

EMBEDDED INCREMENTAL FEATURE SELECTION FOR REINFORCEMENT LEARNING

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Abstract: Classical reinforcement learning techniques become impractical in domains with large complex state spaces. The size of a domain's state space is dominated by the number of features used to describe the state. Fortunately, in many real-world environments learning an effective policy does not usually require all the provided features. In this paper we present a feature selection algorithm for reinforcement learning called Incremental Feature Selection Embedded in NEAT (IFSE-NEAT) that incorporates sequential forward search into neuroevolutionary algorithm NEAT. We provide an empirical analysis on a realistic simulated domain with many irrelevant and relevant features. Our results demonstrate that IFSE-NEAT selects smaller and more effective feature sets than alternative approaches, NEAT and FS-NEAT, and superior performance characteristics as the number of available features increases.

1 INTRODUCTION

Reinforcement learning (RL) is a popular technique for many agent-related problems such as robot control, game playing, and system optimization (Sutton and Barto, 1998). In such problems, an agent strives to learn a policy for selecting actions based on its current state, by performing actions and receiving rewards for the actions it tries. The most common approach to solving an RL problem is to learn a value function which indicates, for a particular policy, the long-term expected reward value of a given state or state-action pair. In domains with large or infinite state spaces, it is infeasible to represent the value function explicitly. Instead, a common strategy is to approximately represent the value function using some parameterized class of functions. Many function approximation methods such as radial basis functions (Keller et al., 2006), adaptive tile coding (Whiteson and Stone, 2007), and neural networks (Whiteson and Stone, 2006) have proven successful on problems with a very small number of state features.

In many real-world problems, the number of features with which to describe the states of the environment can be quite large. When the number of features is large, existing function approximation methods are not only computationally prohibitive, but also prone to over-fitting due to the so-called “curse of dimen-

sionality” - the size of the state space grows exponentially as the number of state features increases. Fortunately, among many possible features, a large portion of them may be irrelevant or redundant regarding learning a policy. Determining which subset of features should be used in order to generate the best performance for the learning task is challenging for either a domain expert or a learning agent.

This work is about automated feature selection for RL. Although feature selection has been extensively studied for supervised learning (Guyon and Elisseeff, 2003; Liu and Yu, 2005), existing methods are either inapplicable or impractical in the RL setting. Filter methods rely on training data, which is not available in RL, to select features. Wrapper methods require repeatedly executing a learning algorithm on each candidate feature subset, and are impractical for RL due to their prohibitively high computational and sample cost. A promising approach is to embed feature selection into the training process of a learning algorithm. However, the embedded approach has to be tailored for the learning algorithm of interest.

In this work, we propose an embedded incremental feature selection algorithm for a neuroevolutionary function approximation algorithm NEAT (NeuroEvolution of Augmenting Topologies) (Stanley and Miikkulainen, 2002), which we call IFSE-NEAT. The main idea of IFSE-NEAT is to embed incremental

subset selection into the neuroevolutionary process of NEAT. Instead of evolving networks with the full set of features as NEAT does, IFSE-NEAT initializes networks with one feature. IFSE-NEAT then iteratively adds features to the current best network that contributes most to its performance improvement while evolving the weights and topology of that network.

Prior to this work, feature selection for reinforcement learning has focused on linear value function approximation (Kolter and Ng, 2009; Parr et al., 2008) and model-based RL algorithms (Kroon and Whiteson, 2009). For neuroevolution algorithms such as NEAT, only random search has been explored (Whiteson et al., 2005). In this light we can see that IFSE-NEAT is a novel approach in feature selection for RL.

Our experimental study has shown several promising results for IFSE-NEAT. We find that the algorithm is nearly unaffected in its ability to select relevant features as the number of irrelevant features grows very large. This, in turn, allows for a better policy to be derived than NEAT and FS-NEAT. Additionally, by using only a few relevant features we are able to learn a good policy while limiting model complexity.

The rest of the paper is organized as follows: Section 2 introduces the NEAT algorithm and the proposed IFSE-NEAT algorithm. Section 3 describes the experimental setup. Section 4 presents and discusses experimental results. Finally, Section 5 concludes this work and identifies some future research directions.

2 APPROACH

2.1 NEAT

Neural networks (NNs) are efficient function approximators that can model complex functions to an arbitrary accuracy. The drawbacks of using NNs in RL domains have been that NN design was a difficult manual process and training was a supervised learning process. Neuroevolutionary approaches, which utilize genetic algorithms to automate the process of training and/or designing NNs, eliminate these drawbacks allowing NNs to be easily applied to RL domains. NeuroEvolution of Augmenting Topologies (NEAT) is a novel RL framework based on neuroevolution. By evolving both the network topology and weights of the connections between network nodes, NEAT solved typical RL benchmark problems several times faster than competing RL algorithms with significantly less system resources (Stanley and Miikkulainen, 2002).

However, one limiting issue with NEAT is that it assumes that all features provided by the environment are relevant and necessary, and attempts to incorpo-

rate all the features into its solution networks. The extraneous features will unnecessarily complicate the networks and severely slow the rate at which NEAT is able to derive an effective policy. In the following section we describe a new algorithm based upon NEAT that builds a small set of required features while learning an effective policy.

2.2 Incremental Feature Selection Embedded in NEAT

To deal with the exponential search space, we adopt sequential forward search (SFS), an efficient search strategy which has proven effective in finding near-optimal subsets in supervised feature selection. Starting from an empty set, SFS iteratively adds one feature at a time to the current best set until a desired number of features k are selected. Since in each of the k iterations, it goes through all N features outside of the current best set, the time complexity of SFS is $O(kN)$. Although SFS does not guarantee the optimal solution, it is capable of selecting relevant features while keeping irrelevant or redundant features out of the final subset. The method is particularly suitable for high-dimensional problems where a large portion of the features are irrelevant or redundant.

Algorithm 1 provides a basic overview about how IFSE-NEAT functions and is able to select a minimal set of features. IFSE-NEAT incrementally adds features to a NN that we call the *BACKBONE*. The *BACKBONE* network utilizes the best discovered feature set and represents the current best derived policy. It is persistent through additions of new features to the feature set and it is what makes IFSE-NEAT an embedded algorithm as opposed to a straightforward wrapper algorithm.

Initially, the *BACKBONE* network consists of only the output nodes (line 5). Then, for each of the individual features available, F_q , a NN is generated by connecting a single input node to every output node (line 13). In parallel, or independently, a population of networks based upon this single-input base network is generated. Each network in the population share the topology of the base network, but have randomly generated weights on the edges joining the nodes. The population of NNs are then evolved via the standard NEAT algorithm for L generations (lines 17-19). At the end of the NEAT process, the champion of each population (the network representing the best policy) is identified. The champions (each corresponding to a candidate feature F_q) are then compared against one another to decide the *BEST_NETWORK* and *BEST_FEATURE* (lines 22-25). It is our hypothesis that the best performing network, *BEST_NETWORK*

Algorithm 1: IFSE-NEAT(N, k, L, p).

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1: //N: set of all available features
2: //k: number of features to select
3: //L: number of generations to evolve
4: //p: population size
5: BACKBONE  $\leftarrow$  outputNodes //initialize the BACKBONE
6: SELECTED_SET  $\leftarrow$  null //initialize the selected feature set
7: for  $i \leftarrow 1 : k$  do
8:   BEST_NETWORK  $\leftarrow$  null
9:   BEST_FEATURE  $\leftarrow$  null
10:  //iterate through all candidate features outside SELECTED_SET
11:  for  $q \leftarrow 1 : N - i$  do
12:    //create new network  $N_b$  based on candidate feature  $F_q$ 
13:     $N_b \leftarrow$  COMBINE( $F_q, BACKBONE$ )
14:    //create a population of  $p$  networks based upon  $N_b$ 
15:    population  $\leftarrow$  INITIALIZE-POPULATION( $N_b, p$ )
16:    //evolve population using NEAT for  $L$  generations
17:    for  $j \leftarrow 1 : L$  do
18:      NEAT-EVOLVE(population)
19:    end for
20:    //select the champion from population
21:    champion  $\leftarrow$  BEST-QUALITY(population)
22:    if champion > BEST_NETWORK then
23:      BEST_FEATURE  $\leftarrow$   $F_q$ 
24:      BEST_NETWORK  $\leftarrow$  champion
25:    end if
26:  end for
27:  Add BEST_FEATURE to SELECTED_SET
28:  BACKBONE  $\leftarrow$  BEST_NETWORK
29: end for

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WORK, will point to the most relevant feature. Therefore, the *BEST_FEATURE* that produced the *BEST_NETWORK* is then added to the *SELECTED_SET* (line 27), and the *BEST_NETWORK* becomes the *BACKBONE* (line 28) for subsequent iterations where the algorithm will determine the next features to add to the feature set.

In the subsequent iterations the remaining features are independently combined with the *BACKBONE* network and then re-evaluated. As in the first feature selection iteration, new populations of NNs, random variations of the base networks, are again evolved by NEAT for L generations. The algorithm stops once a desired number of features are selected. Alternatively, the algorithm can stop when one of the populations produces a network that represents a suitable solution to the problem.

The process of combining the *BACKBONE* network, $\text{COMBINE}(F_q, \text{BACKBONE})$ (line 13), is illustrated in Figure 1. In this process a new base network N_b is created for a candidate feature F_q by connecting F_q to each of the output nodes. The weights of these new edges are assigned zero to preserve the policy of the *BACKBONE* network. Preserving the policy of the *BACKBONE* network bootstraps the successive networks and improves IFSE-NEAT’s ability to deter-

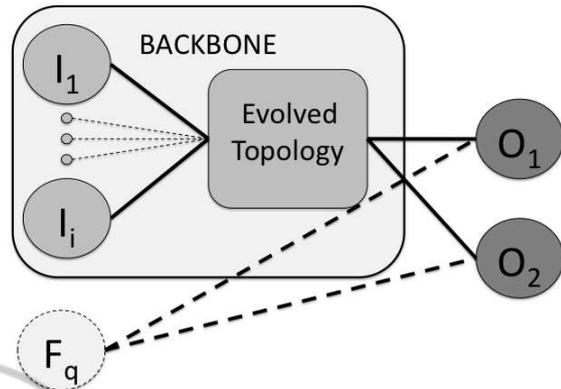


Figure 1: This figure illustrates how a candidate feature is incorporated into the current *BACKBONE* network to create a new base candidate network, N_b . The new feature F_q is introduced to the network and is provided connections, the dashed lines, to every output node. *BACKBONE* in this figure represents the best evolved solution network using only the previously selected features.

mine the relevance of potential new features.

Analysis of Algorithm 1 shows that time complexity of IFSE-NEAT is $O(kN)$ times the NEAT process, which itself is dependent on the population size p and the number of generations L . In practice, however, we can do better than this. For the first few selected features, L can be very small and the algorithm can still identify relevant features, allowing a significant speedup to roughly $O(N)$ times NEAT. Once the *BACKBONE* network is reasonably fit, L must be increased to allow new features enough time to have an impact in the more complex network.

3 EXPERIMENTAL SETUP

We analyze the performance of our IFSE-NEAT algorithm from two perspectives: (i) the quality of the derived policy, and (ii) the ability of the algorithm to select relevant features. We measure the quality of the derived policy by a problem-specific *fitness function*. The composition of the selected subset in terms of the *fraction of relevant features among selected ones* quantifies an algorithm’s ability to select a good feature subset. Finally, we verify that the performance of our algorithm (measured by the above metrics) does not degrade as the number of irrelevant features increases.

We compare IFSE-NEAT to the baseline NEAT as well as FS-NEAT, a competing feature selection algorithm we describe in Section 3.1. All three algorithms are evaluated in a challenging race track domain that is capable of providing many relevant and irrelevant

features for the algorithms to work with. The details of this environment as well as the specific parameters used by the algorithms are given in Section 3.2.

3.1 FS-NEAT

Feature Selective NEAT, or FS-NEAT, is an embedded feature selection algorithm within the NEAT framework (Whiteson et al., 2005). One limiting assumption standard NEAT makes, discussed in Section 2.1, is that all input features are relevant and are fully incorporated into all solution networks. FS-NEAT assumes that few features are actually relevant. Networks are initialized with only a single connection between a randomly selected pair of input and output nodes. Through subsequent mutations other input nodes may add a connection to the rest of the network and hence be selected into the model.

3.2 RARS

We conducted our experimental analysis using version 0.91.2 of the Robot Auto Racing Simulator (RARS)¹. RARS provides a detailed physical simulation of a racetrack and vehicles and allows users to define their own artificial agents to control the racers.

The goal of the simulation is to learn a path around the track that covers the most distance in a limited time while minimizing damage received by the car. Damage is calculated by RARS based on the amount of time the car spends off the track. The racers are controlled by supplying a desired speed and direction at every time step in the simulation.

Table 1: Different problems used in the RARS experiments broken down by the number of relevant, irrelevant, and total number of features.

Relevant	Irrelevant	Total
5	5	10
5	25	30
5	45	50
5	95	100

We implemented a rangefinder system in the simulation to provide vehicle position information to the learning algorithm as in Figure 2. In our experiments we placed N range sensors evenly around the front of the car as in (Whiteson et al., 2005), starting from the left side of the car and finishing at the right to provide a full view of the track. The range finders, together with the velocity of the car, are used by the learner to

¹<http://rars.sourceforge.net/>

provide two continuous control outputs, corresponding to the desired speed and direction of the car.

To make the RARS environment challenging from a feature selection point of view we added *irrelevant* features to the set. Irrelevant features simply return a random value in $[-1,1]$. We developed several challenging problems with different combinations of relevant and irrelevant features as shown in Table 1. These combinations allow us to examine the robustness of each of the three algorithms in comparison w.r.t. increasing numbers of irrelevant features.

All three algorithms tested are neuroevolutionary algorithms that require a fitness function to provide the feedback that guides learning. We adopt the fitness function used by (Whiteson et al., 2005), $S = 2d - r$, where d is the distance the car has traveled from the start and r is the amount of damage received. Trials end after the learner either has observed 2000 time steps or the car registers too much damage.

All experiments took place on the *clkwis.trk* track that is bundled in the RARS package, shown in Figure 3. This track was selected because it exhibits several driving scenarios such as straightaways, turns and an S-curve. The experiments were conducted in the RARS environment according to the following setup.

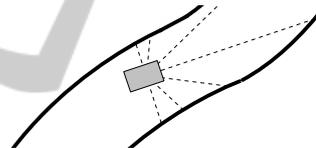


Figure 2: Rangefinders (dashed lines) extending outwards from the vehicle to the edges of the track (solid lines) encode vehicle location information in this environment.

- Three algorithms were tested, NEAT, FS-NEAT, and IFSE-NEAT
- For each tested combination of features 10 runs were conducted with each algorithm, results presented are the **average** of these 10 runs.
- Each run lasted 200 generations
- IFSE-NEAT split the 200 generations into five $L(\cdot)$ periods with $L(1) = 3$, $L(2) = 7$, $L(3) = 20$, $L(4) = 50$, $L(5) = 120$
- The NEAT population size was set to 100

We set the number of generations allowed to 200 since the algorithms appeared to converge by that point and there was no need to carry the experiment further. The particular values of the $L(\cdot)$ function are not important, and we experimented with other values which yielded similar results.

All three algorithms in comparison rely on NEAT for generation of the neural networks to allow learning. In our experiments we make use of Another

NEAT Java Implementation (ANJI) for the NEAT algorithm (James and Tucker, 2004). We followed the settings given in (Whiteson et al., 2005) of 0.10 and 0.02 for add-connection and add-neuron respectively to set the parameters for the FS-NEAT algorithm. For NEAT, and IFSE-NEAT we set the add-connection mutation probability to 0.02 and the add-neuron mutation to 0.01. In our experiments we found the parameters used with FS-NEAT to be too aggressive for NEAT and IFSE-NEAT.

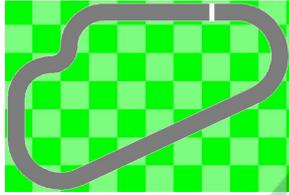


Figure 3: A top-down view of the clockwise track used in the experiments.

4 RESULTS

Figure 4a shows the results of running the three algorithms for the problem with 5 relevant features and 45 irrelevant features as the algorithms search for the optimal policy. Fitness of the derived policies is measured in terms of the value of the fitness function defined in Section 3.2. We can see that IFSE-NEAT converges to the best fitness of the three algorithms, and this convergence occurs at around generation 50. Both NEAT and FS-NEAT exhibit a slower rate of convergence than IFSE-NEAT. In this situation, NEAT is limited by the need to assign correct weights to many features. Since all available features are used in the NEAT neural network, NEAT has to evolve through many generations to find the right weights for links associated with the relevant features while keeping weights for irrelevant features low in order to limit their impact on the network output. The FS-NEAT algorithm suffers from its random search policy. Since there are many irrelevant features in the problem, they have a higher chance of being included in the network than a relevant feature does, causing the algorithm to be slow to learn an effective policy.

In Figure 4b we see the composition of the selected subsets by the three algorithms. IFSE-NEAT clearly has the highest percentage of relevant features per selected group, at around 90% on average. This number begins at 100% for 1 selected feature and slowly decreases as features are added to the set. Figure 4a shows that IFSE-NEAT achieves optimal fitness early, and then even relevant features do not appear helpful, causing some irrelevant features to be

incorrectly selected in some of the 10 runs of the algorithm. FS-NEAT slowly adds new features to the set, many of which are irrelevant, causing low scores in both measures. It should be noted that IFSE-NEAT and FS-NEAT select around 5 features by the 200th generation in all tested settings.

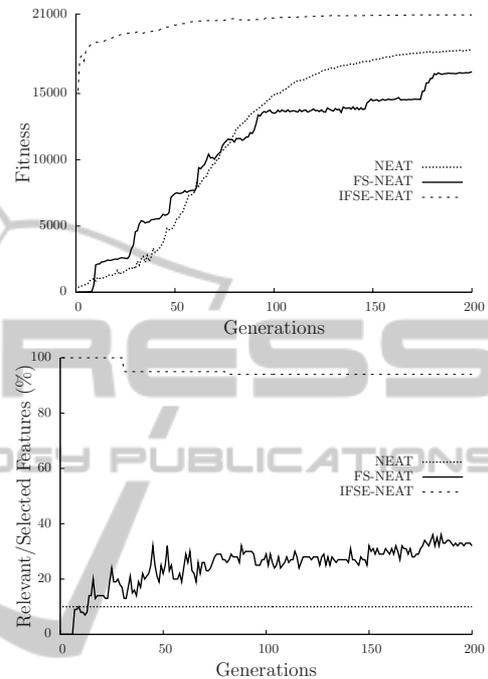


Figure 4: Two performance metrics: fitness (a) and the fraction of relevant features among the selected ones (b), for NEAT, FS-NEAT, and IFSE-NEAT across 200 generations on the problem with **5 relevant** features and **45 irrelevant** features.

We now further study how the three algorithms scale with an increasing number of irrelevant features. Figure 5b shows the fraction of relevant features among the selected ones by each algorithm. We can see that for each of the problems, IFSE-NEAT selects on average at least four relevant features in five feature selection steps. This validates our feature ranking and selection criteria, and supports the consistently good fitness values seen in Figure 5a. As predicted, NEAT's fitness degrades as the number of irrelevant features increases and the fraction of relevant features decreases. It always includes all the irrelevant features, which increases the complexity of the networks and slows down learning. FS-NEAT's fitness shows a variable trend caused by the random selection mechanism. Despite starting with more irrelevant features in the problem with 50 features, the fitness of the final policy actually improved over the problems with 10 and 25 features, as shown in Fig-

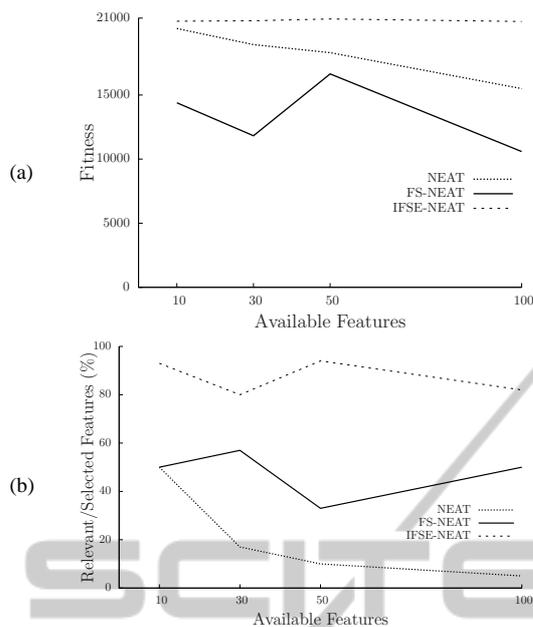


Figure 5: Two performance metrics: fitness (a) and the fraction of relevant features among the selected ones (b), for NEAT, FS-NEAT, and IFSE-NEAT at the 200th generation across 4 problems with 5 relevant features and 5, 25, 45, and 95 irrelevant features.

ure 5a. This is most likely the result of the network weights being randomly improved by chance and more trials should remove this effect.

5 CONCLUSIONS AND FUTURE WORK

In this work, we have developed an embedded feature selection algorithm which incorporates a sequential forward search into the neuroevolutionary function approximation method NEAT for reinforcement learning. Our results demonstrate the effectiveness of IFSE-NEAT at identifying relevant features and eliminating irrelevant ones. This ability enables IFSE-NEAT to converge upon higher quality policies using simpler networks in fewer generations than either NEAT or FS-NEAT.

These contributions do come at a cost. Although IFSE-NEAT is more efficient than wrapper methods, the incremental search for relevant features adds significant computational cost when compared to the other NEAT variants. Possible future directions include investigating the parallelization of the algorithm to help mitigate this cost, and further study on the generalization ability of the simple NN solutions found by IFSE-NEAT.

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