# IMAGE DENOISING BASED ON LAPLACE DISTRIBUTION WITH LOCAL PARAMETERS IN LAPPED TRANSFORM DOMAIN

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Abstract: In this paper, we present a new image denoising method based on statistical modeling of Lapped Transform (LT) coefficients. The lapped transform coefficients are first rearranged into wavelet like structure, then the rearranged coefficient subband statistics are modeled in a similar way like wavelet coefficients. We propose to model the rearranged LT coefficients in a subband using Laplace probability density function (pdf) with local variance. This simple distribution is well able to model the locality and the heavy tailed property of lapped transform coefficients. A *maximum a posteriori* (MAP) estimator using the Laplace probability density function (pdf) with local variance is used for the estimation of noise free lapped transform coefficients. Experimental results show that the proposed low complexity image denoising method outperforms several wavelet based image denoising techniques and also outperforms two existing LT based image denoising schemes. Our main contribution in this paper is to use the local Laplace prior for statistical modeling of LT coefficients and to use MAP estimation procedure with this proposed prior to restore the noisy image LT coefficients.

## 1 INTRODUCTION

Images are often contaminated by noise in its acquisition or transmission. The noise mainly arises from the imaging devices and channels during transmission. The aim of denoising is to eliminate the noise while keeping the signal features as much as possible.

In the past several years, considerable work has been reported on the wavelet based image denoising techniques (Michak et al., 1999)(Fan and Xia, 2001)(Kazubek, 2003) (Eom and Kim, 2004)(Sendur and Selesnick, 2002)(Rabbani and Vafadust, 2008). In wavelet based image denoising, one approach is to design a statistically optimal threshold parameter for non linear thresholding or shrinkage function like soft and hard thresholding. Another approach is to estimate the noise free coefficients from the noisy coefficients with Bayesian estimation methods. If the maximum a posteriori (MAP) or minimum mean square estimation (MMSE) estimator is used for this problem, the solution requires a priori knowledge about the distribution of the noise free coefficients. The shrinkage function is obtained for the corresponding distribution.

Recently, a few approaches on lapped transform based image denoising (Yang and Nguyen, 2003)(Duval and Nguyen, 2003)(Duval and Nguyen, 2004)(Raghvendra and Bhat, 2006) has been proposed. The motivation of image denoising in lapped transform (Malvar, 1989) domain is that, lapped transforms have good energy compaction and are robust to oversmoothing. The lapped transforms are orthogonal transforms, thus signal and noise statistics can be modeled precisely in the lapped transform domain. Since, the lapped transforms are block transforms, the lapped orthogonal transform (LOT) (Malvar, 1989) coefficients are first rearranged into a wavelet like structure (Malvar, 2000)(Yang and Nguyen, 2003)(Duval and Nguyen, 2003)(Duval and Nguyen, 2004)(Xiong et al., 1996), then the rearranged lapped orthogonal transform coefficients subband statistics are modeled in a similar way like wavelet coefficients. In (Yang and Nguyen, 2003), Yang et al., set up a maximum a posteriori estimation problem to reduce the compression artifacts and the additive white gaussian noise in images. In (Duval and Nguyen, 2003), Duval et al. proposed to extend the use of hidden markov tree (HMT) model in lapped orthogonal transform domain (LOT-HMT). In (Duval and Nguyen, 2004), Duval et al. further proposed to improve the results of LOT-HMT denoising by combining it with a redundant decomposition.



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Raghvendra et al. (Raghvendra and Bhat, 2006) proposed to model the LT coefficients using mixture of Laplace distributions which does not use local parameters. The pdf's without local parameters are not good in capturing the spatial clustering property of LT coefficients. The spatial clustering property shows that if a lapped transform coefficient is large, then its adjacent coefficients are also more likely to be large. The LT coefficients which represent edges or other important signal features tends to cluster locally in a subband, like wavelet coefficients. The pdf's with local parameters can better exploit the local statistics and captures the intrascale dependencies of the LT coefficients.

In this paper, we model the LT coefficients using a simple Laplace pdf with local variance. The proposed model is well capable of capturing the clustering and the heavy tailed property of LT coefficients. A MAP estimator which employs the Laplace pdf with local variance is used to estimate the noise free LT coefficients. The paper is organized as follows. In Section 2, an introduction on lapped transforms and its wavelet like representation is presented. In Section 3, we explain the proposed image denoising scheme based on local Laplace prior. In Section 4, the performance of proposed scheme is evaluated and is compared with other image denoising schemes. Finally, the concluding remarks are given in Section 5.

## 2 LAPPED TRANSFORMS

The lapped orthogonal transform (LOT) (Malvar, 1989)(Malvar, 1992) has been proposed to overcome the blocking artifacts of the DCT and has increased coding gain. The lapped transforms has extended basis functions which overlaps across the block boundaries. In lapped transforms, the input signal length is two times its output signal length.

$$L = 2M \tag{1}$$

where $M$ is the output signal length and $L$ is the input signal length. The initial LOT matrix $P$ which may not be necessarily optimal is given by

$$P = \frac{1}{2}\begin{pmatrix} D_e - D_o & D_e - D_o \\ J(D_e - D_o) & -J(D_e - D_o) \end{pmatrix} \tag{2}$$

where $D_e$ and $D_o$ are the *M x M/2* matrices containing the even and odd DCT functions respectively and $J$ is the counter identity matrix. The optimal LOT matrix (Malvar, 1989) is given by

$$P_0 = PZ \tag{3}$$

for an optimal $Z$. The covariance matrix of LOT coefficients is given by

$$R_0 = Z'P'R_{xx}PZ \tag{4}$$

where $R_{xx}$ is the given signal covariance matrix (Malvar, 1989).

$$R_{xx} = \begin{pmatrix} 1 & \rho & \rho^2 & \dots & \rho^L \\ \rho & 1 & \rho & \dots & \rho^{L-1} \\ . & & . & & . \\ . & & . & & . \\ \rho^{L-1} & \dots & \rho & 1 & \rho \\ \rho^L & \dots & \rho^2 & \rho & 1 \end{pmatrix} \tag{5}$$

We assume the signal model to be first order markov model with $\rho = 0.95$. From equation no. (4), when columns of $Z$ are the eigen vectors of $P'R_{xx}P$ that is $R_0$ is diagonal, the transform coding gain (Malvar, 1989) is maximized. The LOT matrix $P_0$ is optimal for such $Z$. The lapped transforms can be viewed as critically sampled multirate filter banks.

Like block DCT coefficients (Xiong et al., 1996), the LT coefficients also can be rearranged in a wavelet like structure with $N = \log_2 M$ decomposition levels. Fig.1 shows the rearrangement of LOT (M=8) coefficients into a 3 level wavelet like pyramid structure for the Barbara image. The image shown in Fig. 1(b) is composed of 8x8 blocks and the image in Fig. 1(c) is its wavelet like representation.

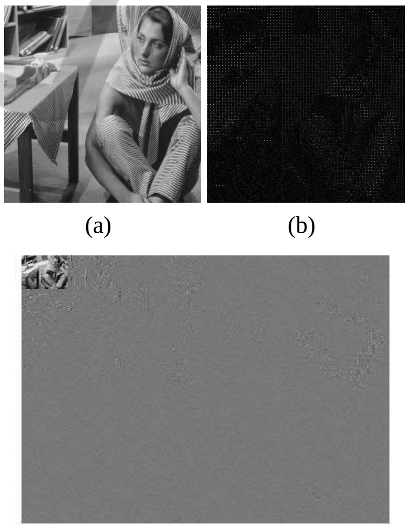
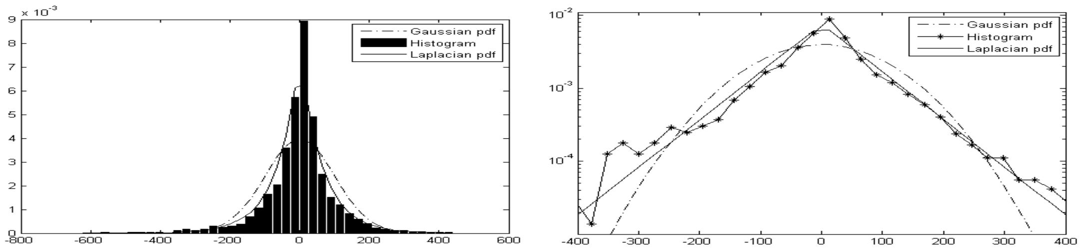



Figure 1: (a) Barbara image (b) LOT (M=8) block decomposition. (c) Rearrangement into 3-level wavelet like pyramid structure.

## 3 PROPOSED DENOISING METHOD

Many wavelet based image denoising schemes (Michak et al., 1999)(Fan and Xia, 2001)(Kazubek,




(a) (b)

Figure 2: (a) The Gaussian and Laplacian pdf fitted to the histogram of the rearranged LT coefficients in a particular subband of Lena (512x512) image. (b) The histogram, Gaussian and Laplace pdf in the log domain.

2003)(Eom and Kim, 2004) assume local distribution of the transform coefficients to be Gaussian with spatially varying variance and employ linear minimum mean square error (MMSE) estimator locally to restore the noisy coefficients. The Laplace priors are also widely used for the statistical modeling of wavelet transform coefficients. Recently, Rabbani (Rabbani, 2009) proposed to use the local Laplace prior for modeling of Steerable Pyramid coefficients. Fig. 2(a) shows the histogram of a specific subband of rearranged LOT coefficients for Lena image. The histogram of coefficients in each subband of rearranged LOT coefficients has a sharp peak around zero and its tails decays to zero much slower than the Gaussian pdf. Fig. 2(a) also indicates that Laplace distribution better fits the histogram of data. The same issue can be better studied from the image in Fig. 2(b) , where the histogram, Gaussian and Laplace pdf's are plotted in the log domain. The image in Fig. 2(b) shows that Gaussian pdf fails to match with the histogram, particularly in tails. In this paper, we propose to model the rearranged LT coefficients in a subband using Laplace pdf with local variance. The MAP estimator using this local Laplace prior is used for the estimation of noise free LT coefficients. Fig. 3 shows the block diagram of the proposed LT based image denoising method.

We assume that the image is corrupted by additive white Gaussian noise with variance $\sigma_n^2$. The orthogonal LT coefficients of the noisy image are given by

$$y(k) = x(k) + n(k) \tag{6}$$

where *x(k)* denotes the clean LT coefficients and *n(k)* is additive white Gaussian noise. When we use a MAP estimator to estimate *x(k)* from the noisy observation *y(k)*, we have (Rabbani and Vafadust, 2008)

$$\hat{x}(k) = \arg\max_{x(k)} P_{x(k)|y(k)}(x(k)|y(k)) \tag{7}$$

The above equation can also be easily written as

$$\hat{x}(k) = \arg\max_{x(k)} \left[P_n(y(k)-x(k))P_{x(k)}(x(k))\right] \tag{8}$$

In this paper, we have assumed the noise to be zero mean gaussian with variance $\sigma_n^2$

$$p_n(n(k)) = \frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[-\frac{n^2(k)}{2\sigma_n^2}\right] \tag{9}$$

Substituting Eq. no. (9) in Eq. no. (8), we have

$$\hat{x}(k) = \arg\max_{x(k)}\left[-\frac{(y(k)-x(k))^2}{(2\sigma_n^2)} + f(x(k))\right] \tag{10}$$

where $f(x_k) = \log\left(P_{x(k)}(x(k))\right)$ . Thus, the MAP estimate of *x(k)* is achieved by setting the derivative with respect to $\hat{x}(k)$ equals to zero

$$\frac{y(k)-\hat{x}(k)}{\sigma_n^2} + f'(\hat{x}(k)) = 0 \tag{11}$$

In this paper, we propose to model the LT coefficients using Laplace pdf with local variance, thus

$$P_{x(k)}(x(k)) = \frac{1}{\sigma_x(k)\sqrt{2}}\exp\left(-\frac{\sqrt{2}\,|x(k)|}{\sigma_x(k)}\right) \tag{12}$$

For this particular case, we have

$$f(x(k)) = -\log\left(\sigma_x(k)\sqrt{2}\right) - \frac{\sqrt{2}\,|x(k)|}{\sigma_x(k)} \tag{13}$$

We have

$$f'(x(k)) = -\frac{\sqrt{2}}{\sigma_x(k)} sign(x(k)) \tag{14}$$

So,

$$y(k) = \hat{x}(k) + \frac{\sqrt{2}\sigma_n^2}{\sigma_x(k)} sign\left(\hat{x}(k)\right) \tag{15}$$

The above equation can also be written as

$$\hat{x}(k) = sign(y(k))\left(|y(k)| - \frac{\sqrt{2}\sigma_n^2}{\sigma_x(k)}\right)_+ \tag{16}$$

From the above equation, $(b)_+$ can be defined as

$$(b)_+ = \begin{cases} 0, b < 0 \\ b, otherwise \end{cases} \tag{17}$$

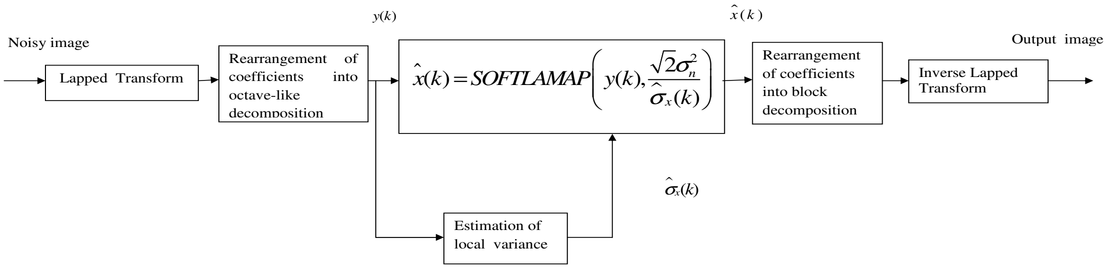



Figure 3: Block diagram of the proposed LT based image denoising method based on local Laplace prior.

If the *SOFTLAMAP* operator (Rabbani and Vafadust, 2008)(Rabbani, 2009) is defined as

$$SOFTLAMAP(p,\eta) = sign(p)(|p|-\eta)_+ \quad (18)$$

Now, Eq. no. (16) can be written as

$$\hat{x}(k) = SOFTLAMAP\left(y(k), \frac{\sqrt{2}\sigma_n^2}{\sigma_x(k)}\right) \quad (19)$$

For each noisy LOT coefficient, an estimate of $\sigma_x^2(k)$ is formed based on its local neighborhood $Z(k)$. In this paper, we use a square window $Z(k)$ centered at *y(k)*. The estimate for $\sigma_x^2(k)$ is given as (Michak et al., 1999)

$$\hat{\sigma}_x^2(k) = \max\left(0, \frac{1}{M}\sum_{j\in Z(k)} y^2(j) - \sigma_n^2\right) \quad (20)$$

where $M$ is the number of coefficients in $Z(k)$.

## 4 EXPERIMENTAL RESULTS

We tested our algorithm on a large number of test images, but report results only for Lena, Barbara, Boat and Fingerprint. Table 1 shows the peak signal to noise ratio (PSNR) comparison between the proposed method and two existing LT based image denoising schemes. The PSNR values of proposed method are averaged over six runs. The proposed low complexity image denoising scheme based on local Laplace prior outperforms both the LOT-HMT (Duval and Nguyen, 2003) method which is based on complex hidden markov tree model and also the LLMDM (Lapped Laplace Mixture Distribution Model) method (Raghvendra and Bhat, 2006) which uses computationally expensive Laplace mixture model (with out local parameters). Table 2 shows the PSNR comparison between the proposed method (M=8 and M=16) and several wavelet (orthogonal discrete wavelet transform) based image denoising schemes. The PSNR results for (Bhuiyan et al., 2008) are obtained using Bayesian MMSE estimator only. The authors of (Bhuiyan et al., 2008) have reported results only for Lena, Barbara and Boat images, therefore we compare our results with (Bhuiyan et al., 2008) method only for these three images. The proposed LT domain method using local Laplace prior is referred to as LOT-Lap. We call the implementation of local Laplace prior in wavelet domain as WT-Lap (Sendur and Selesnick, 2002; Rabbani and Vafadust, 2008; Rabbani, 2009). An orthogonal wavelet transform with four levels of decomposition and Daubechies length-8 wavelet is used for the implementation of WT-Lap, (Michak et al., 1999) and (Chang et al., 2000) algorithms. The proposed method consistently outperforms (Chang et al., 2000), (Bhuiyan et al., 2008) and WT-Lap algorithms in terms of PSNR for all the test images. The LOT-Lap even outperforms the wavelet domain local Wiener filtering method for all the tested images at almost all the tested noise levels. The PSNR results are encouraging especially for highly textured images. Fig. 4 shows the visual results. The proposed method provides images with less distortions in the smooth regions and near the edges.

## 5 CONCLUSIONS

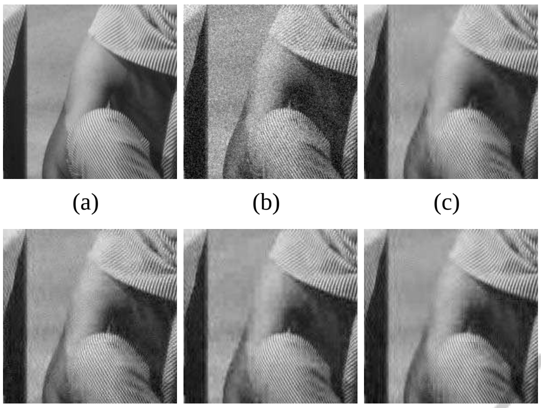
In this paper, we model the rearranged LOT coefficients in each subband using Laplace pdf with local variance. The experimental results show that the proposed low complexity image denoising scheme outperforms all the previously published results on LT based image denoising schemes and several wavelet based image denoising schemes. The proposed denoising scheme shows encouraging results for test images with high amount of textures. The experimental results indicates that the local Laplace pdf is more appropriate model than the models used earlier for modeling the rearranged LT coefficients. The Laplace mixture model with local parameters may further improve the results but at increased computational cost.

Table 1: PSNR (*in dB*) comparison between proposed LOT [M=8] based image denoising scheme and two existing LT based image denoising schemes.

| | Lena (512x512) | | | |
|---|---|---|---|---|
| $\sigma_n$ | Noisy | (Duval and Nguyen, 2003) | (Raghvendra and Bhat, 2006) | Proposed |
| 7.7 | 30.42 | 33.80 | 35.0 | **35.32** |
| 15.5 | 24.33 | 29.60 | 31.70 | **32.03** |
| 23.1 | 20.88 | 27.20 | 29.80 | **30.01** |
| 33.1 | 17.72 | 24.90 | 28.20 | **28.29** |
| | Barbara (512x512) | | | |
| $\sigma_n$ | Noisy | (Duval and Nguyen, 2003) | (Raghvendra and Bhat, 2006) | Proposed |
| 7.7 | 30.39 | 33.30 | 33.70 | **34.20** |
| 15.5 | 24.34 | 29.10 | 29.70 | **30.23** |
| 23.1 | 20.85 | 26.60 | 27.60 | **28.05** |
| 33.1 | 17.76 | 24.20 | 25.70 | **26.13** |

Table 2: PSNR (*in dB*) comparison between proposed LOT [M=8 and M=16] based image denoising scheme and several wavelet based image denoising schemes.

| | Lena (512x512) | | | |
|---|---|---|---|---|
| $\sigma_n$ | 10 | 15 | 20 | 25 |
| Bayes-shrink (Chang et al., 2000) | 33.23 | 31.20 | 29.95 | 28.99 |
| LAWML (Michak et al., 1999) | **34.12** | 31.93 | 30.38 | 29.18 |
| Ref. (Bhuiyan et al., 2008) | 33.89 | 32.01 | 30.71 | 29.75 |
| WT-Lap | 34.04 | 32.05 | 30.69 | 29.64 |
| LOT-Lap (M=8) | 34.06 | 32.20 | 30.82 | 29.69 |
| LOT-Lap (M=16) | 34.09 | **32.26** | **30.85** | **29.76** |
| | Barbara (512x512) | | | |
| $\sigma_n$ | 10 | 15 | 20 | 25 |
| Bayes-shrink (Chang et al., 2000) | 31.10 | 28.72 | 27.12 | 25.90 |
| LAWML (Michak et al., 1999) | 32.51 | 30.07 | 28.38 | 27.09 |
| Ref. (Bhuiyan et al., 2008) | 31.84 | 29.49 | 27.89 | 26.70 |
| WT-Lap | 32.19 | 29.75 | 28.10 | 26.89 |
| LOT-Lap (M=8) | 32.69 | 30.43 | 28.83 | 27.57 |
| LOT-Lap (M=16) | **32.89** | **30.68** | **29.18** | **28.06** |
| | Boat (512x512) | | | |
| $\sigma_n$ | 10 | 15 | 20 | 25 |
| Bayes-shrink (Chang et al., 2000) | 31.89 | 29.76 | 28.33 | 27.25 |
| LAWML (Michak et al., 1999) | **32.50** | **30.36** | 28.86 | 27.67 |
| Ref. (Bhuiyan et al., 2008) | 32.16 | 30.22 | 28.83 | 27.82 |
| WT-Lap | 32.30 | 30.28 | 28.87 | 27.77 |
| LOT-Lap (M=8) | 32.38 | **30.35** | **28.95** | **27.86** |
| LOT-Lap (M=16) | 32.32 | 30.26 | **28.94** | 27.84 |
| | Fingerprint (512x512) | | | |
| $\sigma_n$ | 10 | 15 | 20 | 25 |
| Bayes-shrink (Chang et al., 2000) | 30.98 | 28.73 | 27.21 | 26.09 |
| LAWML (Michak et al., 1999) | 31.30 | 28.97 | 27.36 | 26.15 |
| WT-Lap | 30.77 | 28.55 | 27.02 | 25.86 |
| LOT-Lap (M=8) | 31.13 | 28.93 | 27.38 | 26.17 |
| LOT-Lap (M=16) | **31.45** | **29.15** | **27.55** | **26.39** |



(a) (b) (c)

(d) (e) (f)

Figure 4: A fragment of Barbara image, (a) Original image, (b) Noisy image (SSIM (Wang et al., 2004) =0.7770), (c) Image denoised by WT-Lap method (SSIM=0.8949), (d) Image denoised by LAWML (Michak et al., 1999) (SSIM=0.9001), (e) Image denoised by LOT-Lap (M=8) (SSIM=0.8974), (f) Image denoised by LOT-Lap (M=16) (SSIM=0.9011).



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