RELATIVITY AND CONTRAST ENHANCEMENT

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Abstract: In this paper we present a novel mathematical model for color image processing. The proposed algebraic structure is based on a special mapping of color vectors into the space of bi-quaternions (quaternions with complex coefficients) inspired by the theory of relativity. By this transformation, the space of color vectors remains closed under scalar multiplication and addition and limited by upper and lower bounds. The proposed approach is therefore termed Caged Image Processing (KIP). We demonstrate the usability of the new model by a color image enhancement algorithm. The proposed enhancement algorithm prevents information loss caused by over saturation of color, caused when using Logarithmic Image Processing (LIP) approach. Experimental results on synthetic and natural images comparing the proposed algorithm to the LIP based algorithm are provided.

1 INTRODUCTION

Digital images are defined as functions with real (usually discrete) bounded values over a non-empty spatial domain $D \in \mathbb{Z}^2$. They can be categorized by their values as either scalar (gray-scale) images or vector(color) images. In the scalar images, the value in a point (*i.e.*, pixel) is the measure of the luminosity in that pixel and it is referred as the gray level or intensity. Vector images are usually color images where the value of each pixel is a 3D vector representing the red (R), green (G), and blue (B) luminosity.

The often used mathematical model for manipulating digital images is the classical one, based on real number algebra. This approach places no limitation on the image values, implicitly regarding the image values as the whole Euclidean space. Practically, image values are truncated as soon as they go out of the bounds (higher or lower). This approach, however, inherently causes information loss.

The Logarithmic Image Processing (LIP) approach is a well established mathematical model that aims to define a bounded algebraic structure that is closed under addition and scalar multiplication. It has been proved (Pinoli, 1997) that the LIP model is consistent with several properties of the Human Visual System (HVS). The classical LIP model (Pinoli, 1997) is designed for gray level images and sets an upper bound on the image level range. An extension to color images was presented by Patrascu and Buzuloiu (Patrascu and Buzuloiu, 2001) with an additional lower bound.

In this paper we introduce a new mathematical model for representing and manipulating image color values. We map the color vector to a normalized bi-quaternion (quaternion with complex coefficients) number. By this transformation, the space of color vectors remains close under scalar multiplication and addition and limited by upper and lower bounds. The proposed approach is therefore termed Cages Image Processing (KIP).[1] The proposed mapping is based on the theory of amplitude limited vectors (Coleman, 2006). In this work he developed the theory of ampli-

[1] To distinguish it from CIP (Color Image Processing).

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tude limited vectors and proved its connection to special relativity. This principle has been extended for the electromagnetic field in (Coleman and Kolaman, 2008). In this paper, we show that the LIP model is a particular case of our model and hence establish a similar connection to the HSV.

To summarize, in this paper we make the following contributions. First, we introduce a new mathematical model for manipulating color images. Second, we prove the connection to the LIP model and to the HSV.

Third, we use the new representation for enhancement of color images and show its advantage over existing approaches.

This paper is organized as follows: Section 2 reviews related works on Quaternion Image Processing (QIP) and Logarithmic Image Processing. Section 3 present the mathematical details of our approach. Based on the proposed mathematical model we present a color enhancement algorithm in Section 4. The experimental results are reported in Section 5. Finally, we draw conclusions in Section 6.

2 RELATED WORK

In this section we review the most relevant references for our presentation.

Quaternion Image Processing (QIP) defines each color pixel as a pure Quaternion number (see Section 3.1), *i.e.*,

$$\mathbf{v}_{rgb}(m,n) = r(m,n)\cdot i + g(m,n)\cdot j + b(m,n)\cdot k, \quad (1)$$

where $r(m,n), g(m,n), b(m,n)$ represent red green and blue values respectively, $\mathbf{v}_{rgb}(m,n)$ the full color image and (n,m) the pixel location. Fourier transform (Ell and Sangwine, 2006), color correlation (Moxey et al., 2003) and principle component analysis (Le Bihan and Mars, 2004) have been extended to quaternion arithmetic. Other examples for utilizing the quaternion representation are given in (Ell and Sangwine, 2008).

While this approach has many advantages, it still lacks the ability to be amplitude limiting and thus is not bounded under addition and subtraction. In this work we propose a different approach that will use the advantages of QIP together with amplitude limitation as will be presented in the following section.

The addition between two gray level images, in the classical LIP model (Pinoli, 1997), is defined by

$$f(n,m) \oplus g(n,m) = f(n,m) + g(n,m) - \frac{f(n,m)g(n,m)}{M}, \quad (2)$$

where M is the maximum gray level value. With the definition of subtraction (see (Pinoli, 1997)) the space of gray tone images under the LIP model is bounded from above, *i.e.* $f(n,m) \in (\infty, M)$.

In (Patrascu and Buzuloiu, 2001) Patrascu redefines the addition/subtraction operators, such that its result will be bounded by upper/lower values $(-1,1)$ using the following equation

$$x[\pm]y = \frac{x \pm y}{1 \pm x \cdot y}. \quad (3)$$

As mentioned above, Logarithmic Image Processing (LIP) algebraic structure was proven to have direct connection to Human Visual System (HVS). This model has many applications such as High Dynamic Range (HDR) compression, Segmentation (Ji et al., 2006), image restoration (Debayle et al., 2006) and contrast enhancement(Deng, 2009), to name a few.

3 BI-QUINOR REPRESENTATION OF RGB PIXELS

In this section we present our novel representation of an RGB pixel as a biquaternion with unit norm. The following presentation is consistent with a recent publication on amplitude limited vectors(Coleman and Kolaman, 2008).

3.1 Quaternions and Bi-quaternions

Quaternion space is the origin of modern vector analysis. it was first presented by Hamilton (Hamilton, 1866), 162 years ago. Many Color Image Processing (CIP) algorithms have been adopted to the quaternion representation, (see Section 2).

A quaternion $q \in \mathbb{H}$ number, has a real part and three imaginary parts and can be written as

$$q = a + b\cdot i + c\cdot j + d\cdot k, \quad (4)$$

where $a,b,c,d \in \mathbb{R}$ and i,j,k are its basis elements. The addition and multiplication of quaternion numbers are associative as in familiar algebra. The multiplication is, however, not commutative, and is defined by the product rule of its basic elements:

$$i^2 = j^2 = k^2 = ijk = -1 \quad (5)$$

and by the regular use of the distributive law.

It is common to refer to a in (4) as the quaternion scalar part, denoted by $S(q)$, and to $bi + cj + dk$ as its vector part, denoted by $V(q)$. In case that $a = 0$ the quaternion number is called *pure-quaternion*.

Similar to complex numbers, quaternion conjugate is defined by

$$q^* = a - bi - cj - dk, \tag{6}$$

and quaternion norm, which is used in the following definition, is given by

$$||q|| = qq^* = s^2 + a^2 + b^2 + c^2. \tag{7}$$

Definition 1. *A quaternion $q \in \mathbb{H}$ for which $||q|| = 1$, called* quinor *(quaternion of unit norm).*

The set of quinors, $\mathbb{H}^u = \{q \in \mathbb{H}, ||q|| = 1\}$ is a proper subset of the quaternions, $\mathbb{H}^u \subset \mathbb{H}$. Any quaternion can be projected into the set of quinors by dividing it by its norm, $q_u = \frac{q}{||q||}$.

A complex or complexified quaternion is a quaternion number with complex coefficient (Sangwine, 2002). In the literature they are also known as biquaternions (Hamilton, 1866). The biquaterions can be considered as a tensor product between $\mathbb{C}$ and $\mathbb{H}$ and is denoted by $\mathbb{H}_{\mathbb{C}}$. This means that any biquaternion, $q \in \mathbb{H}_{\mathbb{C}}$, can be expanded to the form of

$$q = (s_r + is_i) + (a_r + \hat{i}a_i) \cdot i + (b_r + \hat{i}b_i) \cdot j + (c_r + \hat{i}c_i) \cdot k \tag{8}$$

where $s_r, s_i, a_r, a_i, b_r, b_i, c_r, c_i$ are real coefficients and $\hat{i} = \sqrt{-1}$ denotes the regular complex basis element. Note that regular quaternion is a special case of a biquaternion when $s_i, a_i, b_i, c_i = 0$. Similar to regular quaternions we have the following definition.

Definition 2. *A bi-quaternion $q \in \mathbb{H}_{\mathbb{C}}$ for which $||q|| = 1$, is called bi-quinor.*

The set of all bi-quinors is denoted by $\mathbb{H}^u_{\mathbb{C}}$.

3.2 Amplitude Limited Vector Theory

In this section we define the mapping that attaches a bi-quinor representation to a 3D color pixel.

We commence with a normalized[2] pure Quaternion representation of a pixel color, as in Eq. (1)

$$\mathbf{v}_{rgb} = R \cdot i + G \cdot j + B \cdot k, \tag{9}$$

where R , G , and B are the normalized[3] red, green, and blue color channel values respectively, and $||\mathbf{v}_{rgb}|| < 1$. Note that this work is not limited to a particular color representation. Our experiments make use of the common RGB space, although similar results have been obtained using alternative, formats such as HSI, YCbCr, or YUV.

The bi-quinor representation is presented in the following definition:

[2] $||\mathbf{v}_{rgb}|| < 1$

[3] This can be obtained by dividing r,g,b by their supremum multiplied by square root of three i.e. $R = \frac{r}{sup(r) \cdot \sqrt{3}}$

Definition 3. *Let $\mathbf{v}_{rgb} \in \mathbb{H}$ be a pure-quaternion such that $||\mathbf{v}_{rgb}|| < 1$. The BQ transform, $BQ : \mathbb{H} \to \mathbb{H}^u_{\mathbb{C}}$ is defined by*

$$BQ(\mathbf{v}_{rgb}) = \gamma - \hat{i} \cdot \gamma \cdot \mathbf{v}_{rgb}, \tag{10}$$

where

$$\gamma = \frac{1}{\sqrt{1 - ||\mathbf{v}_{rgb}||}}. \tag{11}$$

The above definition of γ and $BQ(\mathbf{v}_{rgb})$ are directly connected to the theory of special relativity (Coleman and Kolaman, 2008).

It is straightforward to show that $||BQ(\mathbf{v}_{rgb})|| = 1$ for any pure-quaternion with norm less than unity. The inverse transformation, from Eq. 10, back to a pure-quaternion number,$\mathbf{v}_{rgb}$, representing color pixel is given in the following definition.

Definition 4. *Let $BQ(q) \in \mathbb{H}^u_{\mathbb{C}}$ be a bi-quinor obtained using Eq.* (10) *and* (11). *The inverse transformation $BQ^{-1} : \mathbb{H}^u_{\mathbb{C}} \to \mathbb{H}$ is given by*

$$\mathbf{v}_{rgb} = \frac{V(BQ(q))}{S(BQ(q))} \cdot \hat{i}, \tag{12}$$

where $S(\cdot)$ and $V(\cdot)$ are the scalar part and vector part of a quaternion as defined above, and $\mathbf{v}_{rgb} \in \mathbb{H}$ such that $||\mathbf{v}_{rgb}|| < 1$.

It follows, that $\mathbf{v}_{rgb}$ will always obey $||\mathbf{v}_{rgb}|| < 1$ while being represented as a bi-quinor. This limitation of $\mathbf{v}_{rgb}$ in a three dimensional space is termed *Caging* because the vector is inside a cage from which it cannot escape.

The algebra of bi-quinors has two basic algebraic operations:

- composition
- de-composition

Vector addition in bi-quinor form is non-linear, and to distinguish it from regular addition, we will call it composition. Bi-quinor composition is done by multiplying together two bi-quinors.

Definition 5. *Bi-quinor composition:*

$$\mathbf{v}_1 [+] \mathbf{v}_2 = BQ(\mathbf{v}_1) \cdot BQ(\mathbf{v}_2) \tag{13}$$

Definition 6. *Bi-quinor de-composition:*

$$\mathbf{v}_1 [-] \mathbf{v}_2 = BQ(\mathbf{v}_1) \cdot BQ(-\mathbf{v}_2) \tag{14}$$

In general bi-quaternions are noncommutative, including the above operations of composition/decomposition.

Let $q_1, q_2 \in \mathbb{H}^u_{\mathbb{C}}$ where S_1, S_2 are their scalar parts respectfully and $\mathbf{V}_1, \mathbf{V}_2$ are their vector parts respectfully. Their multiplication is defined by standard quaternion multiplication

$$\begin{aligned} q_1 q_2 &= (S_1 + \mathbf{V}_1)(S_2 + \mathbf{V}_2) \\ &= S_1 \cdot S_2 - \mathbf{V}_1 \cdot \mathbf{V}_2 + \\ &\quad S_1 \mathbf{V}_2 + S_2 \mathbf{V}_1 + \\ &\quad \mathbf{V}_1 \times \mathbf{V}_2 \end{aligned} \tag{15}$$

where $x \cdot y$ is the standard dot product, and $x \times y$ is the vector cross product.

3.3 KIP Connection with LIP

Caged Image Processing (KIP) is a general case of LIP, where LIP is a scalar addition between two vectors(*i.e.*, it only limits addition of two vectors having the same direction), while KIP is a true vector addition (*i.e.*, it limits addition for all types of vectors). This can be proven by a general example showing scalar-type composition of two vectors $\mathbf{v}_1 = (x, 0, 0)$ and $\mathbf{v}_2 = (y, 0, 0)$.

KIP composition is defined in Eq. (13) and performed as in Eq. (15). For simplicity we will write the vectors as x and y, and their bi-quinor representation is defined by,

$$BQ(y) = \gamma_y - \hat{i} \cdot \gamma_y \cdot y \cdot i \tag{16}$$

$$BQ(x) = \gamma_x - \hat{i} \cdot \gamma_x \cdot x \cdot i \tag{17}$$

where $\gamma_x = (1 - x^2)^{-\frac{1}{2}}$, $\gamma_y = (1 - y^2)^{-\frac{1}{2}}$, $\hat{i}$ a standard complex coefficient and i a quaternion coefficient.

Composing/Decomposing scalar x with a scalar y using Eq. (13) and bi-quaternion multiplication given in Eq. (15) yield,

$$\begin{aligned} x[\pm]y &= BQ(y)BQ(\pm x) \qquad (18) \\ &= \gamma_x \cdot \gamma_y (1 \pm x \cdot y) - \hat{i} \cdot \gamma_y \cdot \gamma_x (x \pm y) \end{aligned}$$

Extracting the vectors back form the bi-quinor representation using Eq. (14) gives,

$$x[\pm]y = \frac{x \pm y}{1 \pm x \cdot y} \tag{19}$$

where $x, y, x[\pm]y \in [0, 1)$.

Notice that the above equations are the exact formulas used by Patrascu in (Patrascu and Buzuloiu, 2001). Hence, this example proves that LIP is a special case of KIP. The above also proves that KIP has a direct connection to HVS because LIP was proven to have a direct connection to the Weber-Fechner law in (Pinoli, 1997).

KIP and LIP producethe same result when they work on gray-scale image, but produce different results when they process color images. This difference comes from the fact that LIP composes only vectors in the same direction while KIP composes vectors in any direction. This difference becomes apparent when enhancing color images which have strong color patterns which are close to the fully saturated values. This is experimentally demonstrated in Section 5.2.

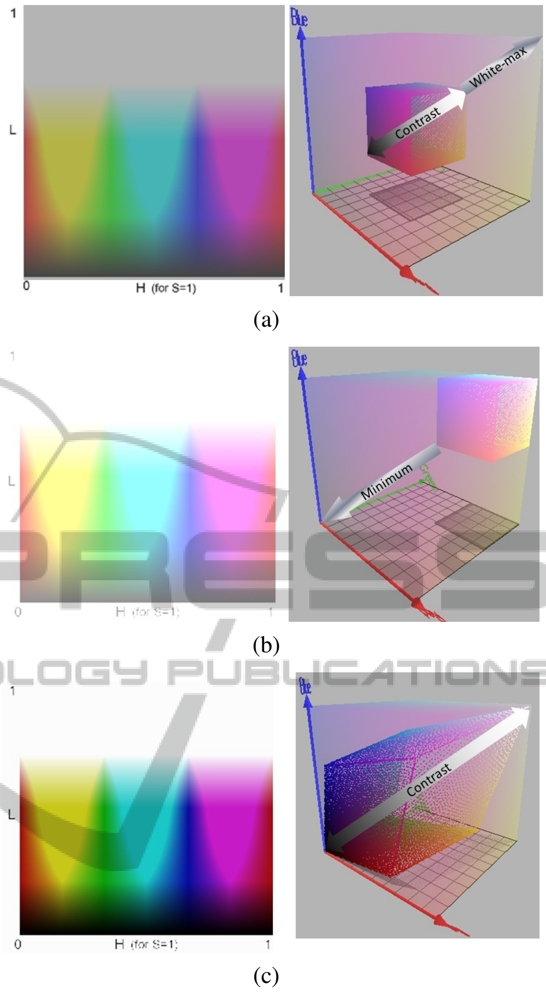

Figure 1: Color enhancement; (a) Low contrast image and its RGB scatter plot of pixel colors; (b) image after global white balance and its RGB scatter plot (c) enhanced image using the proposed scheme and its RGB scatter plot.

4 CONTRAST ENHANCEMENT ALGORITHM

Our proposed contrast enhancement algorithm uses the amplitude limited vectors principledescribed above, to improve contrast, and has no need for parameter adjustment.

The color enhancement algorithm has three stages:

1. Global maximum luminance measure.
2. Linearly adding the difference between the maximum luminance and the measured one.
3. De-composing every pixel, using Eq. (14), with the global minimum luminance.

We choose to demonstrate these stages on a synthetic example depicted in Fig 1. Let the test case image be denoted by I, and its maximum luminance value by I_{max} (Fig. (a)). We first calculate the difference between I_{max} and the maximum luminance possible, M:

$$D = M - I_{max}. \tag{20}$$

This vector is depicted in Fig. 1 at the top-right panel. The second step, consist of adding D to all the pixels in the original image I, *i.e.*

$$I_{wb} = I + D \tag{21}$$

This step is depicted in Fig. 1(b). All the operations used in the first two stages are the usual addition and subtraction of real vectors.

The last stage of our algorithm is based on the connection between Weber's law and KIP decomposition proven in section 3.3. We denote the current global minimum Luminance by $min(I_{wb})$, this value is depicted in the right panel of Fig. 1(b). Finally, Contrast enhancement is performed by KIP decomposition as in Eq. (14):

$$\begin{aligned} BQ(I_{contrast}) &= I_{wb}[-]min(I_{wb}) \\ &= BQ(I_{wb}) \cdot BQ(-min(I_{wb})) \end{aligned} \tag{22}$$

The final enhanced image is extracted from $BQ(I_{contrast})$ by using Eq. (12), and can be seen in Fig. 1(c).

5 EXPERIMENTAL RESULTS

5.1 Contrast Enhancement

We demonstrate the validity of the enhancement algorithm on several natural images with low contrast. We compare our enhancement algorithm to the LIP enhancement approach. The implementation of the LIP approach follows the first two steps described in Sec. 4, with the final step replaced by the LIP subtraction operation. Results can be seen in Fig. 2. Comparing the images one can see that using the LIP framework enhances also saturation of the color. This saturation enhancement is a side effect not always needed.

KIP results can be made the same as LIP by using a simple saturation enhancement. This enhancement is linear and quick and can be seen in Fig. 2.

1. Calculate the image luminance using a known method i.e. $Luminance = \frac{r}{3} + \frac{g}{3} + \frac{b}{3}$
2. Calculate the image chrominance i.e. $Chrominance = Image - Luminance$
3. Add together image luminance with image chrominance multiplied by a scalar i.e. $I_{new} = Luminance + 1.6 \cdot Chrominance$

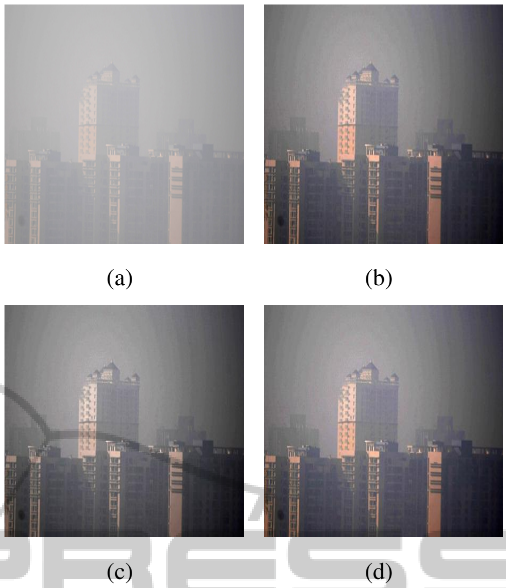

Figure 2: Contrast enhancement using proposed scheme; (a) Low contrast image; (b) contrast stretching using proposed scheme in Logarithmic Image Processing (LIP); (c) enhanced image using the proposed scheme in Caged Image Processing (KIP); (d) Improving saturation of the KIP enhanced image using a simple linear method.

5.2 Clipping Saturation Prevention

Although saturation enhancement can improve the image - it is not always necessary and sometimes degrades the color image by the clipping of color and loss of information.

We chose to compare KIP capabilities in preventing clipping of color caused by the LIP method. Because LIP treats every vector addition as a composition of its its individual elements, it can never distinguish between a vector which has a maximum value in only one of its elements. For example, consider $\mathbf{v}_1 = (0.99, 0, 0)$ with $||\mathbf{v}_1|| = 0.98$, and a vector having maximum value in more than one of its elements, for example $\mathbf{v}_2 = (0.99, 0.99, 0)$ with $||\mathbf{v}_2|| = 1.4$. Clearly $||\mathbf{v}_2|| > ||\mathbf{v}_1||$ but because LIP cannot distinguish between the two, a color clipping may occur.

An example of this can be seen by trying to enhance low contrast color patterns which are nearly saturated. Because LIP treats each color channel on its own (Patrascu and Buzuloiu, 2001), color clipping occurs. This can be seen in Fig. 3 and Fig. 4.

6 CONCLUSIONS

In this paper we introduce a new mathematical model to manipulate color images. This model is based on

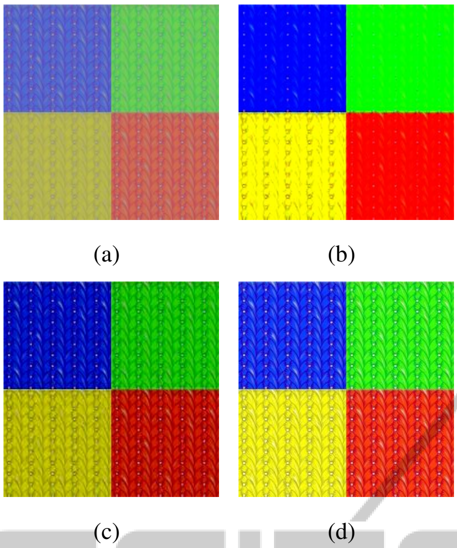

Figure 3: Contrast enhancement using proposed scheme showing clipping of color under LIP framework; (a) Low contrast image; (b) contrast stretching using proposed scheme in Logarithmic Image Processing (LIP) showing loss of information and clipping; (c) enhanced image using the proposed scheme in Caged Image Processing (KIP);(d) original image with full contrast

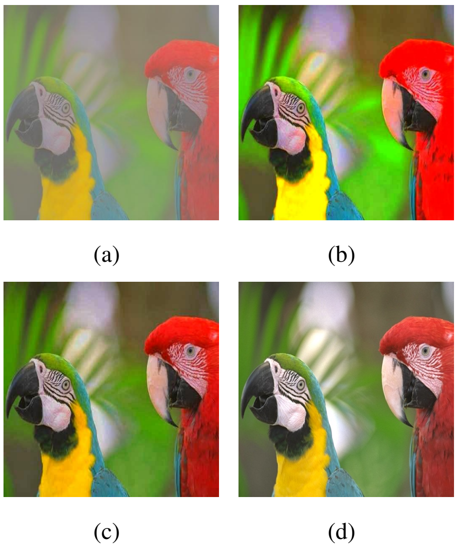

Figure 4: Contrast enhancement using proposed scheme showing clipping of color under LIP framework; (a) Low contrast image; (b) contrast stretching using proposed scheme in Logarithmic Image Processing (LIP) showing loss of information and clipping; (c) enhanced image using the proposed scheme in Caged Image Processing (KIP); (d) original image with full sontrast

principles of special relativity for which the limited amplitude vectors are most relevant to color image processing. We show that the proposed model is consistent with the LIP model and with the HVS. Using this model we introduce a color enhancement algorithm. We demonstrate its validity by several examples, and its advantage over existing methods.

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