OPTIMUM TRAJECTORY PLANNING FOR INDUSTRIAL ROBOTS THROUGH INVERSE DYNAMICS

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Abstract: This paper presents a method for developing robot trajectories that achieve minimum energy consumption for a point-to-point motion under kinematic and dynamic constraints. The method represents trajectories as a fourth degree B-spline function. The parameters of the function are optimised using a multi-parametric optimization algorithm. Actuator torques have been considered for the formulation of the cost function, which utilizes an inverse dynamics analysis. Compared to other trajectory optimization techniques, the proposed method allows kinematic and dynamic constraints to be included in the cost function. Thus, the complexity and computational effort of the optimization algorithm is reduced. A two-link simulated robot manipulator is used to demonstrate the effectiveness of the method.

1 INTRODUCTION

Most of industrial robotic applications are based on repetitive processes, where minimum cycle time is an important factor to reduce the production time and to increase the profit of the production (Zoller and Zentan, 1999). However, the minimum time criterion is not suitable if a smooth path for the motion is required. When the actuators run at high-speeds, they can cause physical vibrations and undesirable shocks to the system. These unwanted vibrations can result in a wide range of problems including loss of accuracy, increased energy consumption and a decrease in actuator life.

Energy requirement has been a significant feature in robotic systems, e.g. robots for space or submarine exploration, or unmanned reconnaissance vehicles (Saravanan and Ramabalan, 2008). This paper focuses on energy minimization in the context of trajectory planning. The cost function in (Gasparetto and Zanotto, 2007) and (Zanotto and Gasparetto, 2007) consists of two terms. The first term is the total execution time and the second is the jerk. Some of the approaches include the travel time in the cost function (LoBianco and Piazzi, 2002). Also the mechanical power of the actuators and energy for gripper action are considered for the formulation of cost function in (Saramago and Ceccarelli, 2004) and just the mechanical power in (Garg and Kumar, 2002).

This paper proposes a path planning trajectory method to generate an optimum path based on minimum torque and/or energy consumption. The proposed method considers an inverse dynamic model of the robot manipulator. The resulting optimization algorithm can be applied to various robots, such as redundant or parallel robots in order to optimize the desired trajectory. The method has the advantage that kinematic and dynamic constraints are included in a sequential manner in the cost function and solving the inverse dynamics is avoided when the constraints are not satisfied.

In this study, the dynamic modelling of the robot is based on Lagrangian dynamics (Wells, 1967), which describes the system in terms of its energy. The *DYSIM* software (Sahinkaya, 2004) is used to construct the equations of motion automatically for both forwards and inverse dynamic analysis of the system. In this study, *DYSIM* was operated in the MatLab/Simulink.

2 OPTIMIZATION

Path planning techniques are associated with the way in which a robot manipulator moves from one point to another in a controlled manner (Niku, 2001). One of the important stages of path planning is that of trajectory optimization. Trajectory optimization problems can be divided into the

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following components:

- Parametric path function
- Path coordinates
- Optimization technique
- Cost function
- Constraints

2.1 Parametric Path Function

Selection of the parametric path function is the first step of the trajectory optimization technique. Generally in optimization, a small number of parameters is preferred without restricting the motion space. Also, for manipulator motions, the trajectory function has to be at least twice differentiable in order to provide smooth and continuous accelerations.

In this study, a fourth order B-spline function is used to define joint motions because of its simplicity and computational efficiency (De Boor, 1978), see (Qin, 2000) for details. A fourth order B-spline function consists of nine parameters. Three of them are used for the start condition (position, and its first and second derivatives). Three parameters are used for the end condition (position, and its first and second derivatives). The remaining three free parameters are calculated by the optimization algorithm.

2.2 Path Coordinates

Trajectory planning can be done either in the joint-space or Cartesian-space. Planning a trajectory in the joint-space has a significant advantage that the control system will be acting on the robot joints rather than on the end effector. In this case, it is easier to set the necessary trajectory in terms of the design requirements. However, the trajectory of the end effectors will not be easily predictable (Niku, 2001). On the other hand, Cartesian-space trajectories are more realistic and very simple to visualize, but these have to be converted to joint space for control purposes. In this paper, joint space trajectories are used. However, the method can handle Cartesian-space trajectories if required.

2.3 Optimization Technique

The selection of the optimization technique is important as a large number of parameters and coefficients may adversely affect the results of optimization, and computational efficiency (Garg and Kumar, 2002). In the proposed method, there is no need to use computationally intensive optimization techniques such as genetic algorithms. Therefore, a sequential quadratic programming technique (the default method in "$fmincon$" function in MatLab (The Mathworks, 2007) is used.

2.4 Cost Function

In the literature, the most common objectives to be optimized are minimum travelling time, minimum energy (or actuator effort, e.g. torque), and minimum jerk (Gasparetto and Zanotto, 2007).

Minimum energy consumption was taken into account here, but other quantitative indicators could be considered according to design objectives. The cost function of actuator effort (or torque) minimization is described by:

$$C = \int_0^T \left(\sum_{i=1}^K g_i^2(t) \right) dt \qquad (1)$$

where C is the actuator cost function, $g_i$the actuator torques/forces applied at joint i along the trajectory, K the number of actuators, and T the total travelling time between initial and final positions. Calculation of the cost function in Eq. (1) requires the solving of the inverse dynamic model for T seconds. Three free parameters of the B-spline function are used to optimize each joint trajectory.

2.5 System Constraints

Robot manipulators will have some physical constraints such as the limits of the position, velocity, acceleration and torque. Using these constraints, unrealistic or unreachable motions of the manipulator are automatically avoided in the optimization procedure. Other constraints can also be added (such as obstacle avoidance, singularity avoidance) to the optimization algorithm for trajectory planning.

The cost function calculations involve running the inverse dynamic model, which is time consuming. In conventional methods the constraint equations are handled separately, and the cost function is called regardless of whether the constraints are satisfied or not. In order to improve computational efficiency in the proposed method, constraints are handled within the cost function calculations and the inverse dynamic analysis is only evaluated when these constraints are satisfied. In order to achieve this, an alternative cost function is formulated to handle constraints as follow:

1. A variable, *c*, is created to count the number of cost function calls where the parameters do

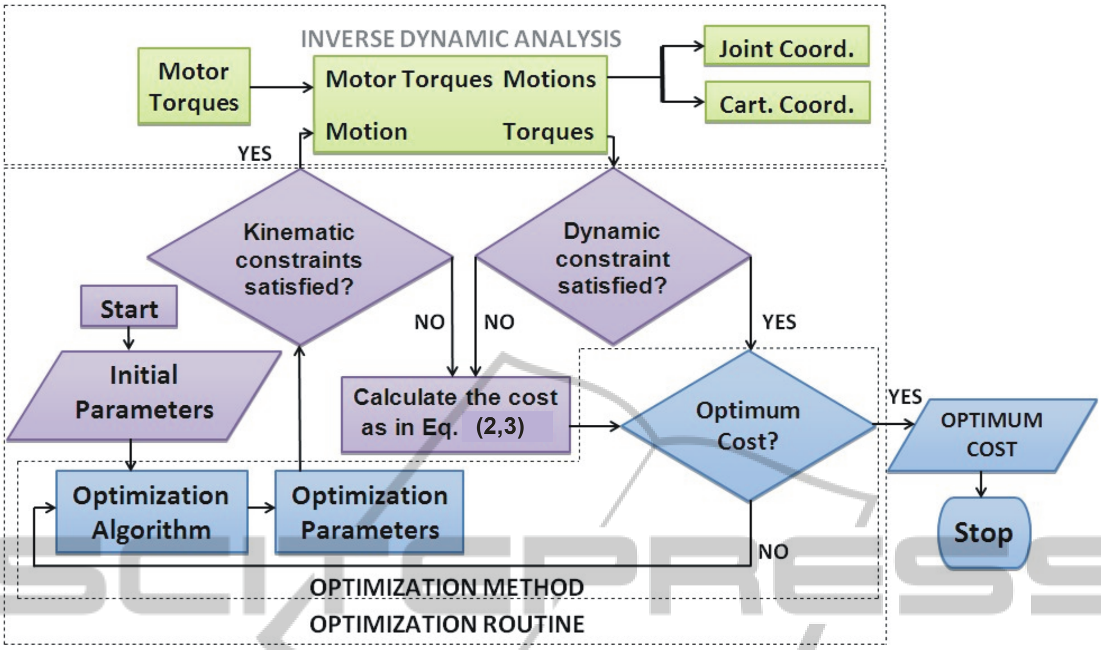


Figure 1: Optimization routine with inverse dynamic programming.

not satisfy the constraint equations.

2. During the cost function call, if any of the constraints are not satisfied, the following alternative cost function is used:

$$c = c + 1 \quad (2)$$

$$C = b * (1 + c/10) \quad (3)$$

where b is a large base value. This formulation ensures that the cost function will result in a higher value than previous violation of constraints to avoid a local minimum to be found outside the constraints. The value $b=10^5$ is used here.

3 PROPOSED ALGORITHM

The steps of the proposed energy minimization algorithm based on inverse dynamics analysis can be summarized as follow:

1. Before executing the optimization algorithm, all kinematic and dynamic constraints of the mechanism have to be identified. In this example, constraints are based on maximum and minimum values of position, velocity, acceleration and torque.
2. Thereafter, the optimization algorithm will start with suitable initial conditions in joint coordinates.
3. When calculating the cost function, the optimization algorithm will check the kinematic constraints.
   a) If the kinematic constraints are not satisfied, the inverse dynamic analysis will not be carried out. The alternative calculation of the cost function will be carried out in accordance with Eqs. (2) and (3).
   b) If the kinematic constraints are satisfied, inverse dynamic simulation will be run in order to calculate the dynamic cost function as in Eq. (1). If the torque limitations or other dynamic constraints are violated, the simulation will be terminated and the alternative cost function in 3 (a) above will be used.
4. This procedure will continue until the optimization algorithm finds the lowest cost value. The procedure of the optimization algorithm is shown in Fig 1.

4 SYSTEM DESCRIPTION AND SIMULATION

This section introduces numerical simulations using a simple 2-DOF planar manipulator with revolute joints as shown in Fig. 2. The simulation is carried out by the program *DYSIM*. Two motors control the motion. The centre of gravity of the links is in the middle of the each link. A load mass of m_l= 1 kg is attached at the end of the second link. The manipulator has two identical links detailed in Table 1. Gears ratios are $R_1 = 100$, $R_2 = 80$. The viscous friction effects of the joints are also included in the simulation.

The manipulator task consists of transporting the load mass from an initial point P_i ($\theta_1 = \theta_2 = 0\ rad$) to a final one P_f ($\theta_1 = \theta_2 = 1\ rad$) in joint space coordinates. The motion duration is specified as T=2s. The initial and final velocities and accelerations are zero for all joints. The limits for each actuator are given in Table 2.

Table 1: 2-DOF robot manipulator data.

Joints	Length	Mass	Inertia	Friction	
Joint 1	$0.6\ m$		$1\ kg$	0.01	0.4
Joint 2	$0.6\ m$		$1\ kg$	0.01	0.4

Table 2: Limit performances of the 2-DOF manipulator.

Conditions	Joint 1	Joint 2
$\theta_i(t)(rad)$	-/+3π/2	-/+3π/2
$\dot{\theta}_i(t)(rad/s)$	-/+6	-/+6
$\ddot{\theta}_i(t)(rad/s^2)$	-/+25	-/+25
$\tau_i(t)(Nm)$	-15/+25	-15/+25

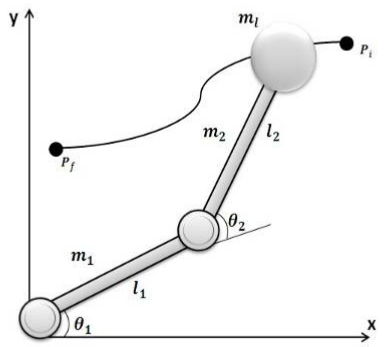


Figure 2: Schematic diagram of the robot and a prescribed trajectory given by initial and final points.

5 RESULTS AND DISCUSSION

The proposed method was implemented in Simulink. The prescribed non-optimized (initial) manipulative task is shown in temporal trajectory position in Fig. 3(a), and the optimum trajectory is traced in Fig. 3(b). The labels '$non-optimized$' and '$optimized$' in the figures denote the result for the case with initial parameter values corresponding to a linear motion in joint space and the case with optimization, respectively.

Figure 4 shows profiles of joint position (rad), velocity (rad/s), torque (Nm) and cost function results. The initial path was a straight line in the joint space between P_i and P_f and cost value for the non-optimized trajectory was 897.683. After optimization, the cost function is reduced to 620.129, which corresponds to 31% energy consumption reduction.

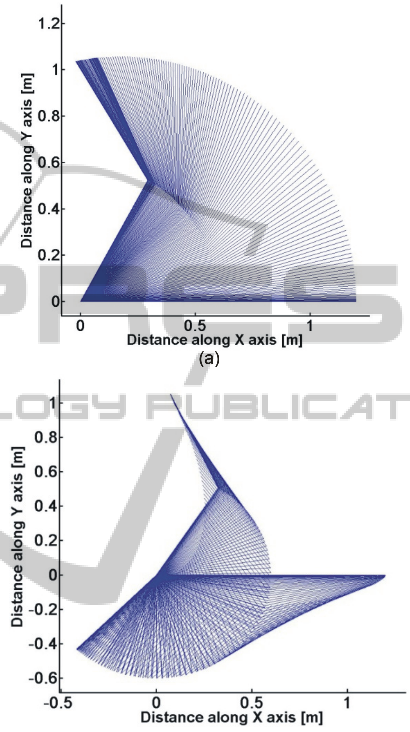


Figure 3: Temporal positions of (a) non-optimized and (b) optimized trajectories.

Figure 4(b) shows that optimized velocity is faster than the non-optimized one. As it can be seen from the Fig. 4(c), non-optimized link-1 has a large peak torque magnitude. Figure 4(d) shows the evolution of the cost functions. There is a sudden ascension on the non-optimized cost curve. Excessive growth of the cost value can be shown to be due to lifting of the first and second arm with minimum joint movements between 0 and 0.5 seconds. On the other hand, optimized cost curve is increasing smoothly by utilising the potential energy of the system.

The optimization was also run by using different friction coefficient values. Temporal position results are shown in Fig. 5. With increasing coefficient of viscous friction in the system, the trajectory of the end effectors has noticeable changed. The trajectory

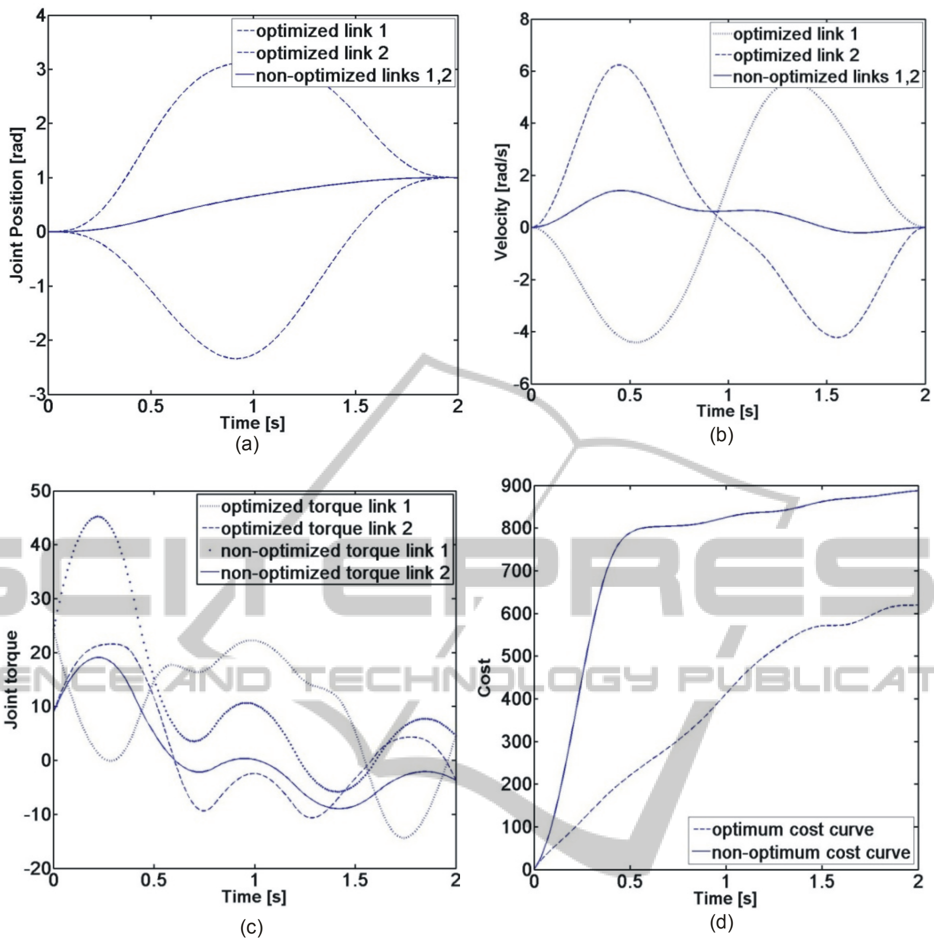


Figure 4: Results for the designed non-optimal and optimal path in terms of time history of (a) joint positions, (b) joint velocities, (c) joint torques, and (d) cost value of the system.

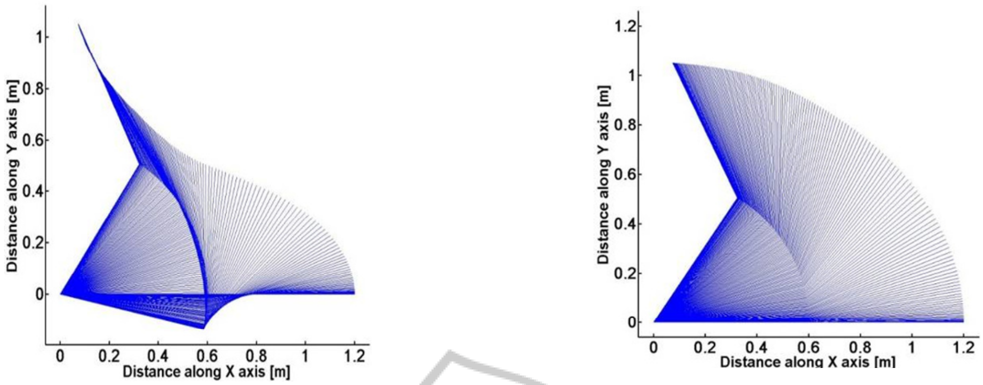
of the end effector moves towards minimum joint displacements in order to minimize energy consumption in the system.

Each cost function call (whether within the workspace or not) requires the inverse dynamic simulations, which has significant effect on the computational efficiency of the optimization. The computational cost in the proposed algorithm is reduced due to not running the inverse model in the alternative cost function calculations when the constraints are not satisfied. To further demonstrate the advantages of the proposed algorithm over the conventional method of handling the constraints, the same optimization was run with Mathlab "fmincon" function with the cost function as in Eq. (1), and the constraints specified separately as nonlinear inequality constraints. It was observed that the optimization algorithm called the cost function even when the parameters did not satisfy the constraints. The number of cost function calls with parameters values outside the permissible workspace was significant (82 out of 243 iterations), resulting unnecessary solving of the inverse dynamics.

In addition to computational efficiency, in cases where the B-splines are used to describe the trajectory in Cartesian coordinates, an additional nonlinear constraint has to be added to make sure that the end point does not fall outside the circle of radius $l_1 + l_2$ during the motion. The conventional constraint handling would still call the inverse dynamics model when this constraints was not satisfied. This would cause the inverse dynamics simulation to crash or terminate prematurely as the required motion cannot be physically achieved. The proposed algorithm avoids this problem.

6 CONCLUSIONS

A methodology for optimal trajectory planning of robotic manipulators has been described in this



(a) friction coefficient of 1.5 (b) friction coefficient of 5

Figure 5: Results for the designed optimal path in terms of temporal positions for increasing friction values.

paper. A fourth degree B-spline function was used to define the trajectory and hence the continuity of velocity and accelerations were guaranteed for the desired trajectory. An inverse dynamic analysis of a two degree of freedom manipulator is performed by using Lagrangian dynamics and an in-house software package *Dysim*. In the proposed optimization method, all the constraints are built in the cost function. Therefore, computational complexity is reduced by avoiding inverse dynamic analysis when the parameters produce a motion that does not satisfy the constraints. The proposed algorithm also avoids the problems with cases where the inverse dynamics model cannot be run when some specific constraints are not satisfied.

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