

UNREVEALING BIOLOGICAL PROCESS WITH LINEAR ALGEBRA

Extracting Patterns from Noisy Data

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Abstract: Extracting patterns from protein sequence data is one of the challenges of computational biology. Here we use linear algebra to analyze sequences without the requirement of multiples alignments. In this study, the singular value decomposition (SVD) of a sparse p -peptide frequency matrix (M) is used to detect and extract signals from noisy protein data ($M = USV^T$). The central matrix S is diagonal and contains the singular values of M in decreasing order. Here we give sense to the biological significance of the SVD: the singular value spectrum visualized as *scree* plots unveils the main components, the process that exists hidden in the database. This information can be used in many applications as clustering, gene expression analysis, immune response pattern identification, characterization of protein molecular dynamics and phylogenetic inference. The visualization of singular value spectrum from SVD analysis shows how many processes can be hidden in database and can help biologists to detect and extract small signals from noisy data.

1 INTRODUCTION

Many bioinformatics tools are designed to detect patterns in protein or DNA sequences by using statistically based sequence similarity methods. The patterns detected can be associated with the function or structural protein stability, can predict family genes or can be used to describe the evolving relationship of group sequences (Hunter, 1993). Such bioinformatics predictions help experimental determination simpler and more efficient (King *et al.*, 2001). However, to evaluate how two proteins are similar is a complex issue. The standard methods quantify the similarity between two proteins using global or local alignments with their primary sequences. The goal is to find the optimal alignment, quantifying it by some metric. In this work, instead of using alignment analysis, the approach applied is

based on linear algebra algorithms, similar to that used in systems for information retrieval in large textual databases and by Google™ web search engine. The ideas and linear algebra methods applied here are important in several areas of data mining, pattern recognition (for example, classification of hand-written digits), and PageRank computations for web search engines (Eldén, 2006). Our objective is to use singular value decomposition – SVD (Berry *et al.*, 1995) of a sparse tripeptide frequency matrix to detect and extract signals from noisy protein data. Such analysis, when done in micro array gene expression data, associates the number of the most significant singular values from SVD with the gene groups and the cell-cycle structure (Wall *et al.*, 2003).

We will analyze the singular value spectrum to visualize them and to unveil the main

components, the number of process that exists hidden in the database. More specifically, as an application of SVD, we want to show that the number of the most significant singular values is associate with the number of protein families in a sequence database. Such prediction can be used in phylogenetic inference, data mining, clustering etc, making experimental tests more efficient, and avoiding randomly determination for possible outcomes.

2 SYSTEM AND METHODS

Programs implemented for this analysis were written in MATLAB (The Mathworks, 1996), using its inbuilt functions (SVD, sparse matrix manipulation subroutines etc). Four datasets were used in this paper. The first evaluated database had 64 vertebrate mitochondrial genomes composed of 832 proteins from 13 known gene families (ATP6, ATP8, COX1, COX2, COX3, CYTB, ND1, ND2, ND3, ND4, ND4L, ND5 and ND6). This curated protein database was downloaded from online information by Stuart *et al.* paper (Stuart *et al.*, 2002). The second database was composed by sequences from proteins retrieved from GenBank in 19/04/2006. It is a random 100 sequences sample of each protein type: globin, cytochrome, histone, cyclohydrolase, pyrophosphatase, ferredoxin, keratin and collagen and 200 other proteins, totalling 1,000 sequences from ten different types of genes. The third database was the file "pdb_seqres.txt.gz", located in <http://bioserv.rpbs.jussieu.fr/PDB/>. This file has 121,556 redundant protein sequences from PDB (Protein Data Bank), which was reduced to 37,561 non-identical sequences. From this file we recovered all sequences related to six types of enzymes: Ligase, Isomerase, Lyase, Hydrolase, Transferase and Oxidoreductase, which totalled 10,915 proteins. We also recovered a sample of 219 globins from the PDB file that was used as another test set. Besides, we extracted 86 sequences of haemoglobin alpha-chain and a sample from the PDB file with all sequences higher than 47 amino acids (31,906 proteins from several types of genes). Each of the above sequence files was analyzed by MATLAB subroutines that generate twelve tripeptide sparse matrices as described by Stuart (Stuart *et al.*, 2002) and adapted by Couto (Couto *et al.*, 2007).

All sequences were recoded as 3-peptide frequency values using all possible overlapping tripeptide window. With 20 amino-acids it is generated a matrix M (8,000 x n), where n is the

number of proteins to be analyzed. After the generation of the tripeptide frequency matrix (M), the matrix itself is subjected to SVD (Deerwester *et al.*, 1990; Berry *et al.*, 1995) and factorized as $M = USV^T$. Where U is the $p \times p$ orthogonal matrix having the left singular vectors of M as its columns, V is the $n \times n$ orthogonal matrix having the right singular vectors of M as its columns, and S is the $p \times n$ diagonal matrix with the singular values $\sigma_1 \geq \sigma_2 \geq \sigma_3 \dots \geq \sigma_r$ of M in order along its diagonal (r is the rank of M or the number of linearly independent columns or rows of M). These singular values are directly related to independent characteristics within the dataset. Actually, the largest values of (S) provide the meaning of the peptides and proteins in the matrix (M). On the other hand, the smaller singular values identify less significant aspects and the noisy inside the dataset (Eldén, 2006).

In this work our focus is only in the matrix (S) and its diagonal values (s_i) that make up the singular value spectrum. The magnitude of any singular value is indicative to its importance in explaining the data (Wall *et al.*, 2003). Then, the objective here is to visualize the singular value spectrum as plots that help biologists to discover the main components, the process, and the groups hidden in the database. Two graphs were built:

- a) the *scree* plot, with 25 bigger singular values for each database;
- b) the cumulative relative variance (V_i) captured by the i th-singular value:

$$V_i = 1 - (S_i)^2 / \sum_k (S_k)^2; S_i = i\text{th-singular value}; k = 1, 2, \dots n.$$

The visual examination of the *scree* plot looks for a "gap" or an "elbow" that indicates how many significant singular values exist in database. After the "gap" there is no significant value. The second graph helps to understand how much variance is explained by each singular value. Despite the effort for automatic analysis, graphic visual inspection still is one of the most commonly used in practice for dimensionality selection (Zhu and Ghodsi, 2006).

3 RESULTS

When there is only one specific type of protein in database, as haemoglobin alpha-chain, the singular value spectrum obtained shows a "big gap" after the first eigenvalue (Figure 1). Such result is confirmed by the second graph (Figure 2) that indicates more than 90% variance is explained by the first singular value, which is compatible with the database itself.

For the globin matrix (Figures 3 and 4) is more difficult to define exactly where the “gap” or “elbow” is, because there are more than one type protein in database. However, the objective here is not to be very precise, but sufficiently accurate to help biologists in finding an interval with the number of process or groups that exists hidden in the database. Such predictions need validation by experimental determination that becomes simpler. In the globin database for example, is reasonable to define between one and three groups that explains about 60% of the variance in database (Figure 4). After the third singular value there is stability in the singular value spectrum (Figure 3).

For the database with 13 mitochondrial genes (Figures 5 and 6) it is possible to define the number of groups around 10: after this interval the singular value spectrum stabilizes and there is between 50% and 60% explained variance. When the GenBank matrix is analyzed, with ten different types of genes, it is necessary carefully combine both graphs. Despite the fact that there is a “gap” after the sixth singular value (Figure 7), the variance explained until this point is only about 40% (Figure 8). The interval between 10 and 15 singular values corresponds to about 50% of relative variance and the spectrum becomes flat.

The PDB database, with more than 31,000 proteins from several types of genes, presents a singular value spectrum where is necessary more than 20 eigenvalues to explain about 30% of variance. There is an “elbow” between the second and third singular value (Figure 9) that is insufficient to explain most data (Figure 10). Similar result is obtained with the PDB enzymes database that apparently had only 6 types of proteins. The visual analysis of the scree plot and cumulative variance graph (Figures 11 and 12) suggest more than 25 groups hidden under the six enzymes denomination. This is a clue, a possibility that should be analyzed by another bioinformatics tool.

Table 1 summarizes the visualization of all singular value spectrums for each database, plotted in the Figures 1 to 12. The suggested numbers of significant singular values for each dataset is coherent, except the enzymes database, which seems to be actually formed by several quite different sequences. SVD analysis unveils biological motives associated with biological processes and other biological properties in each dataset.

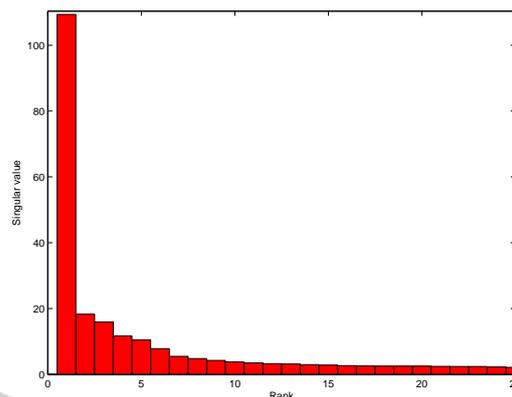


Figure 1: Scree plot showing singular values of haemoglobin α -chain database.

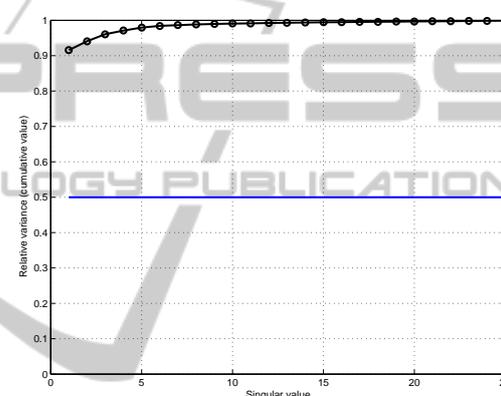


Figure 2: Cumulative relative variance of haemoglobin α -chain database.

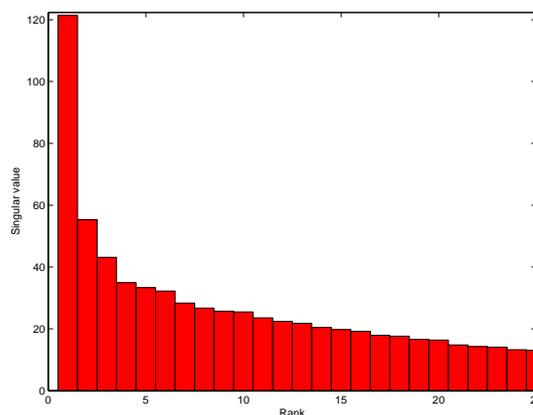


Figure 3: Scree plot showing singular values of globin database.

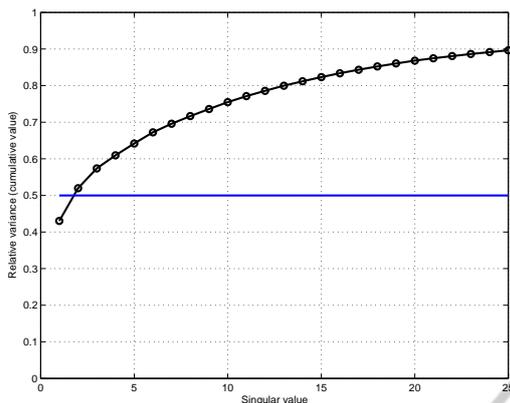


Figure 4: Cumulative relative variance of globin database.

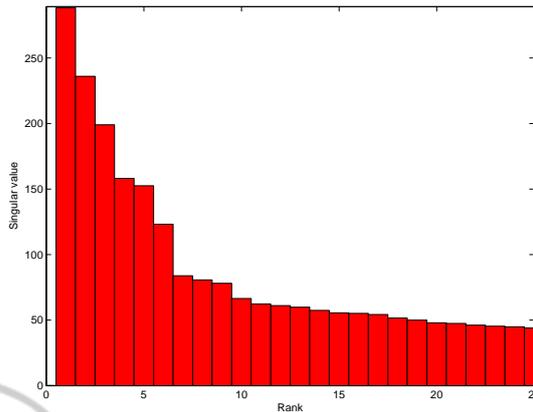


Figure 7: Scree plot showing singular values of sample genes from GenBank.

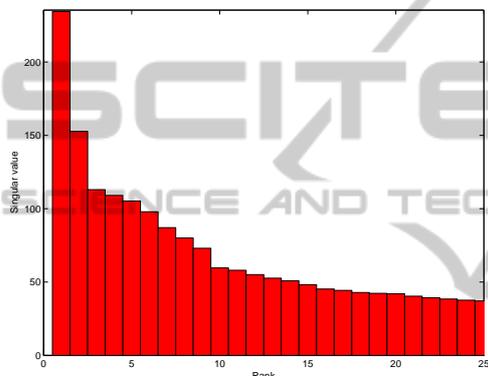


Figure 5: Scree plot showing singular values of mitochondrial genes database.

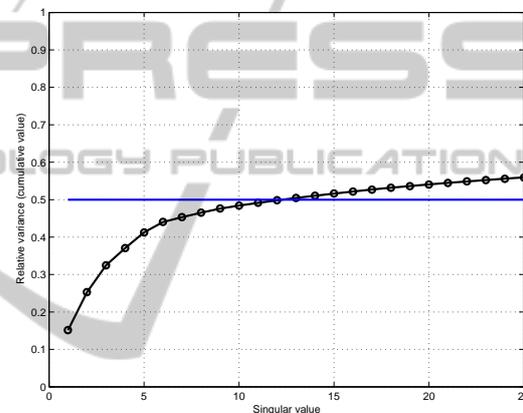


Figure 8: Cumulative relative variance of sample genes from GenBank.

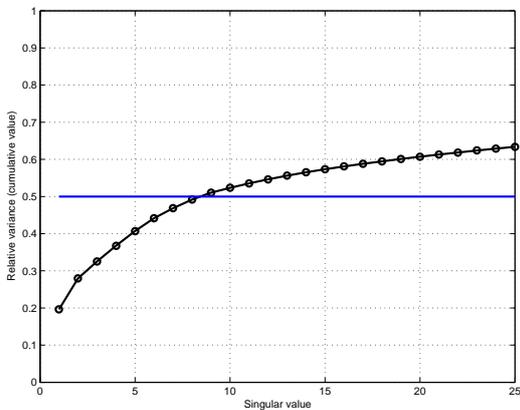


Figure 6: Cumulative relative variance of mitochondrial genes database.

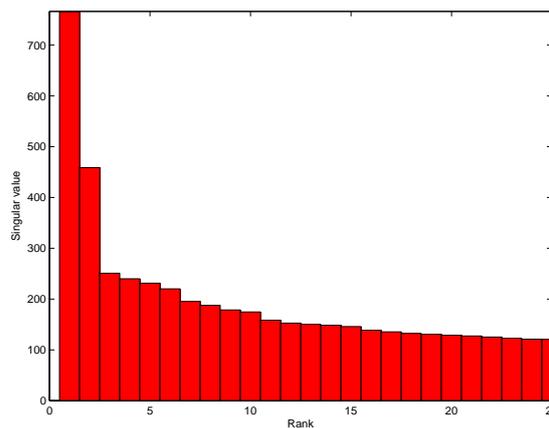


Figure 9: Scree plot showing singular values of random PDB sequences dataset.

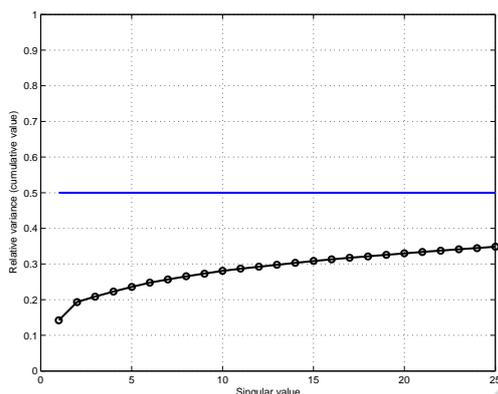


Figure 10: Cumulative relative variance of random PDB sequences dataset.

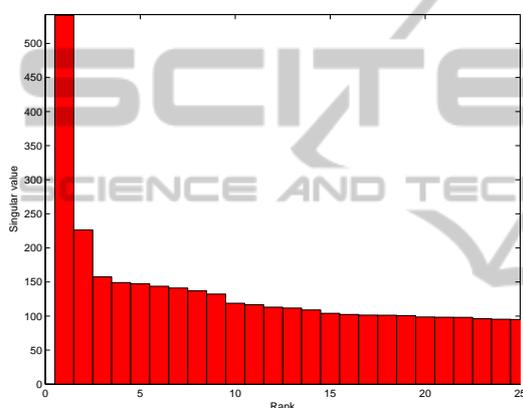


Figure 11: Scree plot showing singular values of PDB enzymes database.

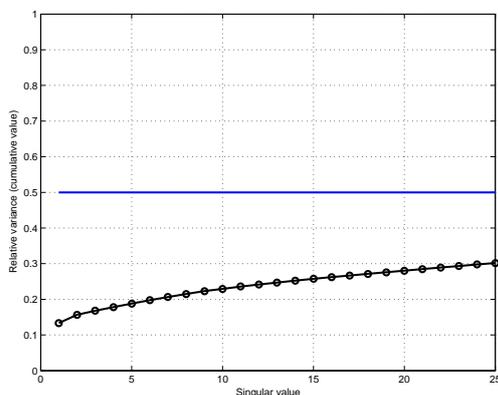


Figure 12: Cumulative relative variance of PDB enzymes dataset.

4 CONCLUSIONS

A biologist could ask: “What is the biological significance of the SVD?” We answered this

question: the visualization of singular value spectrum from SVD analysis shows how many process can be hidden in database. The singular value plot is a suggestion, a clue that helps biologists to detect and extract small signals from noise data.

Table 1: Suggested number of significant singular values.

Dataset	Predefined # groups	Suggested number singular values	
		Min	Max
Haemoglobin α -chain	1	1	1
Globin	1	1	3
Mitochondrial genes	13	9	15
GenBank	10	10	15
PDB sequences	Several	> 20	
Enzymes	6	> 25	

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