AN ASYNCHRONOUS MULTI-AGENT SYSTEM FOR OPTIMIZING SEMI-PARAMETRIC SPATIAL AUTOREGRESSIVE MODELS

Matthias Koch and Tamás Krisztin
Vienna University of Economics and Business, Institute for Economic Geography and GIScience, Vienna, Austria

Keywords: Agent-based modelling, Asynchronous teams, ATeams, Semi-parametric spatial autoregressive modelling, Evolutionary algorithms.

Abstract: Classical spatial autoregressive models share the same weakness as the classical linear regression models, namely it is not possible to estimate non-linear relationships between the dependent and independent variables. In the case of classical linear regression a semi-parametric approach can be used to address this issue. Therefore we propose an advanced semi-parametric modelling approach for spatial autoregressive models. Advanced semi-parametric modelling requires determining the best configuration of independent variable vectors, number of spline-knots and their positions. To solve this combinatorial optimization problem we propose an asynchronous multi-agent system based on genetic-algorithms. Three teams of agents work each on a subset of the problem and cooperate through sharing their most optimal solutions. Through this system we can derive more complex relationships, which are better suited for the often large and non-linear real-world problems faced by applied spatial econometricians.

1 INTRODUCTION

Spatial autoregressive (SAR) models have seen a wide approach in dealing with empirical problems. The key difference between a classical regression and a spatial model is that the latter incorporates a so called spatial lag of the dependent variable. Both model classes assume that the impact of the independent variables on the depended variable can be modelled in a linear fashion. This might not be true in many applied cases. Therefore, the linear regression model was extended by the semi-parametric regression models. These semi-parametric regression models are able to cope with most kind of nonlinearity [see for example Fahrmeir et al]. As a result, we want to extend the SAR-models by semi-parametric modelling techniques.
Our suggested semi-parametric spatial autoregressive (SPSAR) estimation-method is based on so called truncated-splines (for details see Fahrmeir et al., 2009, page 296) and Akaike Information Criteria (AIC) minimization. The truncated spline and the spatial autoregressive estimators will be calculated via a maximum likelihood (ML). In order to model complex nonlinear relationships between the dependent and independent variables we will first choose suitable combinations of the independent variables and use them to as argument for the truncated splines. The truncated spline has an optimized number and position of knots. Therefore, we are partly faced with a combinatorial optimization problem.

In the first section of the paper we will introduce asynchronous multi-agent systems and discuss their characteristics and why they are well-suited for solving the combinatorial optimization problem at hand. The next section details the nonlinear spatial autoregressive models, the SPSAR estimation method and the nature of the optimization problem. The third section outlines the asynchronous agent architecture, while the fourth section introduces our proposed testing methodology.

2 ASYNCHRONOUS MULTI-AGENT SYSTEMS

This section provides a brief introduction to agents, multi-agent systems (MAS) and a more specific

Koch M. and Krisztin T..
AN ASYNCHRONOUS MULTI-AGENT SYSTEM FOR OPTIMIZING SEMI-PARAMETRIC SPATIAL AUTOREGRESSIVE MODELS.
DOI: 10.5220/0003292704830486
In *Proceedings of the 3rd International Conference on Agents and Artificial Intelligence* (ICAART-2011), pages 483-486
ISBN: 978-989-8425-41-6

overview of asynchronous MAS for solving large combinatorial optimization problems.

The definition of agents is laid down by Jennings and Wooldridge (1995), namely an agent is defined by possessing one or more of the four characteristics:

- Autonomy is the agent's ability to work without human interaction and have a control of their own state and actions.
- Social ability is the ability to communicate with other agents.
- Reactivity denotes the ability to respond to actions and to perceive the environment.
- Pro-Activeness is the agent's ability to work towards a goal and take initiative in actions.

A multi-agent system is a collection of loosely coupled agents, who cooperate to solve a problem (Sycara, 1998). In an MAS each agent has only limited information and problem solving capacity, so that the posed problem can only be solved through cooperation. Furthermore there is no central entity that manages the system, instead data and problem-solving are decentralized and managed by individual agents.

Asynchronous problem-solving teams (ATeams), have been proposed by Talukdar et al. (1998). They are a form of MAS, where the system-wide current best solutions of a problem are stored in a central memory. Problem-solving agents try to generate more optimal solutions, each agent with another problem-solving algorithm, while destroyer agents are deleting sub-optimal solutions from the central memory. The architecture of such an ATeam is asynchronous, agents act in an autonomous way and they exchange information through the shared memory.

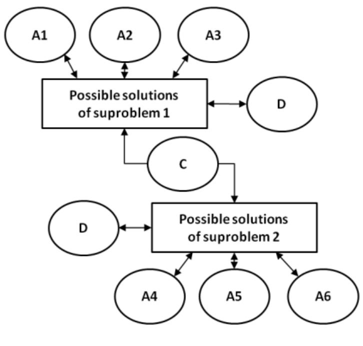

Figure 1: Multiple ATeams cooperating on subsets of a problem.

Fig 1 illustrates this principle, where agents *A1* through *A6* are working to solve the problem using algorithms *a1* through *a6*. They add their most optimal solutions to the solution population in the memory *M*. Other agents review these solutions periodically and try to come up with more optimal solutions. The destroyer agent *D* is checking the population of solutions and deletes any inferior solution which is below a threshold *t*.

For more complicated problems, multiple ATeams can be employed, with each team of agents working on a subset of a problem. Communication between the teams depends on the organization of the problem and by what degree the subsets of the problem depend on each other. Fig. 2 shows two agent teams, the first team is *A1* through *A3* and the second team *A4* through *A6*, cooperating in this way. The second team of agents builds on the population of the first and an coordinator agent *C* provides subproblems for the second team from the solutions of the first team.

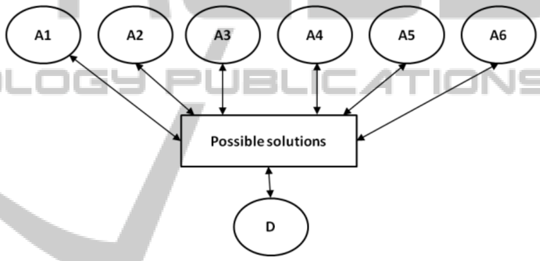

Figure 2: Architecture of an ATeam.

In some sense ATeams are similar to blackboard systems, where problem-solvers cooperate by posting the results of their calculations on a blackboard. In ATeams though the agents operate independently and unlike in blackboard systems, there is no central instance of control and agents work independently from each other (Aydin et al., 2004).

It is possible to combine ATeams with evolutionary methods, such as genetic algorithms. This can be done either with each agent as an instance of an algorithm or each agent performing the individual steps of the algorithm. In such a case one agent would implement the population selection, while others implement crossovers and genetic mutations. It is easy to implement hybrid methods, with different evolutionary algorithms, in an ATeam, since agents can be easily added or substracted from the system; this offers a flexible way of solving complex optimization problems through addition of different selection and crossover methods (Aydin et al., 2004).

Talukdar et al. (1998) used the well-known shortest path problem as a benchmark for ATeams.

By applying this methodology to the problem they demonstrated that the solutions provided by ATeams offer more reachability and can solve the problem in a more efficient manner, than more conventional methods. Other applications of similarly structured asynchronous MAS can be found in the supply-chain literature (Kazemi et al. 2007. Aydin et al. 2004) where they are still popular. We are not aware of any other applications of ATeams in the field applied spatial econometrics.

3 NONLINEAR AND SEMI-PARAMETRIC SPATIAL AUTOREGRESSIVE MODELS

This section introduces nonlinear spatial econometric models and then suggests semi-parametric modelling to account for the nonlinearity. Since this paper focuses primarily on the asynchronous multi Agent systems, this section should only be seen as a sketch for the actual SPSAR-econometric problem.

We consider the following nonlinear spatial autoregressive model (1):

$$\mathbf{Y}_n = \rho \mathbf{W}_n \mathbf{Y}_n + f(\mathbf{X}_{1,n}, \ldots, \mathbf{X}_{k,n}) + \boldsymbol{\varepsilon}_n \quad \text{where } \boldsymbol{\varepsilon}_n \sim i.N(0, \boldsymbol{I}_n \sigma^2) \tag{1}$$

In (1) $\mathbf{Y}_n$ is a *n* by 1 vector containing the dependent variable. $\mathbf{X}_n$ is a *n* by *k* matrix of observations on *k* independent variables, $\mathbf{W}_n$ is a *n* by *n* spatial weighting matrix of known constants, ρ is the spatial autoregressive parameter and $\boldsymbol{\varepsilon}_n$ is an independently normal distributed random vector with zero mean and σ^2 variance. $f(\mathbf{X}_{1,n}, \ldots, \mathbf{X}_{k,n})$ is a nonlinear continuous function with continuous derivatives form $\mathbb{R}^{n\times k} \to \mathbb{R}^{n\times 1}$. Additionally we assume that $\mathbf{X}_n$ only contains metric variables. For notational simplicity we ignore in this section the constant term of the spatial regression model.

We assume that $\mathbf{W}_n$ is either row or maximum row standardized and that the true parameter of ρ is smaller one in absolute value. Therefore, we can solve (1) for $\mathbf{Y}_n$ and get (2)

$$\mathbf{Y}_n = (\mathbf{I}_n - \rho \mathbf{W}_n)^{-1} f(\mathbf{X}_{1,n}, \ldots, \mathbf{X}_{k,n}) + (\mathbf{I}_n - \rho \mathbf{W}_n)^{-1} \boldsymbol{\varepsilon}_n \tag{2}$$

Since we do not know the specific form $f(\mathbf{X}_{1,n}, \ldots, \mathbf{X}_{k,n})$ we first use a finite truncated Taylorseries. Since this series is not practicable, we use a series of truncated splines $g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma)$ of optimized length m, where $\overline{k_\iota}$ is the set containing the optimized knots for the truncated spline and $\overline{\mathbf{X}}_i \in \{\mathbf{X}_{\mathrm{j,n}}^{\mathrm{l}} \odot \mathbf{X}_{\mathrm{o,n}}^{\mathrm{h}} | (\mathrm{j,o}) \in \mathrm{T_k} \times \mathrm{T_k}, (\mathrm{l,h})_{\mathrm{j}\neq\mathrm{o}} \in \mathrm{T_3} \times \mathrm{T_3}\} \cup \{\mathbf{X}_{\mathrm{j,n}}^{1}, \ldots, \mathbf{X}_{\mathrm{j,n}}^{3} | \mathrm{j} \in \mathrm{T_k}\}$ where $\mathrm{T_x} = \{1,2,\ldots,\mathrm{x}\}$. Hence $f(\mathbf{X}_{1,n}, \ldots, \mathbf{X}_{k,n})$ will be approximated by $\sum_{i=1}^{m} g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma)$. Since $g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma)$ represents a truncated spline, $\sum_{i=1}^{m} g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma)$ must have a linear representation: $\sum_{i=1}^{m} g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma) = \mathbf{Z}_{\mathrm{n}} \bar{\gamma}$ for given vectors $\overline{\mathbf{X}}_i$, the set $\overline{k_\iota}$ and the length m. If we use this approximation of $f(\mathbf{X}_{1,n}, \ldots, \mathbf{X}_{k,n})$ we can rewrite (2) to (3)

$$\mathbf{Y}_n \approx \rho \mathbf{W}_n \mathbf{Y}_n + \mathbf{Z}_{\mathrm{n}} \bar{\gamma} + \boldsymbol{\varepsilon}_n \tag{3}$$

Estimators for ρ, $\bar{\gamma}$ and σ^2 (we will denote estimators with ^) in (3) can be found via ML. ML leads to the following maximization problem (4) (Le Sage and Pace, 2009):

$$\begin{pmatrix} \hat{\rho} \\ \hat{\gamma} \\ \hat{\sigma}^2 \end{pmatrix} = \max_{\rho,\bar{\gamma},\sigma^2} \left\{ \frac{1}{(2\pi)^{\frac{n}{2}} \det(\mathbf{S}(\rho)^{-1}\sigma)} \exp\left(-\frac{1}{2\sigma^2} (\mathbf{S}(\rho)\mathbf{Y}_n - \mathbf{Z}_{\mathrm{n}}\bar{\gamma})' (\mathbf{S}(\rho)\mathbf{Y}_n - \mathbf{Z}_{\mathrm{n}}\bar{\gamma}) \right) \right\} \tag{4}$$

where $\mathbf{S}(\rho) = (\mathbf{I}_n - \rho \mathbf{W}_n)$. With the estimators $\hat{\rho}$, $\hat{\gamma}$ and $\hat{\sigma}^2$ we are able to calculate the AIC. We consider $\mathbf{Z}_{\mathrm{n}} \hat{\gamma}$ a good estimator for $f(\mathbf{X}_{1,n}, \ldots, \mathbf{X}_{k,n})$ if we find a minimal AIC. Since most of the econometric issues like ML are already sufficiently solved, the next section discusses the optimization procedure for finding optimal $\overline{\mathbf{X}}_i$ and number and position of the truncated spline knots.

4 SOLUTION METHODOLOGY

To optimize the problem of semi-parametric spatial autoregressive models, which we introduced in the previous section, we propose three asynchronous teams of agents, each working on a subset of the optimization. The first team attempts to optimize the number of splines, the second team adjusts the position of the spline-knots, while the third team tries to find the optimal variable vectors for the selected number of splines and positions. A coordinator agent is responsible for informing the other teams about the current most optimal results of the three problem subsets. Based on the AIC of these results, each team of agents attempts to improve upon the solution. A destroyer agent deletes in each cycle the worst solution from each of the three populations.

The system starts with a pool of randomly selected population samples, with uniform distribution. Each team consists of three agents. These agents encapsulate the selection, crossover and mutation of genetic algorithms. They deposit the new samples into a population pool, shared by all three agents. The agents differ in the crossover methods, which are applied to the members of the population. The three agent teams use the same three genetic algorithms. The termination criteria for the system is either 1000 cycles or the system terminates when no change has been detected in either of optimal solutions of the agent teams, for the last 20 cycles.

Each agent implements the following steps:

1. The first step is the evaluation of the current solution population, according to the AIC criteria.
2. In the second step a subset of individuals are selected from the population, for producing a new generation of solutions. This is done through roulette wheel selection for all agents.
3. The offspring are created by recombining elements of their parents and by mutation. The mutation is done throughout all agent types by stochastical perturbation for the newly created generation.
4. In the final step, the new generation is inserted into the population.

As mentioned 3previously, there are three types of agents in the system; each of them uses different crossover methods:

- The first type of agent (*g1*) uses single-point crossover for creating new solutions,
- the second agent type (*g2*) employs two-point crossover and
- the third type of agents (*g3*) uses a random crossover method, whereby a binary random vector - corresponding to the length of the first parent - is created. Where the vector has a value of 1, the matching value of the first parent is chosen, else the equivalent value of the second parent is selected for the offspring.

The communication agent selects in each cycle the best solution – determined by the AIC value – from each of the three ATeams and informs the other ATeams about the parameters of this selection.

5 CONCLUSIONS

This paper derives an optimization for semi-parametric spatial autoregressive models, through asynchronous multi-agent teams. The agent teams employ genetic algorithms and cooperate to find the optimal solution for this large combinatorial optimization problem.

This agent-based model offers an elegant method for applied spatial econometrics. Through combined agent teams the problem can be subdivided and solved on separate levels. In addition it is also possible to try other then evolutionary methods for the agents, even combining hybrid approaches. Due to the characteristics of ATeams such an extension can be implemented to utilise the proposed methodology for other spatial econometric problems.

REFERENCES

Aydin, E. M., Fogarty, T. C., 2004. Teams of autonomous agents for job-shop scheduling problems: an experimental study. Journal of Intelligent Manufacturing, 15, 455-462.

Fahrmeier, L., Kneib, T., Lang, S., 2009. Regression: Modelle, Methoden und Anwendungen. Springer, Berlin/Heidelberg/New York.

LeSage, J., Pace, R. K., 2009. An introduction to spatial econometrics. Tylor & Francis Group, Boca Raton/London/New York.

Kazemi, A., Fazel Zarandi, M. H., Moattar Husseini, S. M. 2007. A multi-agent system to solve the production-distribution planning problem for a supply-chain: a genetic algorithm approach. International Journal of Advanced Manufacturing Technology, 44, 180-193.

Sycara, K. P., 1998. Multiagent systems. Artificial Intelligence Magazine, 19(2), 79-92.

Talukdar, S., Baerentzen, L., Gove, A., De Souza, P., 1998. Asynchronous teams: cooperation schemes for autonomous agents. Journal of Heuristics, 4(4), 295—321.

Wooldridge, M., Jennings, N. R., 1995. Intelligent agents: theory and practice. Knowledge Engineering Review, 10, 115-152.