \text{THEN } \Delta u(k) = K_P^1[\Delta e(k) + \mu^1 e(k)],\\
&\vdots\\
&\text{Rule 9: IF } e(k) \text{ IS P AND } \Delta e(k) \text{ IS N}\\
&\qquad \text{THEN } \Delta u(k) = K_P^9[\Delta e(k) + \mu^9 e(k)].
\end{aligned} \tag{9}$$

The superscripts presented in (9) indicate the index of the certain rule. For complete rule bases as those presented in (9) the superscripts highlight the possibility to carry out the separate design of a maximum of nine linear PI controllers. The blocks FB-τ will behave like bumpless interpolators between these separately designed PI controllers as shown in the next section.

The unified structures of 2-DOF PI-fuzzy controllers are presented in Figure 3.

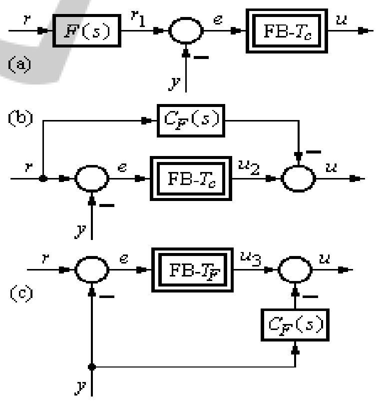

Figure 3: 2-DOF PI-fuzzy controller structures.

They are referred to as set-point filter 2-DOF PI-fuzzy controller (a), the feedforward 2-DOF PI-fuzzy controller (b) and the feedback 2-DOF PI-fuzzy controller (c). The linear blocks can be discretized to ensure the discrete-time treatment of all signals in the 2-DOF PI-fuzzy controller structures to increase the application areas.

3 MODELS OF MAGNETIC LEVITATION SYSTEM

The nonlinear state-space model of the MLSLE is

$$\begin{aligned}
&\dot{x}_1 = x_2,\\
&\dot{x}_2 = -F_{EM1}(x_1,x_3)/m + g + F_{EM2}(x_1,x_4)/m,\\
&\dot{x}_3 = (k_i u_1 + c_i - x_3)/f_i(x_1),\\
&\dot{x}_4 = (k_i u_2 + c_i - x_4)/f_i(x_d - x_1),\\
&y = x_1,
\end{aligned} \tag{10}$$

where x_1 is the sphere position $0 \le x_1 \le 0.016\ m$, x_2 is the sphere speed, x_3 and x_4, are the currents in the upper and lower electromagnetic coil, respectively $0.038\ A \le x_3, x_4 \le 2.38\ A$, u_1 and u_2 are the voltages applied to the upper and lower electromagnet, respectively $0.0049 \le u_1, u_2 \le 1$, g is the gravity acceleration, m is the sphere mass, y is the controlled output, x_d is the distance between electromagnets minus the sphere diameter, and the parameters k_i and c_i set the actuator dynamics. The control signal is applied to the upper electromagnet (EM1), $u=u_1$, and the disturbance input is applied to the lower electromagnet (EM2) $d=u_2$. The numerical values of the parameters are given in (Inteco, 2008).

We are carrying out the linearization of the nonlinear model (10) at several operating points $A^j(x_{10}, x_{2o}, x_3, x_{40})$ (with j – the index of the operating point) to meet the control objectives and also to offer low cost solutions. Accepting u_2=0 (the state variable x_4 is neglected but its effect is not) the following general linearized state-space mathematical model is employed in the design:

$$\begin{aligned}
\Delta\dot{\mathbf{x}} &= \mathbf{A}\,\Delta\mathbf{x} + \mathbf{b}\,\Delta u,\\
\Delta y &= \mathbf{c}^T \Delta\mathbf{x},
\end{aligned} \tag{11}$$

where $\Delta u = u - u_0$ and $\Delta y = y - y_0$ are the differences of the variables u and y with respect to their values at the operating point, u_0 and y_0, respectively, $\Delta\boldsymbol{x} = [\Delta x_1\ \Delta x_2\ \Delta x_3\ \Delta x_4]^T$ is the state vector, and the superscript T indicates the matrix transposition. For three operating points $A^1(0.007,0,0.3,0)$, $A^2(0.008,0,0.285,0)$, $A^3(0.009,0,0.6,0)$ the expressions of the matrices in (11) are

$$\begin{aligned}
&\mathbf{A}^1 = \begin{bmatrix} 0 & 1 & 0 \\ 244.8 & 0 & -9.5028 \\ 15056 & 0 & -149.6 \end{bmatrix}, \mathbf{b}^1 = \begin{bmatrix} 0 \\ 0 \\ 376.6 \end{bmatrix},\\
&(\mathbf{c}^T)^1 = [1 \quad 0 \quad 0],\\
&\mathbf{A}^2 = \begin{bmatrix} 0 & 1 & 0 \\ 186.8 & 0 & -7.6031 \\ 18125 & 0 & -186.3 \end{bmatrix}, \mathbf{b}^2 = \begin{bmatrix} 0 \\ 0 \\ 468.8 \end{bmatrix},\\
&(\mathbf{c}^T)^2 = [1 \quad 0 \quad 0],\\
&\mathbf{A}^3 = \begin{bmatrix} 0 & 1 & 0 \\ 132.8 & 0 & -3.9339 \\ 20224 & 0 & -186.3 \end{bmatrix}, \mathbf{b}^3 = \begin{bmatrix} 0 \\ 0 \\ 786.8 \end{bmatrix},\\
&(\mathbf{c}^T)^3 = [1 \quad 0 \quad 0].
\end{aligned} \tag{12}$$

4 CONTROLLER DESIGN AND EXPERIMENTAL RESULTS

The cascade CS structures are represented by the unified structure presented in Figure 4, where r_x is the reference input for the inner state feedback control loop, $\mathbf{k}^T$ is the state feedback gain matrix, and the MLSLE process includes the actuators and sensors dynamics.

The unified design approach dedicated to 2-DOF PI-fuzzy controllers consists of six design steps.

- Step I. Apply the pole placement method to the linearized state-space models (11) and obtain the state feedback gain matrix $k^T = [36 \quad 5 \quad 0.0075]$

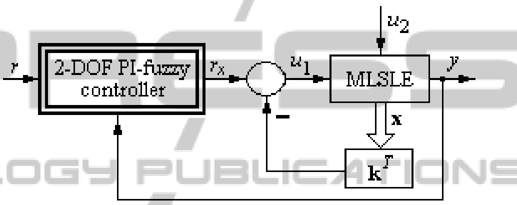

Figure 4: Unified cascade CS structure.

- Step II. Express the t.f.s of the inner state feedback control loops resulted from (12):

$$P^j(s) = \frac{k_P}{(1+T_1^j s)[1 + 2\xi^j T_\Sigma^j s + (T_\Sigma^j)^2 s^2]}, \quad j \in \{1,2,3\}. \tag{13}$$

- Step III. Apply a linear design method to tune the parameters of the 2-DOF linear PI controllers for the processes with the t.f.s (13).
- Step IV. Set the sampling period T_s, according to the requirements of quasi-continuous digital control, (T_s=0.01s in our case), take into account the zero-order hold, and discretize the continuous-time 2-DOF PI linear controllers resulting in the parameters

$$K_P^S < K_P^M < K_P^B, \mu^S < \mu^M < \mu^B. \tag{14}$$

- Step V. Set the parameter Be=0.05 and apply the tuning conditions

$$B_{\Delta e} = \mu^B\, B_e\,, \quad B_{\Delta u} = K_P^S\, \mu^B\, B_e\,, \tag{15}$$

- Step VI. Formulate the rule base (13) of the TS fuzzy block FB-τ:

$$\begin{aligned}
&K_P^1 = K_P^5 = K_P^9 = K_P^S, K_P^2 = K_P^6\\
&= K_P^4 = K_P^8 = K_P^M, K_P^3 = K_P^7 = K_P^B,\\
&\mu^1 = \mu^5 = \mu^9 \mu^S, \mu^2 = \mu^6 = \mu^4\\
&= \mu^8 = \mu^M, \mu^3 = \mu^7 < \mu^B.
\end{aligned} \tag{16}$$

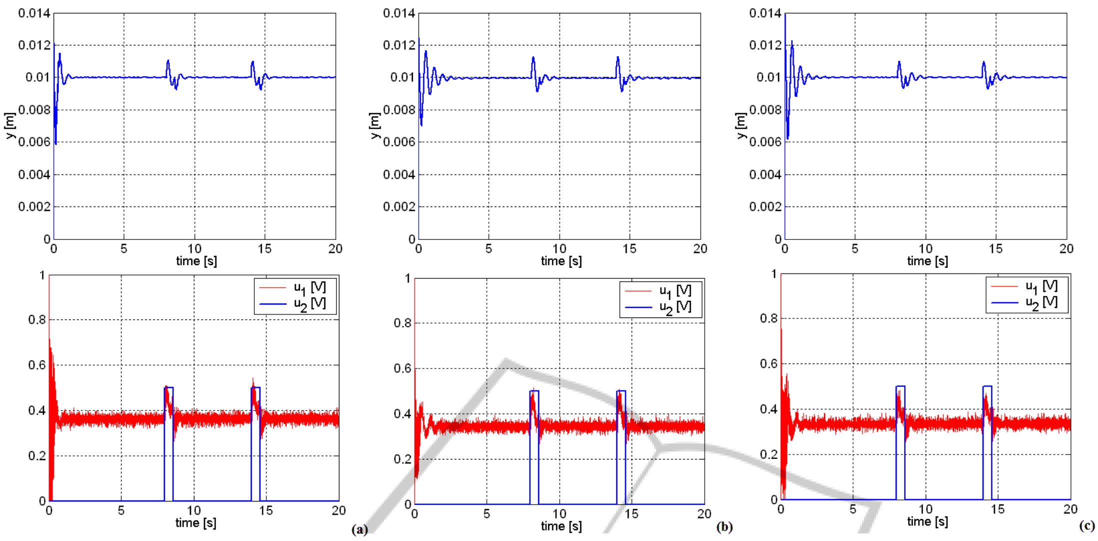

Figure 5: Experimental results for the TS fuzzy CS with the set-point filter (a), with the feedforward (b) and with the feedback (c) 2-DOF PI-fuzzy controller: position (y), control signal $u = u_1$ and disturbance input $d = u_2$ versus time.

The tuning conditions (15) are obtained from the modal equivalence principle in order to guarantee the quasi-PI behaviour of the Mamdani fuzzy block FB-τ and of the TS fuzzy block FB-τ. Both tuning conditions are applied in the tuning of Mamdani fuzzy block FB-τ, and the first one is applied in the tuning of TS fuzzy block FB-τ. The setting of the parameter B_e is important. The experience of CS designer can be taken into consideration but other mathematical or engineering analyses can be taken into consideration including the stability analysis (Škrjanc et al., 2005). The linear blocks in Figure 3 are implemented for one of the linear controllers that correspond to (14), and the results are presented as follows for K_P^S and μ^S.

Some real-time experimental results for the TS fuzzy CSs with the set-point filter 2-DOF PI-fuzzy controller, with the feedforward 2-DOF PI-fuzzy controller, and the feedback 2-DOF PI-fuzzy controller, are presented in Figures 5. The experimental scenario is characterized by the application of a step reference input $r = 0.01\,\mathrm{m}$ and of a pulse width modulated disturbance input.

The results presented in Figure 5 show very good CS performance indices, therefore our new fuzzy controllers are validated. The best performance indices (in terms of overshoot and settling time) are exhibited by the fuzzy CS with the set-point filter 2-DOF PI-fuzzy controller.

5 CONCLUSIONS

This paper has suggested a new generation of 2-DOF PI-fuzzy controller structures that consists of three fuzzy CS structures. A unified approach to the design of these fuzzy controller structures that enables the design of both Mamdani and TS fuzzy controllers has bee offered with focus on the position control of an MLSLE.

Our approach is justified because of the process nonlinearities. Therefore very good CS performance is ensured by means of low cost fuzzy controllers.

Future research will be focused on the convenient proof of the stability of the 2-DOF fuzzy control structures. Extensions to other models and processes are targeted.

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