THE MULTI-AGENT PLANNING PROBLEM

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Abstract: The purpose of this paper is to present a Multi-Agent planner for a team of autonomous agents. The approach is demonstrated by the Multi-Agent Planning Problem, which is a variant of the classical Multiple Traveling Salesmen Problem (MTSP): given a set of *n* goals/targets and a team of *m* agents, the optimal team strategy consists of finding *m* tours such that each target is visited only once and by only one agent, and the total cost of visiting all nodes is minimal. The proposed solution method is a Genetic Algorithm Inspired Steepest Descent (GAISD) method. To validate the approach, the method has been benchmarked against MTSPs and routing problems. Numerical experiments demonstrate the goodness of the approach.

1 INTRODUCTION

Improving the capability of a system to plan and to act autonomously represents an important direction in the field of autonomy and artificial intelligence. Many applications, from space exploration (http://marsrovers.nasa.gov, ; Hayati et al., 1997; Baumgartner, 2000) to search and rescue problems (Birk and Carpin, 2006; Sariel and Akin, 2005; Jacoff et al., 2002; Carpin et al., 2006), have underlined the need for autonomous systems that are able to plan strategies either with or without human feedback.

However, many 'autonomous' vehicles are provided with no decision or planning capability. They are typically able to execute given commands (i.e. reaching a particular point or using a required instrument), but they are not able to decide by themselves a sequence of tasks or a plan to achieve. Mission strategy and the goals to accomplish are usually decided by human operators.

Our long-term goal is to realize a multi-agent planner for a team of autonomous vehicles to cooperatively explore their environment (Giardini and Kalmár-Nagy, 2007). To achieve this goal, we require that the vehicles/agents be able to compute a coordinated mission strategy. Specifically, the overall planning problem can be formulated as finding a near-optimal set of paths that allows the team of agents to visit a given number of targets in the shortest amount of time.

This problem is quite similar to the well-known Multiple Traveling Salesmen Problem (MTSP) (Mitrovic-Minic and Krishnamutri, 2002; Bektas, 2006; Hong and Padberg, 1977; Singh and Baghel, 2009), a generalization of the Traveling Salesman Problem (TSP) (Johnson and McGeoch, 1997; http://www.tsp.gatech.edu/, ; Gutin and Punnen, 2002) that can be stated as follow:

Given n nodes (targets) and m salesmen (agents) located at the same node (the depot), the MTSP consists of finding m closed tours (that start and end at the depot), such that each target is visited only once and by only one agent and the total cost of visiting all nodes is minimal.

MTSP has been used for modeling many real situations, from scheduling activities of companies and industries to cooperative planning problems. See for example (Carter and Ragsdale, 2002), where MTSP is used for modeling the pre-printed insert scheduling problem. Planning problems have also been investigated through MTSP formulations, specifically in (Stentz and Brummit, 1998; Stentz and Brummit, 1996), where a dynamic mission planning system for multiple mobile robots operating in unstructured environments is presented (analysis of planetary exploration), or in (Yu et al., 2002), where the MTSP for-

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mulation is used to describe a path planning problem for a team of cooperative vehicles.

An important and well-studied extension of the MTSP is the Vehicle Routing Problem (http://neo.lcc.uma.es/radi aeb/WebVRP/, ; Pereira et al., 2002), where a fleet of vehicles of different capacities, based at either one or *several* depots, must deliver different customer demands (the number of vehicles is often considered as a minimization criterion in addition to total traveled distance).

In this work, the problem of planning a set of strategies for cooperatively exploring the environment with a fleet of vehicles is modeled as a variant of the classical MTSP, referred to as the Multi-Agent Planning Problem (MAPP):

Given n nodes (targets) and m salesmen (agents) located at different depots, the MAPP consists of finding m tours such that each target is visited only once and by only one agent, while minimizing a given cost function.

Here we present a multi-agent planner for obtaining good quality MAPP solutions. The method is based on a Genetic Algorithm Inspired Steepest Descent method, simply called GAISD.

The paper is organized as follows: after the MAPP formulation (Section 2), and an overview of how similar problems (MTSP above all) are solved (Section 3), the proposed GA-Inspired Steepest Descent method is described in Section 4. Results are then reported in Section 5, conclusions in Section 6.

2 MAPP FORMULATION

Graph theory has been instrumental for analyzing and solving problems in areas as diverse as computer network design, urban planning, and molecular biology. Highly regarded books on graph theory include (Bondy and Murty, 1976; Diestel, 2005).

In this Section we define the problem of computing an optimal strategy for a team of agents to visit a set of known targets.

Let $T = \{t_1, \ldots, t_n\}$ be the set of *n targets* (goals) to be visited. The *i*-th target t_i is an object located in Euclidean space and its position is specified by the vector $\mathbf{r}(t_i)$. Let $A = \{a_1, \ldots, a_m\}$ denote the set of *m* agents, each one specified by the $\mathbf{r}(a_i)$ in Euclidean space.

For the *i*-th agent, the augmented vertex set is given by $V_i = T \cup a_i$ and the configuration space of the problem is the complete graph $K_{n+1}(V_i)$.

The weight associated with each edge is given by the Euclidean distance between the corresponding locations, i.e. $w(v_i, v_j) = w(v_j, v_i) = \| \mathbf{r}(v_i) - \mathbf{r}(v_j) \|$, with $v_i, v_j \in V$, rendering $K_{n+1}(V_i)$ a *weighted* and *symmetric* graph.

Let P_i denote a path of length k_i starting at vertex a_i, the Multi-Agent Planning Problem is equivalent to finding a set of *m pairwise disjoint* paths P_i

$$\mathbb{P} = \{P_1, \ldots, P_m\}, \quad \sum_{i=1}^{m} k_i = n, \tag{1}$$

such that the length of the longest path

$$\mathcal{W}_m(\mathbb{P}) = \max_i \mathcal{W}(P_i) \tag{2}$$

is minimal (this is the so-called *minmax* problem). In other words, *m* agents have to visit *n* targets in the least amount of time, with the constraint that every target is only visited once. Note that the agents all have different starting positions (paths are pairwise disjoint) and can visit a different number of targets (no constraints are imposed on the path lengths k_i).

The Multi-Agent Planning Problem is a variant of the classical Multiple Traveling Salesman Problem, that can be formulated as follows. Let $T = \{t_1, \ldots, t_n\}$ be the set of *n* targets to be visited and let *a* denote the *unique* depot the *m* agents share. The augmented vertex set is given by $V = T \cup a$ and the configuration space of the problem is the complete graph $K_{n+1}(V)$.

Let C_i denote a cycle of length k_i starting and ending at vertex *a* (the depot). The Multiple Traveling Salesmen Problem can be formulated as finding *m* cycles C_i of length k_i

$$\mathbb{C} = \{C_1, \ldots, C_m\}, \quad \sum_{i=1}^{m} k_i = n + m, \tag{3}$$

such that each target is visited only once and by only one agent and the sum of the costs of all the *m* tours C_i

$$\mathcal{W}(\mathbb{C}) = \sum_{i=1}^{m} \mathcal{W}(C_i) \tag{4}$$

is minimal.

3 OVERVIEW OF SOLUTION METHODS

The obvious difficulty with the Multiple Traveling Salesman Problem and consequently the Multi-Agent Planning Problem is their combinatorial nature (they are NP-hard, and there is no known deterministic algorithm that solves them in polynomial time).

A common approach is to *transform* the studied

MTSP into an equivalent Traveling Salesman Problem, for which solutions can be found by applying both well-known exact methods (e.g. branch-and-bound algorithms and linear programming (Tschoke et al., 1995; Balas and Toth, 1985; Schrijver, 1986)) and approximate algorithms such as Genetic Algorithms, Simulated Annealing and Ant System (Carter, 2003; Kulich et al., 2004). For example, in (Gorenstein, 1970; Tang et al., 2000) the authors proposed to transform the MTSP into an equivalent TSP by adding *dummy* cities and edges with ad-hoc null or infinite costs. However, as stated in (Gavish and Srikanth, 1986; Junjie and Dingwei, 2006; Kara and Bektas, 2006), transforming the MTSP into an equivalent TSP can result in a harder problem to solve (i.e. the equivalent TSP can be more arduous than solving an ordinary TSP). Similar approaches are investigated in (Singh and Baghel, 2009; Orloff, 1974; Bellmore and Hong, 1974).

The first attempt to solve large-scale MTSPs is given in (Gavish and Srikanth, 1986), where a branch-and-bound method (the most widely adopted technique for solving these combinatorial problems (Laporte et al., 1987)) is applied to both Euclidean (up to 100 cities and 10 salesmen) and non Euclidean problems (up to 500 cities and 10 salesmen). Branch-and-bound is also applied in (Ali and Kennington, 1986) for solving an asymmetric MTSP up to 100 cities.

Other solution methods have also been proposed, from simulated annealing (Stentz and Brummit, 1998) (here coupled with a general-purpose interpreted grammar) to evolutionary algorithms (i.e. in (Sofge et al., 2002), different evolutionary algorithms, ranging from Genetic Algorithms to Particle Swarm and Monte-Carlo optimization, are compared). In (Junjie and Dingwei, 2006) the MTSP with ability constraint is solved with an Ant Colony Optimization (ACO) algorithm, where the MTSP is not translated into an equivalent TSP and the ACO algorithm is opportunely modified for dealing with the characteristics of the original problem (the MTSP). In (Junjie and Dingwei, 2006) results are compared with a modified Genetic Algorithm, that solves the equivalent TSP. In (Kara and Bektas, 2006), Linear Programming is used, and similar to (Junjie and Dingwei, 2006), the original MTSP is analyzed and solved (no TSP translation). In both (Kara and Bektas, 2006) and (Junjie and Dingwei, 2006), the authors conclude that the original MTSP is easier to solve than the derived TSP.

An important work is (Na, 2006), where different local search heuristics are presented and compared.

In (Carter and Ragsdale, 2002; Carter and Ragsdale, 2006; Carter, 2003; Brown et al., 2007), Genetic Algorithms are used, and the sum of the salesmen path lengths is minimized, as is the maximum distance traveled by each salesmen (to balance the agent workload). (Tang et al., 2000) uses a modified Genetic Algorithm on the equivalent TSP.

4 MAPP: PROPOSED METHOD

Given $m \geq 1$ agents and n known targets/cities to visit, the optimal team strategy (also called *Team Plan*), is sought that allows the fleet to visit every target only once.

We represent the Team Plan as a *collection* of m distinct subtours. Thus, given m agents and n targets, Team Plan $\mathbb{P}$ is defined as $\mathbb{P} = \{P_1, \ldots, P_m\}$, where P_i is the path of the i-th agent visiting $k_i < n$ targets. Note that this representation allows the individual subtours to have different lengths. Moreover, the Multi-Agent Planning Problem can also be rewritten for each i-th agent to find the best possible Subtour of length $k_i < n$ that satisfies the imposed cost function.

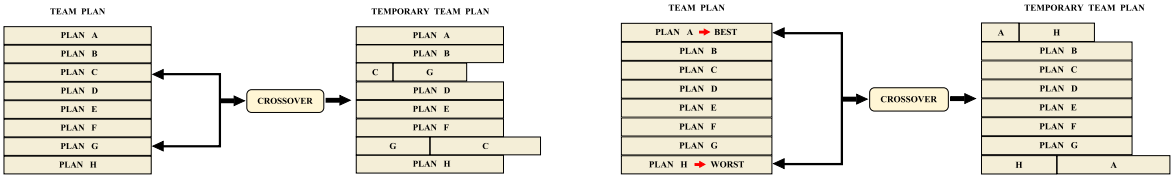
The proposed optimization technique is a Genetic Algorithm Inspired Steepest Descent (GAISD) method. Briefly, a Genetic Algorithm (GA) is an optimization technique used to find approximate solutions of optimization and search problems. Genetic Algorithms are a particular class of evolutionary methods that use techniques inspired by Darwin's theory of evolution and evolutionary biology, such as inheritance, mutation, selection, and crossover. In these systems, *populations* of solutions compete and only the fittest survive.

Similar to the classical GA, GAISD consists of two phases: *initialization* and *evolution*. In the initialization phase, the starting Team Plan is created (see Section 4.1), while the evolution phase (see Section 4.2) evolves it toward a near-optimal final solution.

4.1 Initialization Phase

During the initialization phase, the starting Team Plan is created, and thus the starting set of subtours is planned. The initialization phase is an important step in the optimization process, since it defines the starting point for the evolutionary search, and consequently the effectiveness of the algorithm.

Let $T_1 = \{t_1, \ldots, t_n\}$ and $A = \{a_1, \ldots, a_m\}$ be the set of n targets and the m agents, respectively. The initial Team Plan is created in a sequential order, and, without loss of generality, we assume that the order of planning is $a_1, a_2, \ldots, a_m$ and that the starting subtours have similar lengths. For feasible plans, subtours that are pairwise disjoint need to be created.



(a) Random selection. (b) Deterministic selection.

Figure 1: Crossover operator: two subtours are randomly (a) or deterministically (b) selected and mated through the crossover operator. In the deterministic case, the best and the worst subtours are selected.

As a first step, a Subtour P_1 of length k_1 is computed. At the end of the a_1 initialization phase, agent a_2 plans its own Subtour P_2 considering a target set T_2 that is simply obtained by discarding from T_1 the targets visited by agent a_1. In general, the i-th agent plans a Subtour P_i on the targets not yet allocated by the previous agents. Obviously, this process yields a feasible Team Plan $\mathbb{P} = \{P_1, \ldots, P_m\}$.

Based on this mathematical formulation, two initialization methods are proposed:

- **Greedy Initialization:** the initial Team Plan is created using a greedy approach to form feasible starting subtours. Each agent selects the targets to visit following a Nearest Neighbor method: given a target i, target $i+1$ is the nearest not yet visited one.
- **TSP-based Initialization:** for those problems where the positions of the m agent are not imposed (the m agents can start from any target $t_i \in T$), a feasible starting Team Plan $\mathbb{P} = \{P_1, \ldots, P_m\}$ can be generated by *clustering* the TSP solution computed on the complete graph $K_n(T)$. 'Clustering' is carried out by discarding m edges from the TSP tour, in order to 'equally' subdivide it, and having m starting subtours with similar costs. This initialization method introduces a degree of complexity in the overall system, since a TSP solution must be computed.

4.2 Evolution Phase

The evolution phase evolves the Team Plan, trying to design a strategy where the mission time is reduced (minimizing the cost $\mathcal{W}_m(\mathbb{P})$, see cost function (2)).

This phase has the same mechanism of a classical Genetic Algorithm (Goldberg, 1989) while introducing an important difference: only *one* Team Plan $\mathbb{P}$, not a population of them, is evolved.

At every evolution/generation step, a set of *operators* (see Section 4.3) is applied to the subtours $P_i \in \mathbb{P}$, either improving the Team Plan or not. If $\mathbb{P}$ improves, it is used at the next generation step, otherwise the Team Plan *before* the application of the operators is restored. No *elitism* is considered, and the unique Team Plan changes only when its cost decreases (Steepest Descent). An *evaluation* phase evaluates, at each generation step, the Team Plan.

The logical flowchart of the GAISD evolution phase is shown in Figure 2.

4.3 Team Plan Operators

The evolution of the unique solution $\mathbb{P}$ toward a near-optimal multi-agent strategy is accomplished by combining the genetic materials of its subtours through the application of *genetic-like* operators.

Three different operators have been designed: the *crossover*, the *mutation* and the *migration*. The operators are applied in a predefined order, and their application depends on a given probability, as shown in Figure 2. In addition to these operators, the heuristic 2-opt method (Matayoshi et al., 2004; Bentley, 1990; Sengoku and Yoshihara, 1998) to directly improve the goodness of the single agent plans is introduced (it replaces subtours with better ones from their 'neighborhood').

The *Crossover Operator* combines the genetic materials of two selected subtours (called *parents*), replacing them with the two newly created ones (called *offspring*). At every generation step the crossover operator is applied only once, thus only two subtours are chosen and mated. Depending on a given probability ($p_{selection}$), parents can be chosen either randomly (see Figure 1(a)) or deterministically (see Figure 1(b)). In the deterministic case, the mating process always occurs between the subtours with maximum and minimum costs. Once selected, parents are mated in a classical way (Goldberg, 1989): they are randomly halved (not necessarily at the same target) and their halves are simply swapped.

The *Mutation Operator* changes the Team Plan by randomly swapping two genes between two different subtours. The *Migration Operator* consists of moving a randomly chosen target from a Subtour $P_i \in \mathbb{P}$ (of length k_i) to a Subtour $P_j \in \mathbb{P}$ (of length k_j). Note that the lengths of the subtours change: k_i decreases while k_j increases.

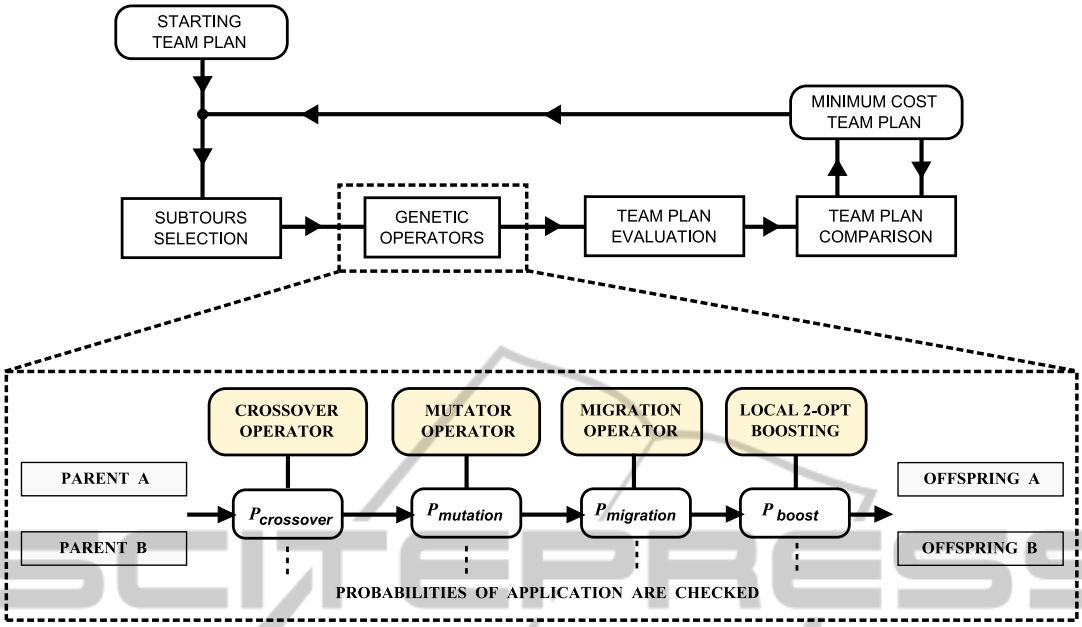


Figure 2: Flowchart of the evolution phase, together with the sequence of operators.

With probability p_{boost}, each Subtour is processed with a *heuristic boosting technique*. The local search method adopted here is the 2-opt method (Matayoshi et al., 2004; Bentley, 1990; Sengoku and Yoshihara, 1998) that replaces solutions with better ones from their 'neighborhood'.

Let us consider a set T of n targets and the corresponding complete and weighted graph $K_{n+1}(V)$ ($V = T \cup a$ with a being the agent). Let P denote the considered Subtour, with $1 \leq k \leq n$, coded as a sequence of targets $\mathbf{s} = (x_1, \ldots, x_k)$ (where $x_i \in T$). The 2-opt method determines whether the inequality

$$w(x_i, x_{i+1}) + w(x_j, x_{j+1}) > w(x_i, x_j) + w(x_{i+1}, x_{j+1})$$

between the four vertices x_i, x_{i+1}, x_j and x_{j+1} of P holds, in which case edges (x_i, x_{i+1}) and (x_j, x_{j+1}) are replaced with the edges (x_i, x_j) and (x_{i+1}, x_{j+1}), respectively. This method provides a shorter path without intersecting edges.

5 RESULTS

An extensive set of simulations were run to test the performance of the proposed method. Where necessary, for the possibility of comparing our results with other referenced ones, some constraints have been introduced.

Unless otherwise specified, simulations were run for 150000 generation steps, while the crossover, mutation, migration and boosting (2-opt) operators were applied with a $p_{crossover} = 70\%$, $p_{mutation} = 40\%$, $p_{migration} = 60\%$ and $p_{boost} = 30\%$, respectively. When the crossover operator is applied, the probability of deterministically selecting two parents is equal to $p_{selection} = 50\%$.

For each simulation, the initialization method is always specified, and can be either the greedy or the TSP-based one.

5.1 Comparison with Structured and Well-known Solutions

In this Section, GAISD has been compared with known literature results. In order to make the comparison meaningful, we introduce, where necessary, the same constraints adopted in the referenced works.

5.1.1 Comparison with Evolutionary Algorithms

We compare our method with the results reported in (Junjie and Dingwei, 2006), where an Ant Colony Optimization algorithm is compared against the Modified Genetic Algorithm (MGA) described in (Tang et al., 2000). Six MTSPs, all derived from well-known TSPLIB instances (Reinelt, 1991), are solved. The number of agents is fixed, $m = 5$, and they all share the same starting location (that is, the first target in the corresponding problem data file). In addition, the maximum length of the cycles, k_{max}, is limited (note that since MTSPs are considered here, we need to 'modify' the Team Plan using cycles instead

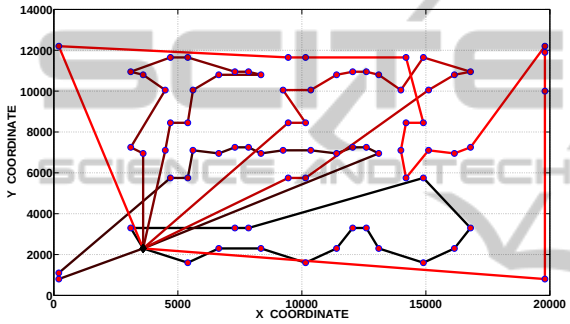
of paths).

In both (Junjie and Dingwei, 2006; Tang et al., 2000), rounded costs are considered. Therefore, for meaningful comparison, we need to minimize the following cost function:

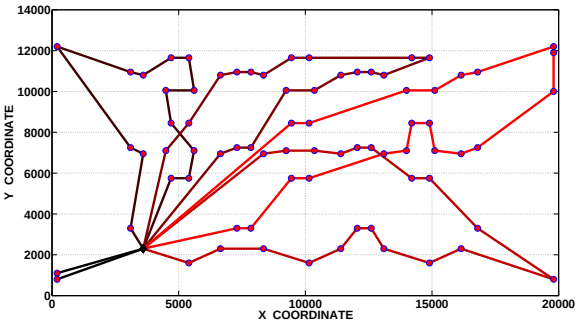
$$\mathcal{W}(\mathbb{C}) = \sum_{i=1}^{m} \text{round}\,(\mathcal{W}(C_i)). \quad (5)$$

Finally, since a single depot is considered, only the greedy initialization method is used.

Results are shown in Table 1, and they are based on an average of 100 simulations. In this case, the obtained results globally outperform the literature ones. Figure 3 shows also the solution obtained for the pr76 TSPLIB problem.



(a) pr76 TSPLIB problem: starting Team Plan.



(b) pr76 TSPLIB problem: final Team Plan.

Figure 3: The pr76 TSPLIB target configuration is here adopted, and the MTSP is solved. The maximum plan length is constrained to $k_{max} = 20$ targets and $m = 5$ agents are considered. Cost function (5) is minimized, and the comparison between the starting and the final Team Plans is shown. In (a) the starting Team Plan, with cost equal to 194618, is shown, while in (b) the final Team Plan, with cost equal to 152722, is reported. This solution is 15.4% and 2.6% better than the referenced ones, obtained by implementing an Ant Colony Optimization method and a Modified Genetic Algorithm, respectively.

5.1.2 Comparison with Heuristics

We compare our method against the results reported in (Na, 2006), where different heuristics methods are proposed and compared for solving MTSP instances.

In (Na, 2006), a *no-depot* MTSP variant is considered (the agents do not have a predefined starting location), and the *minmax* optimization problem is solved (cost function (2) is minimized). In addition, the number of salesmen is fixed and no constraints on the plan lengths are imposed. Note that since we are comparing our method with MTSP results, we need to modify our Team Plan accordingly, using cycles instead of paths. In addition, since in (Na, 2006) rounded distances are considered, for meaningful comparison cost function (2) is opportunely modified:

$$\mathcal{W}_m(\mathbb{C}) = \max_i \text{round}\,(\mathcal{W}(C_i)). \quad (6)$$

For each test case, our results are compared with only the best ones of (Na, 2006), without considering all the other (worse) solutions. We also report the name of the Heuristic with which each referenced solution has been obtained (please refer to (Na, 2006) for a more detailed description of the adopted heuristic methods). Tables 2 and 3 show the results obtained by initializing the Team Plan either with the greedy or the TSP-based initialization methods, respectively. Results are averaged over 100 simulations.

In each test case, GAISD returns good solutions, independent of the applied initialization method. In general, our best solutions are closer to the literature ones, and, in a few cases (for example in the berlin52 or the pr264 problems), even better. The greedy initialization method seems to better initialize the Team Plan in those problems where the distribution of targets has a well defined structure (the pr264 problem), while in 'small-size' problems (i.e. berlin52) the TSP-based method is preferable (even if the obtained improvement does not justify its required complexity, time and computational costs).

Figure 4 shows the solutions obtained by applying the greedy (Figure 4 (a)) and the TSP-based (Figure 4 (b)) initialization methods for the kroA100 problem (with $m = 5$ agents). In this case, if compared with (Na, 2006), the TSP-based method and the greedy method allow for final solutions that are only 3.6% and 7.2% worse than the cited one, respectively.

5.2 Comparison with Other Software

In this Section we test GAISD against a Matlab MTSP solver, based on a Genetic Algorithm (www.mathworks.com/matlabcentral/fileexchange/, 2008) freely available online.

Table 1: Comparison between the proposed method and the Ant Colony Optimization (ACO) and the Modified Genetic Algorithm (MGA) methods. Results are averaged over 100 simulations. The number of targets can be easily derived from the TSPLIB problem name. k_{max} is the maximum number of targets an agent can visit. For each case, a fixed number of $m = 5$ agents is considered. Only the greedy initialization method is used (one-depot problem).

TSPLIB Problem	k_{max}	ACO		MGA		Proposed Method		Difference of Means	
		Min	Mean	Min	Mean	Min	Mean	ACO	MGA
pr76	20	178597	180690	157444	160574	152722	156503.9	−15.4%	−2.6%
pr152	40	130953	136341	127839	133337	114698	126128.8	−8%	−5.7%
pr226	50	167646	170877	166827	178501	152198	158073.9	−8%	−12.9%
pr299	70	82106	83845	82176	85796	70059	71705.1	−16.9%	−19.6%
pr439	100	161955	165035	173839	183698	136169	138655.5	−15.6%	−24.5%
pr1002	220	382198	387205	427269	459179	311492	319240.4	−17.5%	−30.5%

Table 2: Heuristic comparison: for each case, 100 simulations have been run. The initialization phase is based on the *greedy* method. The number of targets can easily be derived from the TSPLIB problem name.

TSPLIB Problem	Number of Agents	Heuristics			Greedy Method		Errors	
		Method	Minimum	Mean	Minimum	Mean	Minimum	Mean
berlin52	4	bisection	2182	2204.3	2183	2422.4	0.04%	9.8%
	5	k-split	1713	1739.7	1825	2160.4	6.5%	24.2%
	6	SGH	1531	1585	1611	1905.8	-	20.2%
		bisection	1476	1643.1			9.1%	-
kroA100	4	SGH	5955	6096.7	6000	6690.3	0.7%	9.7%
	5	k-split	4629	5025.9	4964	5620.1	7.2%	11.8%
	6	k-center	4200	4429.4	4370	5038.4	4%	-
		k-means	4230	4234.6			-	18.9%
bier127	4	bisection	32423	32757.5	32434	35740.9	0.03%	9.1%
	6	k-split	22815	23071.7	24608	26993.3	7.8%	16.9%
pr264	4	bisection	12705	12705	12196	13830.3	−4.1%	8.8%
	6	k-means	8526	9131.6	9000	10256.3	5.5%	-
		bisection	8739	9051.6			-	13.3%

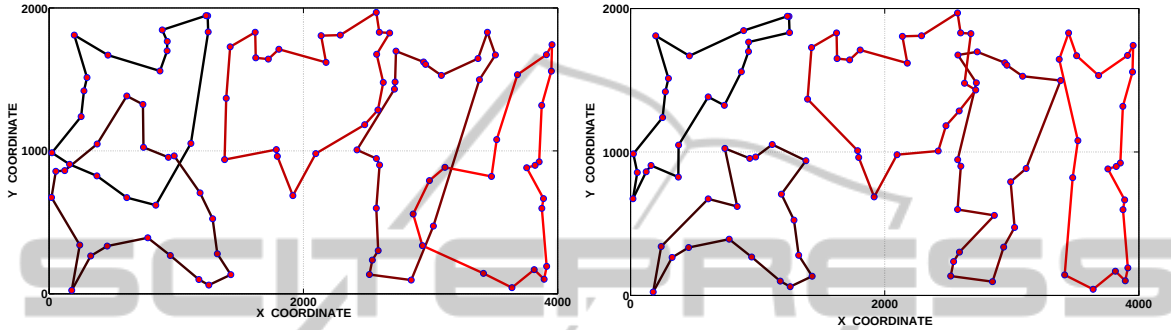
Table 3: Heuristic comparison: for each case, 100 simulations have been run. The initialization phase is based on the *TSP-based* method. The number of targets can easily be derived from the TSPLIB problem name.

TSPLIB Problem	Number of Agents	Heuristics			TSP-based Method		Errors	
		Method	Minimum	Mean	Minimum	Mean	Minimum	Mean
berlin52	4	bisection	2182	2204.3	2088	2359.6	−4.5%	7%
	5	k-split	1713	1739.7	1804	2014.8	5.3%	15.8%
	6	SGH	1531	1585	1576	1801.2	-	13.6%
		bisection	1476	1643.1			6.7%	-
kroA100	4	SGH	5955	6096.7	6000	6443.6	0.7%	5.7%
	5	k-split	4629	5025.9	4796	5314.6	3.6%	5.7%
	6	k-center	4200	4429.4	4310	4693.9	2.6%	-
		k-means	4230	4234.6			-	10.8%
bier127	4	bisection	32423	32757.5	32948	34606	1.6%	5.6%
	6	k-split	22815	23071.7	24290	26497.1	6.4%	14.8%
pr264	4	bisection	12705	12705	13400	19135.3	5.4%	50.6%
	6	k-means	8526	9131.6	8629	14395.8	1.2%	-
		bisection	8739	9051.6			-	59%

The number of agents is fixed, and the minimum path length, $k_{min} = 2$, is imposed (this way, solutions with paths composed of only one target are avoided). Cost function (2) is minimized, and targets are randomly generated over the unit square map. In all simulations, the greedy initialization method is adopted. Since the Matlab MTSP solver is based *only* on a Genetic Algorithm, and no heuristics is used, we

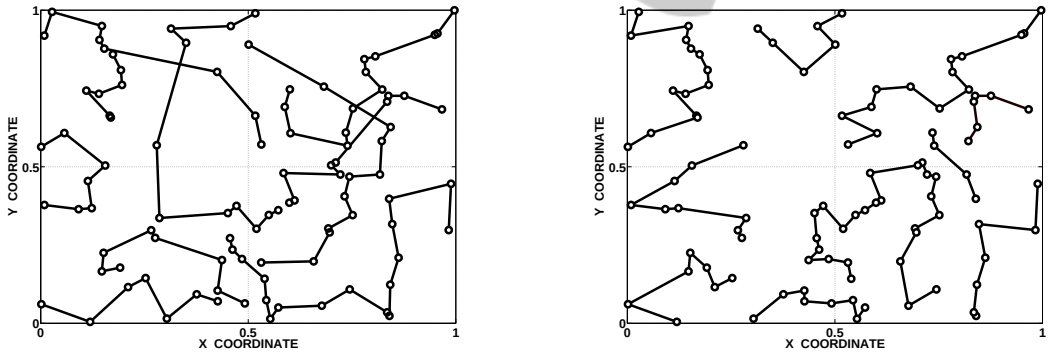
Table 4: Comparison between the proposed method against a Matlab MTSP code. For each test, $k_{min} = 2$ and the greedy initialization method is used. Targets are randomly generated over the unit square map.

Test case	Matlab Code			Proposed Method				Cost Variation		
	Min	Mean	Max	p_{boost}	Min	Mean	Max	Min	Mean	Max
100 targets 10 agents	7.14	8.19	9.21	30%	6.37	6.72	7.11	−10.7%	−17.9%	−22.8%
				0%	6.67	7.14	7.58	−6.5%	−12.8%	−17.7%
200 targets 20 agents	15.64	17.5	19.65	30%	9.55	9.89	10.25	−39%	−43.5%	−47.8%
				0%	10.05	10.6	11.23	−35.7%	−39.4%	−42.8%



(a) kroA100 problem: greedy initialization. (b) kroA100 problem: TSP-based initialization.

Figure 4: For the same problem, the MTSP has been solved by applying both the greedy and the cluster initialization methods. In both cases, $m = 5$ agents are considered, and the target set is the kroA100 TSPLIB one. In (a), the greedy initialization method is used and the final obtained Team Plan is shown. The final Team Plan is the best computed one and its cost is equal to 4964, that is 7.2% worse than the referenced one. In (b), the TSP-based initialization method is used and the obtained final Team Plan is shown. In this case, its cost is equal to 4796, that is only 3.6% worse than the referenced one.



(a) Matlab solution. (b) Proposed method.

Figure 5: GAISD has been compared with a Matlab MTSP solver. In this example, MAPP is solved. 100 targets are randomly distributed over the unit square map. $m = 10$ agents are considered. The minimum path length is $k_{min} = 2$. Cost function (2) is minimized. In (a) the solution obtained by running the MTSP Matlab solver is shown, and its cost is $\mathcal{W}_t = 7.92$. In (b) our solution is proposed. In this case, the total cost is $\mathcal{W}_t = 6.37$, that is 19.5% better than the MTSP Matlab solver one.

also run a set of tests with $p_{boost} = 0\%$.

Table 4 shows the obtained results, averaged over 100 simulations. Our method clearly outperforms the Matlab one, both with and without the application of the 2-opt method (clearly, with the 2-opt method the results are better). Figures 5 shows two examples where the Matlab MTSP solver solution is compared with the GAISD ones.

6 CONCLUSIONS AND FUTURE WORKS

This paper describes the Multi-Agent Planning Problem, a variant of the classical Multiple Traveling Salesman Problem where the agents do not have the constraint of returning to their starting location. The solution method is based on a simplified Genetic

Algorithm, that is initialized in two different ways: a greedy (Nearest Neighbor) and a TSP-based approach. The importance of this work is to provide a team of agents the ability to plan a common set of strategies, the Team Plan, by sharing in an optimal way a set of given targets/goals.

The results presented here show the success of the approach, demonstrating how a simple method can solve otherwise hard combinatorial problems.

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