

ESTIMATION-DECODING ON LDPC-BASED 2D-BARCODES

W. Proß¹, M. Otesteanu¹ and F. Quint²

¹Faculty of Electronics and Telecommunications, Politehnica University of Timisoara, Timisoara, Romania

²Faculty of Electrical Engineering and Information Technology, University of Applied Sciences, Karlsruhe, Germany

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Abstract: In this paper we propose an extension of the Estimation-Decoding algorithm for the decoding of our Data Matrix Code (DMC), which is based on Low-Density-Parity-Check (LDPC) codes and is designed for use in industrial environment. To include possible damages in the channel-model, a Markov-modulated Gaussian channel (MMGC) was chosen to represent everything in between the embossing of a LDPC-based DMC and the camera-based acquisition. The MMGC is based on a Hidden-Markov-Model (HMM) that turns into a two-dimensional model when used in the context of DMCs. The proposed ED2D-algorithm (Estimation-Decoding in two dimensions) is implemented to operate on a 2D-LDPC-Markov factor graph that comprises of a LDPC code's Tanner-graph and a 2D-HMM. For a subsequent comparison between different barcodes in industrial environment, a simulation of typical damages has been implemented. Tests showed a superior decoding behavior of our LDPC-based DMC decoded with the ED2D-decoder over the standard Reed-Solomon-based DMC.

1 INTRODUCTION

In 1952 the first barcode system was patented by J. N. Woodland and B. Silver (Woodland and Silver, 1949). Today one dimensional barcodes are more and more replaced by their two dimensional (2D) successors. They offer a high information-density as well as an integrated error-correction capability in most cases. One of the most successful 2D-barcode is the *Data Matrix Code* (DMC) which is internationally standardized in (ISO/IEC, 2000). A DMC is formed by the three major components, as shown in Figure 1. The finder-pattern is comprised of the solid border and the broken border. The L-shaped solid border helps in locating the DMC whereas the alternating pattern of the broken border allows to determine the DMC's size. The data region contains the encoded information. Thereby a binary one is represented by a black squared module and a binary zero by a white squared module. This is only true if the DMC is printed black on a white surface. When used in industrial environment the codes get stamped, milled and laser-etched on different kinds of material. Furthermore there are different kinds of interferences that may disturb the barcode. Thereby decoding is much more challenging.

2 LDPC-BASED DMC

Considering the application in industrial environment, a 2D-barcode based on *Low-Density-Parity-Check* (LDPC) codes was developed. The outer appearance of our barcode is similar to that of the DMC since the finder-pattern that surrounds a DMC has been adopted.

PEG-LDPC Codes: The information is encoded in our barcode by use of a regular PEG-LDPC code unlike the standard DMC that is based on Reed-Solomon (RS) codes. The first introduction of LDPC codes has already been in 1962 by Gallager (Gallager, 1962). Since the rediscovery of LDPC codes by MacKay and Neal in 1995 (MacKay and Neal, 1995), many further developments have been published, making LDPC codes a serious competitor to RS codes and the more recent Turbo codes for many fields of application. One important contribution was the introduction of the *Progressive-Edge-Growth* (PEG) construction of the LDPC codes underlying Parity-Check-Matrix (Hu et al., 2005) that made LDPC codes attractive for short block length applications as well. A single LDPC codeword is used to fill the data region of the DMC because it is well known that the decoding performance of LDPC codes increases with the codeword-length.

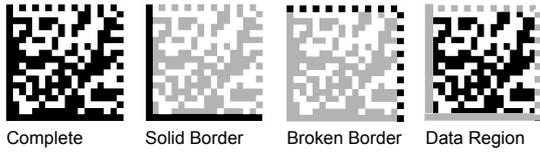


Figure 1: Three major parts of a DMC.

Symbol-placement: The procedure of placing the LDPC codeword's symbols within the available grid of the DMC's data region is done with respect to typical interferences that may occur in industrial environment. The probability that damages caused by dirt, rust, scratches, unequal illumination etc. affect a contiguous part of the DMC is very high. Considering this, the symbol-nodes connected to the same check-node in the LDPC code's *Tanner graph* (Tanner, 1981) are placed as far as possible from each other in the data region under the constraint of the limited area occupied by the code. This way each check-node is affected by the fewest possible number of disturbed symbol-nodes. The placement procedure is based on an optimization process and is explained in detail in (Proß et al., 2010). As an example, Figure 2 depicts the symbol-placement of three symbol-nodes connected to the same check-node.

Image-processing: For the localization of the DMC, the already known standard procedures are applied. In contrast to the RS code used by the original DMC, the LDPC decoder uses soft-decisions (SDs) as an input. For the computation of these SDs a correlation coefficient r_{ij} is calculated for each module in row i and column j of the DMC as follows:

$$r_{ij} = \frac{\sum_{k=1}^h \sum_{l=1}^v (x_{kl}^{ij} - \bar{x}^{ij})(y_{kl} - \bar{y})}{\sqrt{\sum_{k=1}^h \sum_{l=1}^v (x_{kl}^{ij} - \bar{x}^{ij})^2 \sum_{k=1}^h \sum_{l=1}^v (y_{kl} - \bar{y})^2}} \quad (1)$$

k and l are the indices for the h horizontal and v vertical pixels in each module respectively. Considering one module in row i and column j of the DMC, x_{kl}^{ij} denotes one pixel in row k and column l of the module. y_{kl} stands for one pixel in the reference module. The reference module is generated based on an averaging of all modules that belong to the DMC's finder-pattern and represent a binary one. \bar{y} and \bar{x}^{ij} are the means of all the pixels referring to the reference module and the module in row i and column j of the DMC, respectively.

3 DESIGN OF THE DECODER

The choice of an appropriate channel-model is essential for the decoding success of the new designed

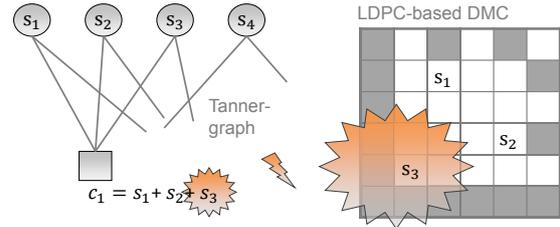


Figure 2: Symbol placement.

LDPC-based DMC. The channel-model has a high impact on:

1. The design of the employed LDPC code;
2. The computation of the SDs passed to the LDPC decoder;
3. The decoding procedure.

Therefore one has to study the DMC environment and carefully choose an appropriate channel-model to represent everything in between the embossing and the capturing of a DMC.

3.1 Channel-model in Absence of Damages

In order to describe the distribution of the correlation-coefficients computed by equation (1), one has to find an appropriate channel-model. The correlation-coefficients are separated into two data-sets referring to one-modules and zero-modules respectively. Then one has to choose a Probability-Density-Function (PDF) for each of the two data-sets that together describe the channel.

The distribution is mainly affected by the embossing-technique, the material and the camera-based system. Thus various pictures of DMCs milled on different types of material like aluminum, copper, steel, brass and different colored plastic have been analyzed. Thereby several cutting depths have been considered as well. Dependent on the material, the acquisition of the codes was done in a bright or a dark field. This can be seen in the two examples in Figure 3. In Figure 3(a) the DMC was milled on a plate of steel and the illumination setting caused the cavities to reflect the light directly into the camera's lens. Opposed to that the surface reflects the light into the camera in Figure 3(b) where the DMC was milled into a white plate of plastic.

The test of the null hypotheses that the one-samples and the zero-samples belong to a Gaussian distribution was done based on a 5% Shapiro-Wilk (SW) test as well as a 5% Anderson-Darling (AD) test. Only in a few cases the null hypotheses have not been rejected. Furthermore this was only true for

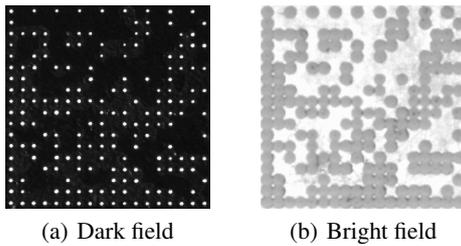


Figure 3: DMC milled on a) steel b) white plastic.

the zero-modules. Because of that, another analysis was done by use of the Johnson distribution (Johnson, 1949) (Johnson et al., 1994) that provides a system of curves with the flexibility of covering a wide variety of shapes. Although the fitting to lots of different shaped samples works very good, the Gaussian approximation was chosen in the context of DMCs. The reason for that is explained using the example of Figure 3(b) and the corresponding histograms shown in Figure 4. Figure 4(a) shows the histogram of the zero-modules and the one-modules of the DMC under the situation of correct labeling of the modules. According to the employed SW-test and the AD-test the zero-modules belong to a Gaussian-distribution whereas the hypothesis for a Gaussian distribution of the one-modules has been rejected. Thus the histogram of the zero-modules on the left side in Figure 4(b) has been approximated by a Gaussian curve whereas the curve on the right side stems from a Johnson fitting to the histogram of the one-modules. It is well seen that the two histograms as well as their fitted curves overlap each other.

In contrast to the above, the codeword is not known when considering a common decoding-process of a DMC. For the purpose of estimating the PDF of the two types of modules we provisionally separate the modules into two classes by applying a threshold to the correlation coefficients. The only difference between Figure 4(b) and Figure 4(c) is that the fitting curve to the histogram of the one-modules in Figure 4(b) is obtained by a Johnson fitting whereas the approximation in Figure 4(c) has been done by using a Gaussian fitting. The zero-samples have been approximated by a Gaussian-PDF in both cases as the hypothesis-test has been successful. As seen in Figure 4(b), the approximation of the histogram with the Johnson PDF leads to an overfitting. This suggests a good confidence for correlation values just below the tentative threshold to belong to zero-modules. In reality, this is not the case since the histograms of the two classes heavily overlap. The suggested high confidence leads to large log-likelihood ratios (LLRs) which are used in the subsequent decoding algorithm. In this case the advantage of soft-decoding is lost. Furthermore, when calculating the LLRs based on a

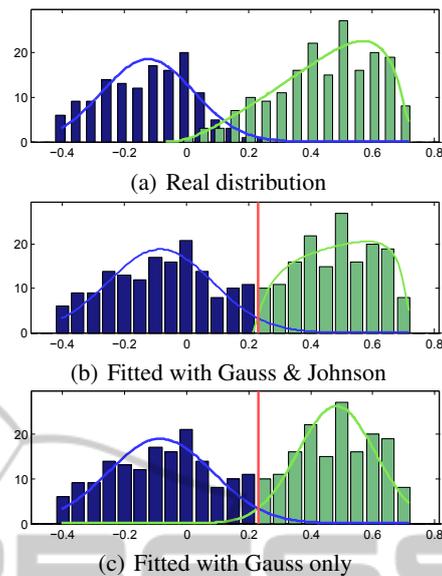


Figure 4: Histograms of correlation-coefficients separated into one-modules and zero-modules.

Gaussian approximation, the histogram-skewness that in many cases is responsible for the failure of the hypothesis test, does not have a critical effect.

3.2 Channel-model for Damaged DMCs

The situation changes a lot when taking possible damages into account. In industrial environment, these are typically blots, scratches, dirt and rust as well as effects caused by unequal illumination or soiled camera-lenses. This leads to a change of the gray-value distributions. In most cases one can observe a stretching of the histograms that refer to ones and zeros respectively. Because of that a two-state Markov-channel is utilized that includes possible effects caused by damages. The resulting channel-model can be seen in Figure 5. The two states of the Hidden-Markov Model (HMM) represent the following two sub-channels:

Good Channel: This sub-channel is an Additive White Gaussian Noise (AWGN) channel as described in Section 3.1. This channel does not consider the above mentioned damages and thus is referred to as good channel.

Bad Channel: The second sub-channel is denoted as bad channel and takes damages into account. It turns out to be an AWGN channel as well, but with larger variances compared to the good channel.

Thus the whole model represents a channel with memory whose behavior is dependent on the current underlying channel state. Moreover it is a Markov-modulated Gaussian channel (MMGC) since it can be

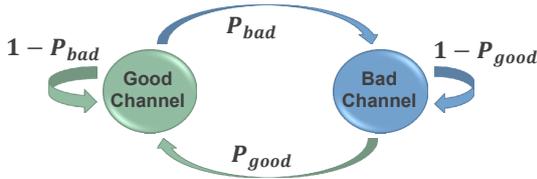


Figure 5: Channel-model based on a two-state hidden-markov model.

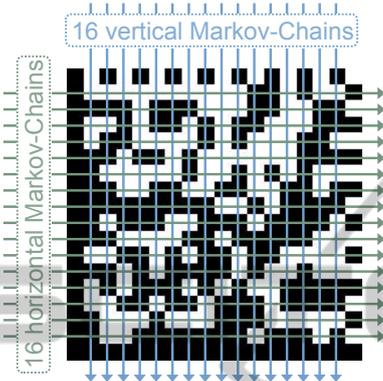


Figure 6: Two-dimensional HMM.

described as a memoryless AWGN channel parameterized by the noise variances. The probabilities for a transition from the good to the bad sub-channel and vice versa are denoted as P_{bad} and P_{good} respectively.

3.3 ED2D-algorithm

The random-like connection of symbol-nodes with check-nodes in a LDPC-code's Tanner graph can be interpreted as a build in interleaver. In traditional approaches channel interleavers are used to obtain a channel which is assumed to be memoryless. However, it has been shown (Wadayama, 2000) (Garcia-Frias, 2004) (Ratzer, 2002) (Eckford, 2004) that significant improvement is obtained by use of an Estimation-Decoding (ED) algorithm that takes the channel's memory into account.

The ED-algorithm is based on the so called Markov-LDPC factor graph which comprises two subgraphs, namely the LDPC code's Tanner-graph and the Markov chain. On the Markov-subgraph a state-estimation is computed by use of the Forward-Backward algorithm that is similar to the BCJR-algorithm (Bahl et al., 1974). This algorithm is bit-wise connected with the Belief-Propagation - algorithm (BPA) (Gallager, 1962) on the LDPC-subgraph to form the ED-algorithm. So far ED of LDPC codes has only been applied to time dependent and thus one-dimensional systems.

Considering the application of Estimation-Decoding on DMCs the one-dimensional timescale

turns into a geometry of two dimensions. This leads to a replacement of one Markov-chain by several Markov-chains. In our Estimation-Decoding in two dimensions (ED2D), we assign a sub-Markov-chain to each row and each column of the DMC's data region as depicted in the example in Figure 6. This way the state-estimation referring to a single module is based on a horizontal Markov-chain and a vertical Markov-chain. The complete 2D-Markov-LDPC factor graph that the ED2D-algorithm is based on is depicted in Figure 7 and the messages of one sector are shown in Figure 8. For clarity purposes only the messages for the horizontal Markov-chain are depicted. r and c are the indices for the rows and columns of the 2D-Markov-subgraph. The check-nodes c and the symbol-nodes x are part of the LDPC-subgraph whereas the state-nodes s and the channel-nodes (black squares) belong to the Markov-subgraph. The soft-decisions (SDs), that the ED2D-algorithm receives from our image-processing part (Section 2) are denoted by y . The noise added to the binary value of a DMC-module in row r and column c is assumed to stem either from the good sub-channel or the bad sub-channel of the MMGC which is dependent on the state that the state-node $s_{r,c}$ is estimated to be in. $S = \{G, B\}$ is the set of states a state-node $s_{r,c}$ can be in, where G and B represent the good and the bad sub-channel respectively. The forward and backward messages are represented by α and β respectively. The channel-message ζ is sent from the 2D-Markov-subgraph to the LDPC-subgraph. χ is the extrinsic information passed from the LPDC-subgraph to the 2D-Markov-subgraph. The messages of the 2D-Markov-subgraph are computed as follows.

Forward-message α :

$$\alpha_{r,c+1}^h(s_{r,c+1}) = \sum_{s_{r,c} \in S} Pr(s_{r,c+1} | s_{r,c}) \alpha_{r,c}^h(s_{r,c}) \cdot \sum_{x_{r,c} \in \{0,1\}} Pr(x_{r,c} | \chi_{r,c}) Pr(y_{r,c} | x_{r,c}, s_{r,c}) \quad (2)$$

Backward-message β :

$$\beta_{r,c}^h(s_{r,c}) = \sum_{s_{r,c+1} \in S} Pr(s_{r,c+1} | s_{r,c}) \beta_{r,c+1}^h(s_{r,c+1}) \cdot \sum_{x_{r,c} \in \{0,1\}} Pr(x_{r,c} | \chi_{r,c}) Pr(y_{r,c} | x_{r,c}, s_{r,c}) \quad (3)$$

where $Pr(s_{r,c+1} | s_{r,c})$ is one of the four transition probabilities of Figure 5. The computation of the messages α^v and β^v for the vertical Markov-chains of Figure 6 are likewise. The channel-message ζ passed to the LDPC-subgraph is computed based on the mes-

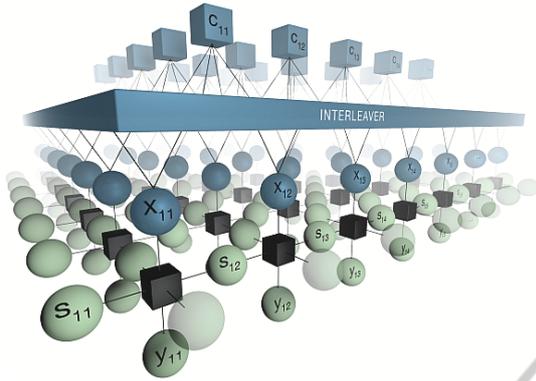


Figure 7: 2D-Markov-LDPC factor graph.

sages α^h and β^h of the horizontal Markov-Chain and the messages α^v and β^v of the vertical Markov-Chain:

$$\zeta_{r,c} = \log \frac{Pr(x_{r,c} = 0 | \alpha_{r,c}^h(s_{r,c})\beta_{r,c+1}^h(s_{r,c+1}))}{Pr(x_{r,c} = 1 | \alpha_{r,c}^h(s_{r,c})\beta_{r,c+1}^h(s_{r,c+1}))} + \log \frac{Pr(x_{r,c} = 0 | \alpha_{r,c}^v(s_{r,c})\beta_{r+1,c}^v(s_{r+1,c}))}{Pr(x_{r,c} = 1 | \alpha_{r,c}^v(s_{r,c})\beta_{r+1,c}^v(s_{r+1,c}))} \quad (4)$$

with

$$Pr(x_{r,c} = 0 | \alpha_{r,c}^h(s_{r,c}), \beta_{r,c+1}^h(s_{r,c+1})) = \sum_{s_{r,c} \in \mathcal{S}} \sum_{s_{r,c+1} \in \mathcal{S}} Pr(y_{r,c} | x_{r,c} = 0, s_{r,c}) \cdot Pr(s_{r,c+1} | s_{r,c}) \alpha_{r,c}^h(s_{r,c}) \beta_{r,c+1}^h(s_{r,c+1}) \quad (5)$$

h and v refer to horizontal and vertical rows respectively. Concerning the application of the ED2D-algorithm in the context of DMCs, the DMC's finder-pattern offers another advantage next to the original purpose. Since the values of the finder-pattern are always known, the corresponding messages χ do not change during the iterative ED2D-decoding so that

$$Pr(x_{r,c} | \chi_{r,c}) = \begin{cases} 0 & , x_{r,c} = 0 \\ 1 & , x_{r,c} = 1 \end{cases} \quad \forall x_{r,c} \in \mathcal{F}_1 \quad (6a)$$

and

$$Pr(x_{r,c} | \chi_{r,c}) = \begin{cases} 1 & , x_{r,c} = 0 \\ 0 & , x_{r,c} = 1 \end{cases} \quad \forall x_{r,c} \in \mathcal{F}_0 \quad (6b)$$

with $\mathcal{F}_1 = \{\text{one-modules of the finder-pattern}\}$ and $\mathcal{F}_0 = \{\text{zero-modules of the finder-pattern}\}$. The channel-message ζ and the extrinsic message χ represent the interface from the 2D-Markov-subgraph to the LDPC codes Tanner-graph on which the messages are computed based on a common BPA (Gallager, 1962).

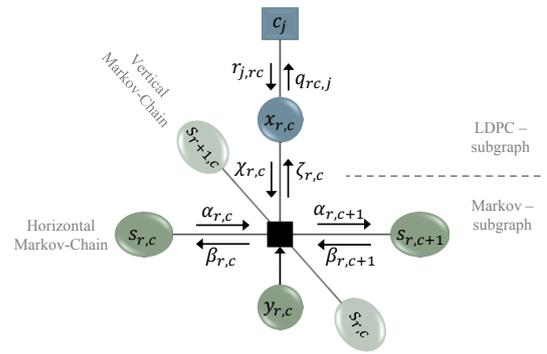


Figure 8: Local messages in the 2D-Markov-LDPC factor graph.

4 IMPLEMENTATION & TEST

The ED2D-algorithm has been tested based on a DMC of size 26×26 . The data has been encoded with a rate 0.61 regular PEG-LDPC code of length $n = 576$. The finder-pattern that surrounds the 24×24 size data region and the code rate referring to the DMC size have been chosen conforming to standard (ISO/IEC, 2000). The 576 bits of the LDPC codeword have been placed in the data region using the optimization technique described in (Probet et al., 2010).

For comparison purposes the new designed LDPC-based DMC and the original RS-based version have been milled one next to the other on three different kinds of material. For both versions, the information to be encoded, the DMC-size and the code rate have been chosen identically. In addition, the conditions of the following acquisition and image processing were exactly the same for all DMC-pairs. To include possible damages, a simulation of water drops and oil drops was integrated. The simulation ensured that both versions of the DMC were interfered with identical damages. An example of the damage-simulation can be seen in Figure 9. For this test plates of brass, aluminum and grey plastic have been used. Each DMC was interfered with 10 simulated versions of oil drops and water drops respectively. Thus, for each material there have been two DMC versions pairwise interfered with 20 different disturbances. All in all 60 interfered LDPC-based DMCs were compared with 60 RS-based DMCs affected by exactly the same damages.

The test results are shown in Figure 10. Successful decodings are counted according to the material and the type of damages. The last line shows the cumulative percentage of succeeded decodings. With a success rate of 92% our LDPC-based DMC decoded with the ED2D-decoder clearly outperforms the standard DMC of which only 53% succeeded in decoding.

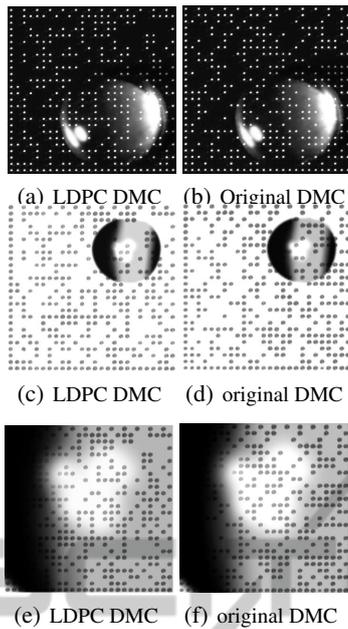


Figure 9: DMC-pairs milled on the same plate of brass and interfered with simulated water drop (a and b) and aluminum and interfered with simulated oil drop of size larger than the DMC (c, d, e and f).

Successfully decoded		ED2D	RS
Aluminum	water	9	4
	oil	8	4
	total	17	8
Brass	water	10	7
	oil	10	10
	total	20	17
Plastic	water	10	4
	oil	8	3
	total	18	7
Total	water	97%	50%
	oil	87%	57%
	Total	92%	53%

Figure 10: comparison results of 60 tests.

5 CONCLUSIONS

For the decoding of LDPC-based DMCs an algorithm called ED2D-algorithm was established. This decoding-algorithm is an extension of the one-dimensional ED-algorithm. ED2D-decoding is based on a two-dimensional Hidden-Markov-Model that has been constructed in order to include possible damages into the underlying channel-model. Two types of damages that are typical in industrial environment were simulated. Based on the damage-simulation a testing showed a superior decoding behavior of our LDPC-based DMCs decoded with the ED2D-algorithm compared to the standard RS-based DMC.

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