DISTRIBUTED JOINT POWER AND RATE ADAPTATION IN AD HOC NETWORKS

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Abstract: Ad hoc networks are dynamic and scalable entities that autonomously adapt to nodes entering the network (i.e. increasing interference) or exiting the network (i.e. due to energy depletion), poor connectivity among others. In such networks, nodes exhibit individualistic behaviours where nodes selfishly compete for the limited network resources (i.e. energy and bandwidth) to maximize their own utilities. This consequently degrades network performance leading to low data rates, poor power efficiency, loss of connectivity etcetera. This paper considers a network utility maximization (NUM) strategy based on coupled interference minimization to adapt the transmission power and data rates in ad hoc networks. The proposed distributive joint power and rate adaptation (JRPA) algorithm employs costing (and reward) mechanisms to promote users' cooperation such that both users' local and network global optimum is always attained. This is similar to a super-modular game hence the optimality and convergence of JRPA is analysed using super-modular game theory. Simulation results show that the proposed algorithm improves network performance since users' are compels to transmit at optimal data rates and power levels just enough to sustain the transmission.

1 INTRODUCTION

Preference of wireless networks (WNs) to fixed networks has incredibly increased in the recent past due to their cost efficiency and ease of setting-up and integrating them with other networks. This has since led to introduction of IEEE standards that support higher data rate e.g. 802.11a/g. However, transmitting at higher data rates reduces connectivity due to decline in communicating range and hence requires that the transmission power be increased to sustain transmission.

To attain spectrum efficiency in WNs, resource sharing and management is critical. Nonetheless, this is not easily attainable in ad hoc networks due to dynamic topology and time-variant channel conditions in such networks hence need for adaptive approaches.

Though reducing the transmit power allows multiple simultaneous transmissions, this results to decrease in SINR performance owing to either weak received signal strength (RSS) or increased interference. As a result, transmissions are sustained at lower data rates. Moreover, such scenarios are vulnerable to hidden node problems resulting from high interference range created by the reduced transmit power. On the converse, transmitting at high power impedes concurrent transmissions. Nonetheless, this mitigates hidden terminal problems and improves SINR thence higher data rates are achievable. In a nutshell, to attain high data rates at minimum transmission power in WNs is a contradictive objective. Huang *et al* in (Huang and Letaief, 2005) shows that adapting transmission parameters (data rate and power) based on link dynamics can solve the aforementioned objective. In such a case, the link dynamics can be estimated based on the RSS, acknowledgment (ACK) history (Kim and Huh, 2006) or SINR (Olwal et al., 2009, Grilo and Nunes, 2003, del Prado Pavon and Choi, 2003). However, SINR based schemes has better performance compared to RSS and ACK since it responds faster to link variations (Olwal et al., 2009).

We propose a joint power and rate adaptation scheme based on NUM problem formulated as a coupled interference minimization such that nodes determine their data rates and transmit power based

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on presumed coupled interference at the receiver. In such a case, users are always aware of channel conditions as they choose their transmission parameters. Further, costing (pricing) effect is imposed on users' choices to encourage cooperation and deter selfish behaviors hence both local and global utility are attainable.

The reminder of this paper is organized as follows: Section 2 reviews related works; Section 3 gives the system model; JRPA algorithm is presented in Section 4 while simulation results are given in Section 5 and finally, conclusion is drawn in Section 6.

2 RELATED WORK

Most protocols proposed in literature ((Luo et al., 2010, Hayajneh and Abdallah, 2004, Grilo and Nunes, 2003) and references therein) considers power control, rate adaptation or joint rate-power control in centralized infrastructures WNs where a centralized station determines and dictates the power/rate for data transmission in the network. Such protocols may not be applicable in ad hoc networks where all stations are at free will to choose their transmission parameter based on their own preferences. This may lead to greedy behavior wherein users adapt their transmission power with sole objective of achieving individual desired throughput without considering others users' interests (Olwal et al., 2009). Such schemes require much power to sustain a stable SINR in deep fading environment and causes high interference. Furthermore, such algorithms tend to diverge in case of no feasible power allocation due to hard SINR requirements. However, this divergence problem is easily solved by adaptive SINR based on coupled interference at the receiver.

Due to the distributed and heterogeneous nature of ad hoc network, it is often challenging to design distributed algorithms that can achieve the global optimal NUM solution. The difficulty in distributed algorithm design often lies in the coupling nature of the NUM problem. NUM problems generally assume that user's utilities are uncoupled, i.e., each utility depends only on local variables (Li Ping et al., 2009). However, in problems where cooperation or competition is modeled using the objective function, each user's utility depends on both its local variables and local variables of other users in the network (Hayajneh and Abdallah, 2004, Wang et al., 2006). In (Chee Wei et al., 2006, Palomar and Mung, 2006), these NUM problems are formulated as coupled optimization. Dual decomposition with significant message passing is used to solve such coupled NUM problems where the coupling in the objective function is transferred to coupling in the constraints. However this requires strict convexity and exhibits slow convergence. In (Huang, 2005, Huang et al., 2006), "reverse engineering" with limited message passing is proposed that solves coupled NUM problems without need for strict convexity.

Similar to (Huang, 2005, Huang et al., 2006), our proposed algorithm considers limited message passing strategy based on "reverse engineering" to solve the formulated coupled interference NUM problem. The proposed JRPA dynamically adjust the users' choices of transmission power to curb the influence of coupled interference. Such dynamic adjustments exploit the locally observable network channel conditions and cost charges attached to that transmit power choice. The users are hence cognizant of the current link condition while determining their data rates. Moreover, due to the ineluctable cooperation, every user's strategy to maximize its utility maximizes the utility of other network users, thus improving global network performance.

Supermodular game theory is used to show the existence, convergence and optimality of user's utility functions (Saraydar et al., 1999) since in such games, each player strives to increase its strategy while increases other players' strategies as well. Such a game contains Nash Equilibrium (NE), and does not necessarily require assumption of convexity in order to attain NE (Ozdaglar, 2010, Levin, 2003).

3 SYSTEM MODEL

3.1 Problem Formulation

Consider an ad hoc network with N stations where node i transmits to node j on a single hop subjected to path loss, shadowing and multi path fading dynamics (Olwal et al., 2009). Assume further that all the nodes in the network are within the transmission range of their neighbors such that a node's transmission interferes with other nodes in the network. Consider a set of transmission power levels p and set of data rates r defined as follows: $p = \{p_{\min}, p_2, p_3, \dots, p_{\max}\}$ and $r = \{r_{\min}, r_2, r_3, \dots, r_{\max}\}$ where $r_{\min}$ and $r_{\max}$ are the minimum and maximum data rates while $p_{\min}$ and $p_{\max}$ are the minimum and maximum transmit power levels possible in the network. These sets are assumed identical to all users in the network. The channel gain on link ij given by G_{ij} derived as below:

$$p_j = G_{ij} p_i \tag{1}$$

where p_i is i's transmit power and p_j is received power at j. Notably, G_{ij} is not necessarily equal to G_{ji} since the channel condition is time variant. Half duplex model is assumed i.e. a user can either receive or transmit but not both simultaneously.

The objective is to determine i's power allocation that maximizes its utility given the coupled interference perceived at j. Utility function $u_n(\gamma_n(p))$ for user $n \in N$ is differentiable, concave and increasing function of the received SINR (Saraydar et al., 1999, Huang et al., 2006) and hence NUM problem based on coupled interference can be formulated as follows:

$$\max \sum_{n \in N} u_n(\gamma_n(p)) \tag{2}$$

such that

$$r_{\min} \le r \le r_{\max} \forall N \tag{3}$$

$$p_{\min} \le p \le p_{\max} \forall N \tag{4}$$

where SINR, $\gamma_n(p)$ is given by

$$\gamma_{ij} = \frac{G_{ij} p_i}{\sum_{k \ne i,j} G_{kj} p_k + n_o} \tag{5}$$

where $\sum_{k \ne i,j} G_{kj} p_k$ is the sum of interference power I_{ij} at node j due to communication of other users in the network other than i. n_o is the thermal noise, G_{ij} is the channel gain while p_i is the transmit power used by i to transmit to j.

3.2 Optimal Power based on Coupled Interference

Due to existence of mutual interference, network users have coupled utility function that depends on both the user's local decision and decisions of other users in the network. The global NUM problem can therefore be formulated from (2) as

$$\max_{\{p: p_i \in P \forall n\}} \sum_{n=1}^{N} u_n\left(\gamma_n\left(p\right)\right) \text{ s.t. (3) and (4)} \tag{6}$$

To solve the coupled objective function in (6), (Palomar and Mung, 2006) proposes consistency pricing which requires significant message passing to attain optimal decision. Moreover, this approach requires convexity in (6) but $U_k(.)$ in (6) is concave in γ_n. Therefore we adopt reverse-engineering based on KKT conditions (Huang et al., 2006, Huang, 2005) to solve (6) by localizing the network objective function and updating users on their neighbors' utility choices by means of limited message passing.

Define p_i as the power profile of user i in the network and p_{-i} as the power profile for user $i's$ opponents i.e. $p_{-i} = \left(p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n\right)$ such that $p \in \{p_i; p_{-i}\}$. Then this utility maximization can be modeled as a power control game $G = \left[N, \{p_i\}, \{u_i\}\right]$ where all the players selects transmit power p_i that maximize their utility u_i whereby $u_i(i)$ represents user $i's$ pay-off or reward. User $i's$ optimal response is p_i that maximizes its utility u_i given by $u_i\left(\gamma_i\left(p_i, p_{-i}\right)\right)$ formulated as (7) (Huang, 2005, Huang and Letaief, 2005, Li Ping et al., 2009).

$$\beta_n(p_{-i}) = arg \max_{p_i \in p} u_i(\gamma_i(p_i, p_{-i})) \tag{7}$$

Assuming that p_{-i} is fixed, the reward $u_i(\gamma_i(p_i, p_{-i}))$ in (7) is strictly increasing with p_i.

In view of a Non Cooperative Game (NCG) where players select optimal power levels to maximize their rewards at the expense of others, then a fixed point $p = p*$ defined by (8) would be the NE.

$$u_i\left(\gamma_i\left(p_i^*, p_{-i}^*\right)\right) \ge u_i\left(\gamma_i\left(p_i^{'}, p_{-i}^{'}\right)\right) \tag{8}$$

where $p^{'} \in p$ is any power chosen by any user i other than $p*$ in view of the fact that each user's reward $u_i(\gamma_i(p_i, p_{-i}))$ is strictly increasing with p_i for fixed p_{-i} (Huang, 2005, Huang et al., 2006, Li Ping et al., 2009).

We seek to improve the NE in (8) by introducing pricing in users' choices since pricing discourages users' selfish behaviors. In effect, every user strives to maximize its pay-off or reward $f_i(\gamma_i)$ in (9) by minimizing the cost c attached to its transmission power choice p_i.

$$u_i(p_i, p_{-i}) = f_i(\gamma_i) - cp_i \tag{9}$$

Considering (9) as cost or penalty imposed on i for generating interference to other network users, user i has to minimize c in (9) to be able to maximize its utility. Since c depends on channel gain G_{ij} and network factor ε_j, surplus function is derived from (9) as follows:

$$S_i(p_i; p_{-i}, \varepsilon_{-i}) = u_i(\gamma_i(p_i; p_{-i})) - p_i \sum_{j \neq i} \varepsilon_j G_{ij} \tag{10}$$

Lemma 1 (KKT conditions) (Huang et al., 2006)**:** For any local optimal p^* of problem (6), there exist unique lagrange multipliers $\mu^*_{1,u}, \ldots, \mu^*_{I,u}$ and $\mu^*_{1,g}, \ldots, \mu^*_{I,g}$ such that for all $n \in N$,

$$\frac{\partial u_i\left(\gamma_i\left(p^*\right)\right)}{\partial p_i} + \sum_{k \neq i} \frac{\partial u_k\left(\gamma_k\left(p^*\right)\right)}{\partial p_k} = \mu^*_{i,u} - \mu^*_{g,u} \tag{11}$$

where

$$\mu^*_{i,u}\left(p^*_i - p^{\max}_i\right) = 0,\ \mu^*_{i,g}\left(p^{\max}_i - p^*_i\right) = 0, \quad \mu^*_{i,u}, \mu^*_{g,u} \geq 0 \tag{12}$$

The KKT set of problem (6) need to contain all the solutions that satisfy (11) and (12) hence we design a distributed algorithm that converges to this set. Substituting (11) in (6), the KKT condition for i can be expressed as follows

$$\frac{\partial u_i\left(\gamma_i\left(p^*\right)\right)}{\partial p_i} \sum_{k \neq i} \varepsilon_j\left(p^*_j, p^*_{-j}\right) G_{i,j} = \mu^*_{i,u} - \mu^*_{g,u} \tag{13}$$

where

$$\varepsilon_j(p_j, p_{-j}) = -\frac{\partial u_j\left(\gamma_j\left(p_j, p_{-j}\right)\right)}{\partial I_j(p_{-j})} \tag{14}$$

In equation (14), $I_j(p_{-j})$ is the total interference received by user j given by $\sum_{i \neq j} p_i G_{ij}$. Notably, the cost function in (14) is always nonnegative and represents j's marginal increase in utility per unit decrease in total interference. The reward is the product of user's transmission power p and the weighted sum of other users' costs in (10) where weights equal to the channel gains between transmitter i and the other users' receivers. If ε_j is the penalty obtruded to other users for generating interference to user i defined in (9), then (14) is an acceptable optimal condition for the problem in which each user i chooses a power level $p_i \in p$ to maximize (i.e. the surplus function in (10)) (Huang, 2005).

At an instance of time t, network users announce their cost in reference to (14) and adjust their transmit power taking into account network dynamics according to (10). The chosen power is constrained to (13) and as a result, an optimal localized distributive power algorithm with costing constrains is derived. The surplus in (10) and cost function (14) are formulated as function of the desired power p_i and SINR as in (14) and (15) respectively.

$$S_i(p_{-i}, \varepsilon_{-i}) = \min\left(\max\left(p_{\min}, \frac{p_i}{\gamma_i(p)}\left(\frac{p_i}{\gamma_i(p)}\left(\sum_{j \neq i} \varepsilon_j G_{ij}\right)\right)\right), p_{\max}\right) \tag{15}$$

$$\varepsilon_i(p) = \frac{\partial u_i(\gamma_i(p))}{\partial \gamma_i(p)} \frac{(\gamma_i(p))^2}{\beta p_i G_{ij}} \tag{16}$$

where β is the spreading factor while $\frac{du_i(\omega_i)}{d\omega_i}$ is given by $\frac{u_i(\omega^t_i) - u_i(\omega^{t-1}_i)}{\omega^t_i - \omega^{t-1}_i}$.

3.3 Convergence and Optimality

Lemma 2 **(**Ozdaglar, 2010)**:** Let $X \subseteq \mathbb{R}$ and $T \subset \mathbb{R}^k$ for some k, a partial ordered set with the usual vector order. Let $f : X \times T \to \mathbb{R}$ be a twice continuously differential function. Then, the following statements are equivalent: *(i)* The function *f* has increasing differences in *(x,t)*, *(ii)* For all $t' \geq t$ and $x \in X$, we have $\frac{\partial f(x,t')}{\partial x} \geq \frac{\partial f(x,t)}{\partial x}$ and, *(iii)* For all $x \in X$, $t \in T$, and all *i=1,…,k*, we have $\frac{\partial^2 f(x,t)}{\partial x \partial t_i} \geq 0$.

Theorem 1: Define $X \subseteq \mathbb{R}$ as a compact set and T as some partially ordered set. Assume that the function $f : X \times T \to \mathbb{R}$ is upper semicontinuous in x for all $_1$and has increasing differences in *(x,t)*. Define $x(t) = \arg\max_{x \in X} f(x,t)$. Then, we have: for all $t \in T$, x(t) is nonempty and has a greatest and least element, denoted by $\bar{x}(t)$ and $\underline{x}(t)$ respectively and, for all $t' \geq t$, we have $\bar{x}(t') \geq \bar{x}(t)$ and $\underline{x}(t')$ and $\underline{x}(t)$

From *lemma 2* and *theorem 1*, every user's utility function $u_i(p_i, p_{-i})$ has increasing differences in (p_i, p_{-i}) given that $\frac{-\gamma_i f_i^{''}(\gamma_i)}{f_i^{'}(\gamma_i)} \geq 1, \forall \gamma_i \geq 0$ hence the convergence.

Assume $(I, (p), (u_i))$ is a supermodular game. Then $\beta_i(p_{-i})$ in (7) as a greatest and least element, denoted by $\overline{\beta_i}(p_{-i})$ and $\underline{\beta_i}(p_{-i})$, and If $p'_{-i} \geq p_{-i}$ then $\overline{\beta_i}(p'_{-i}) \geq \overline{\beta_i}(p_{-i})$ and $\underline{\beta_{-i}}(p'_{-i}) \geq \underline{\beta_{-i}}(p_{-i})$ (Levin, 2003).

This implies that each player's best response is increasing in the actions of other players. The set of strategies that survive iterated strict dominance (i.e. iterated elimination of strictly dominated strategies) has greatest and least elements $\bar{p}$ and $\underline{p}$, which are both pure strategy in Nash Equilibrium.

Definition and formulation of supermodular game theory can be found in (Huang, 2005, Ozdaglar, 2010, Hayajneh and Abdallah, 2004, Levin, 2003).

3.4 Rate Adaptation

From the SINR's of the distributive pricing power control algorithm above, best constellation size for *M-QAM* modulation is determined that is supported by the SINR level. From Shannon theory of communication ((Yu-Chee et al., 2001)) we can deduce the following: $M = 1 + \left(\frac{-\vartheta_1}{ln(-\vartheta_2 BER)} \right) SINR$ where BER is the bit error rate while ϑ_1 and ϑ_2 are modulation type dependent constants. Let $\delta = \frac{-\vartheta_1}{\ln(\vartheta_2 BER)}$, then data rate r_i for transmit power p_i between the sender i and receiver j is a function of SINR estimated as $M = 1 + \delta SINR$ and hence

$$r_i = \frac{1}{T}\log_2\left(1 + \delta SINR\right) \approx r_i = \frac{1}{T}\log_2\left(\delta SINR\right) \qquad (17)$$

where $\delta SINR \gg 1$ while $\frac{1}{T}$ is the bandwidth of the channel used for data transmission. When the signal level is much higher than the interference level or when the spreading gain is large then r_i lies within (3).

4 JOINT POWER AND RATE CONTROL ALGORITHM (JRPA)

The outline of joint power and rate control algorithm is presented as follows:

1. ***Initialization Stage:*** Initialize power p_i and cost ε_{-j} to some non-negative value, and then calculate r_i from (17).
2. ***Cost Advertisement and Transmit Power Adjustment***
   a. *Cost Announcing:* interferers i_{-1} update and advertise their cost ε_{-j} according to (16).
   b. *Power Updating:* based on network cost, user i updates its transmission power p_i according to (15).
   c. *Determine data rate according to (17).*
   d. *Repeat 2 while not end of communication*

5 SIMULATION TEST AND RESULTS

Simulation is performed in MATLAB with 32 nodes randomly placed in a $20m \times 20m$ field free of obstacles. It's assumed that only *Tx* communicates with *Rx* while other network users are actively interfering. Performance metrics are evaluated for 50 independent runs (transmissions). For all the simulations, we assume single hop with the following simulation parameters: path loss model exponent = *1*, AWGN = *-96dB*, P*max* = *10dB*, P*min =1dB*, Initial cost = 0.1 and utility function, $u_i(\gamma_i)$ is given by $\log(\gamma_i)$. It is further summed that all transmissions are successful, channel bandwidth = *20MHz* and spreading factor, β = *5.* We consider 2 scenarios: scenario 1 is a stationary network where users are static while scenario 2 reflects a mobile network where users randomly move after every 2 transmissions at a velocity *20kmph*. In addition, all transmissions are assumed to be successful. The performance of JRPA is compared to IEEE 802.11 and adaptive auto response joint power and rate control algorithm – LP proposed by (Chevillat et al., 2005).

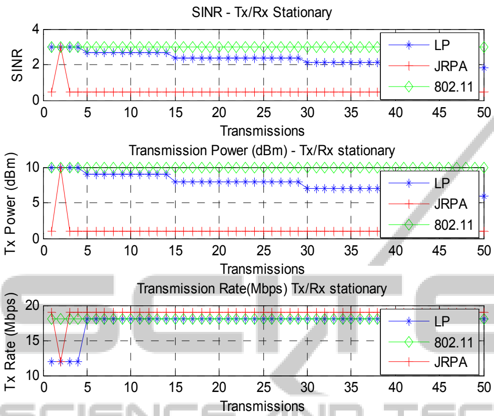

Figure 1: Stationary users.

In all the runs, it’s observed that JRPA attains the highest data rates at minimal transmission power followed by LP and the legend 802.11 protocols. The costing mechanism drives the power selection response in JRPA to the most cost effective option. At the beginning, transmission power hikes due to limited information available at *Tx* about the channel conditions. As the other network users advertise their network costs, *Tx* determines the most feasible transmission power for the subsequent transmissions till most optimal transmission power is attained. This is the NE. LP and 802.11 transmit at higher power levels and hence achieve higher SINR than JRPA. Nonetheless, JRPA attains highest data rate. The improvement on JRPA compared to LP and 802.11 is that JRPA operates at optimal power just enough to sustain the required transmission and to decode data packets at the receiver *Rx*.

Figure 2 shows the performance of JRPA in an environment where the network users are assumed to be in random movement. Similar to figure 1, performance of JRPA in terms of data rates and power efficiency is relatively better compared to LP and 802.11. However, low data rates are experienced due to fast fading channel conditions resulting from user mobility. The power level that JRPA settles on is apparently the most optimal power that maximizes both local and global utility considering the network dynamics during data transmission. At such power choices, interference cost function is always minimized while the reward function (data rate) is maximized hence improving network performance.

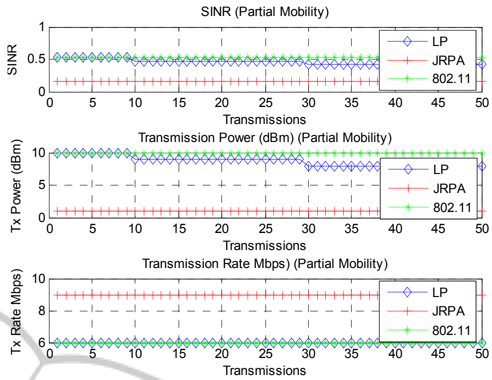

Figure 2: Partial mobility.

6 CONCLUSIONS

This paper proposes distributive algorithm that jointly adapts transmission powers and data rates in ad hoc networks by formulating NUM as a coupled interference minimization problem. The simulation results have shown that penalizing users’ selfish behaviors promotes cooperation such that user aim to optimize both global and local utilities. Future work may consider cross layering optimization to incorporate packet routing in the proposed model.

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