A DYNAMICAL MODEL FOR SIMULATING A DEBATE OUTCOME

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Keywords: Debate, Influence, Decisional power, Choquet integral, Control, Collective decision, Social network.

Abstract: A group of agents is faced with collective decisional problems. The corresponding debate is seen as a dynamical process. A first theoretical model based upon a muticriteria decision framework was proposed in (Rico et al., 2004) but without semantic justifications and explicit dynamical representation. A second descriptive model was proposed in (Imoussaten et al., 2009) where social influences and argumentation strategy govern the dynamics of the debate. This paper aims at justifying the equations introduced in (Rico et al., 2004) with the semantics concepts reported in (Imoussaten et al., 2009) to provide a model of a debate in the framework of control theory that explicitly exhibits dynamical aspects and offers further perspectives for control purposes of the debate.

1 INTRODUCTION

A group of agents is faced with a collective decision. A debate is organized to identify which alternative appears to be the most relevant one after deliberation. This study is limited to the binary but common situation where two options ± 1 are involved. It is assumed that each agent has an inclination to choose one of both alternatives ± 1 which, due to influence of other agents, may be different from the decision of the agent (Grabisch and Rusinowska, 2008). More generally, it can be considered that each time a speaker intervenes in the debate, agents may change their preference due to social influences in the group. When agents' preferences do not change anymore, the deliberation process ends and a group decision is made. The aim of the debate is that every agent knows the arguments of all the others at the end of the deliberation process and makes his final decision with full knowledge of the facts.

The deliberation is seen as a dynamical process with its own dynamics where beliefs and preferences of agents evolve when arguments are exchanged. The deliberation outcome thus depends on the order the agents intervene in the debate to explain their opinion and on the influence an agent may exert on a social network.

Social influence is here related to statistical notion of decisional power of an individual in a social network as proposed in (Hoede and Bakker, 1982) and (Grabisch and Rusinowska, 2008).

One of the conclusions of (Grabisch and Rusinowska, 2008) concerns the integration of dynamical aspects in the influence model. Indeed, the authors' framework is a decision process after a single step of mutual influence. In reality, the mutual influence does not stop necessarily after one step but may iterate. This paper proposes a possible extension of (Grabisch and Rusinowska, 2008) in the dynamical case. The evolutions of agents' beliefs during the debate change or reinforce the agents' convictions relatively to their initial preference. Intuitively, among others, the social influence of an agent depends on the more or less marked convictions of the other agents. Thus, the idea is to define influence as a time-varying variable itself in our model.

(Rico et al., 2004) introduces the concepts of influence and conviction in the simulation of a debate. This article follows prior works proposed in (Bonnevay et al., 2003). In (Rico et al., 2004),

Imoussaten A., Montmain J., Rico A. and Rico F..
A DYNAMICAL MODEL FOR SIMULATING A DEBATE OUTCOME .
DOI: 10.5220/0003126400310040
In *Proceedings of the 3rd International Conference on Agents and Artificial Intelligence* (ICAART-2011), pages 31-40
ISBN: 978-989-8425-40-9

coalitions of agents are modeled with capacities; the change of conviction during the debate was computed with a symmetric Choquet integral which is an aggregation function usually used in multicriteria decision making (Grabisch and Labreuche, 2002). The main drawback in (Rico et al. 2004) is its lack of semantic justifications.

Thus, (Grabisch and Rusinowska, 2008) provides a formal framework to define the notion of influence and (Rico et al., 2004) introduces the revision equations of agents' convictions and preferences. Finally, (Imoussaten et al., 2009) suggests a cybernetic interpretation to merge both models. This paper is the continuation of (Rico et al., 2004) in the light of (Imoussaten et al., 2009). The main contribution of this paper is to propose the state equations of the cybernetic interpretation to describe the way agents' convictions may evolve in time. To achieve this goal, a capacity is introduced to model the relative importance of agents in the debate that is based upon the decisional power of agents using the generalized Hoede-Bakker index (Grabisch and Rusinowska, 2008), (Hoede and Bakker, 1982). Hence some simulations are proposed to illustrate the collective decision making process.

The paper is organized as follows. Section 2 recalls briefly the main concepts of models in (Rico et al., 2004) and (Grabisch and Rusinowska, 2008). Based upon this formal framework, section 3 establishes the state equations that model the dynamical relationships between convictions and influences when a pair of *speaker-agent, listener-agent* is isolated. Section 4 associates the revision of convictions and the changes of preferences. Section 5 proposes some illustrations. Finally, the conclusion evocates the use of the model for debates controlling purposes.

2 CONCEPTS AND NOTATIONS

2.1 Notion of Influence in a Debate

The assumption behind our model is that the influence of an agent is related to his capacity to alter the group decision. It evocates the concept of «weight» of an agent's choice in a collective vote procedure. This «weight» cannot be a static parameter, because it should evolve with the preferences of agents that make the formation of certain coalitions more probable than other ones. To tackle this issue the definition of decisional power as proposed in (Grabisch and Rusinowska, 2008) is first summarized.

We consider a set of agents denoted $\{a_1,...,a_N\}$ or $\{1,...,N\}$ to simplify the notations and the power set is denoted $2^{\{a_1,...,a_N\}}$. It is assumed that each agent has an inclination to choose +1 or -1 which, due to influence of other agents, may be different from the decision of the agent. The point of departure is the concept of the Hoede-Bakker index—the notion which computes the overall decisional 'power' of an agent in a social network (n agents). This index was provided in 1982 (Hoede and Bakker, 1982).

Definition: the *Hoede-Bakker* index of agent a_j is defined by:

$$GHB_{a_j}(B, gd) = \frac{1}{2^{N-1}} \cdot \sum_{\{i/i_{a_j}=+1\}} gd(Bi) \qquad (1)$$

- i is an inclinations vector in $I = \{-1,+1\}^N$ that models the agents' inclinations, more precisely, we have $i = (i_{a_1},...,i_{a_N})$ where $i_{a_j} \in \{-1,+1\}$ the $j-th$ coordinate of i is the inclination of the agent a_j.
- $B: I \to I$ is the influence function and for any inclination vector i the decision vector Bi is a n-vector consisting of ones and minus ones and indicating the decisions made by all agents.
- $gd: B(I) \to \{-1,+1\}$ is the group decision function, having the value +1 if the group decision is +1, and the value −1 if the group decision is −1.

The main drawback of the Hoede-Bakker index is that it hides the actual role of the influence function, analyzing only the final decision in terms of success and failure. The decision is successful for an agent as soon as his inclination matches the group decision.

In (Grabisch and Rusinowska, 2008), the authors separate the influence part from the group decision part, and propose a first modified index of decisional power where the decision of the agent must coincide with the group decision to be a success for the agent. Lastly, the authors provide a second modified decisional power, which allows the inclinations vectors to be unequally probable.

Definition: Let $p: I \to [0,1]$ be a probability distribution, $p(i)$ is the probability i occurs. The modified decisional power is then:

$$\phi_{a_j}(B, gd, p) = \sum_{\{i/(Bi)_{a_j}=+1\}} p(i).gd(Bi) \\ -\sum_{\{i/(Bi)_{a_j}=-1\}} p(i).gd(Bi).$$

To conclude this summary, for each agent a_j the probabilities of success and failure are reminded:

$$SUC_{a_j}(B,gd,p)=\sum_{\{b\in I/(b)_{a_j}=gd(b)\}} p\circ B^{-1}(b)$$

$$FAIL_{a_j}(B,gd,p)=\sum_{\{b\in I/(b)_{a_j}=-gd(b)\}} p\circ B^{-1}(b)$$

Note that we have

$$\phi_{a_j}(B,gd,p)=SUC_{a_j}(B,gd,p)-FAIL_{a_j}(B,gd,p)$$

2.2 Convictions and Preferences in a Debate

This section presents the dynamical model of the debate proposed in (Rico et al., 2004). The influence an agent may have on the others is modeled by a capacity over $2^{\{a_1,\ldots,a_N\}}$.

Definition: A capacity υ over $2^{\{a_1,\ldots,a_N\}}$ is a set function $\upsilon: 2^{\{a_1,\ldots,a_N\}}\to[0,1]$ such that $\upsilon(\emptyset)=0$, $\upsilon(\{a_1,..,a_N\})=1$ and $\forall A,A'\subseteq\{a_1,..,a_N\}$, $A\subseteq A'\Rightarrow \upsilon(A)\le\upsilon(A')$.

The profile of an agent a_j includes his preference, his importance (his capacity $\upsilon(a_j)$), his conviction $c_{a_j}\in[0,1]$ related to his preference. It is stated as a rule that agents speak in turns. The agent a_s (*speaker-agent*) who speaks and any agent a_l (*listener-agent*) are isolated which introduces a capacity υ_{a_l,a_s} upon the pair of agents (a_l,a_s). More precisely, the following capacity is defined $\upsilon_{a_l,a_s}(a_l)=\dfrac{\upsilon(a_l)}{\upsilon(a_l,a_s)}$, $\upsilon_{a_l,a_s}(a_s)=\dfrac{\upsilon(a_s)}{\upsilon(a_l,a_s)}$ and $\upsilon_{a_l,a_s}(a_l,a_s)=1$.

The change of conviction is then modeled with the symmetric Choquet integral also called Sipos integral. The definition of Choquet integral and Sipos integral are now provided.

Definition: *Let* $c=(c_{a_1},\ldots,c_{a_N})\in[0,1]^N$ *be a vector of convictions, () be a permutation on* $\{1,\ldots,N\}$ *such that* $c_{a_{(1)}}\le\ldots\le c_{a_{(N)}}$ *and* υ *be a capacity on* $2^{\{a_1,\ldots,a_N\}}$.

The Choquet Integral of c *with respect to* υ *is defined by:*

$$C_\upsilon(c)=\sum_{i=1}^{N}\left[c_{a_{(i)}}-c_{a_{(i-1)}}\right].\upsilon(\{(i),\ldots,(N)\})\ \textit{with}\ c_{a_{(0)}}=0$$

Definition: *Let* $c=(c_{a_1},\ldots,c_{a_N})\in[-1,1]^N$ *be a vector which can take negatives values, () be the permutation on* $\{1,\ldots,N\}$ *such that* $c_{a_{(1)}}\le\ldots\le c_{a_{(p)}}\le 0\le c_{a_{(p+1)}}\le\ldots\le c_{a_{(N)}}$ *and* υ *be a capacity on* $2^{\{a_1,\ldots,a_N\}}$.

The symmetric Choquet Integral of c *with respect to* υ *is defined by:*

$$\breve{C}_\upsilon(c)=\sum_{i=1}^{p-1}\left[c_{a_{(i)}}-c_{a_{(i+1)}}\right]\upsilon(\{(1),\ldots,(i)\})+c_{a_{(p)}}\upsilon(\{(i),\ldots,(p)\})$$

$$+c_{a_{(p+1)}}\upsilon(\{(p+1),\ldots,(N)\})+\sum_{i=p+2}^{N}\left[c_{a_{(i)}}-c_{a_{(i-1)}}\right]\upsilon(\{(i),\ldots,(N)\})$$

In this paper the Sipos integral is defined on the set of agents $\{a_l,a_s\}$. It is denoted $\breve{C}_{\upsilon_{a_l,a_s}}$. The changes of convictions proposed in (Rico et al., 2004) can be summarized as follows with $\breve{C}_{\upsilon_{a_l,a_s}}$:

- If agents a_l and a_s have the same preference,

When $c_{a_s}>c_{a_l}$ the new conviction is:

$$\breve{C}_{\upsilon_{a_l,a_s}}(c_{a_s},c_{a_l})=c_{a_l}+(c_{a_s}-c_{a_l}).\upsilon_{a_l,a_s}(a_s)$$

When $c_{a_l}>c_{a_s}$ the new conviction is:

$$\breve{C}_{\upsilon_{a_l,a_s}}(c_{a_s},c_{a_l})=c_{a_s}+(c_{a_l}-c_{a_s}).\upsilon_{a_l,a_s}(a_l).$$

- If agents a_l and a_s do not have the same preference, the new conviction is:

$$\breve{C}_{\upsilon_{a_l,a_s}}(c_{a_s},c_{a_l})=-c_{a_s}.\upsilon_{a_l,a_s}(a_s)+c_{a_l}.\upsilon_{a_l,a_s}(a_l)$$

The main drawback to this model is its lack of semantics justifications with regard to capacity υ (influence is merely a normalized relative importance), the concept of conviction is not formally defined and the revision equations are not provided in an appropriate formalism where time would appear explicitly (dynamical aspects).

3 THE DYNAMICAL MODEL

This section presents our dynamical model for simulating a debate outcome. To begin note that in the framework of this paper, influence function B used in (Grabisch and Rusinowska, 2008) is perceived as a disturbance function applied to the set of all the possible inclination vectors.

3.1 Decisional Power and Capacities

This section proposes to design a capacity based upon the decisional power for the above model.

For any $i \in I$, the group decision is modeled by $gd(Bi)$ and belongs to $\{-1,+1\}$. Furthermore, $\phi_{a_j}(B, gd, p) \in [-1,1]$.

- If the decisional power of an agent is close to -1, it means that the agent scarcely chooses the alternative the collective finally chooses: he fails most of the time ($FAIL$).

- In revenge, when his decisional power is close to 1, the agent is most of the time successful $(SUCC)$; his decisional power is high.

For example, without further information, the importance of an agent a_j, *i.e.*, his capacity $\upsilon(a_j)$, can be defined as: $\upsilon(a_j) = \frac{1}{2}.\phi_{a_j}(B, gd, p) + \frac{1}{2}$, $\upsilon(a_j) \in [0,1]$ with $\upsilon(a_j) = 0$ if and only if $\phi_{a_j}(B, gd, p) = -1$ and $\upsilon(a_j) = 1$ if and only if $\phi_{a_j}(B, gd, p) = 1$.

It thus defines a function $\upsilon : \{a_1,..,a_N\} \to [0,1]$. From this function, a capacity υ can be generated over $2^{\{a_1,..,a_N\}}$, with constraints $\forall A, A' \subseteq \{a_1,..,a_N\}$, $A \subseteq A' \Rightarrow \upsilon(A) \le \upsilon(A')$. Without further knowledge, it can be chosen: $\upsilon(A) = \max_{a_j \in A} \upsilon(a_j), \forall A \subset \{a_1,..,a_N\}$ and $\upsilon(\{a_1,..,a_N\}) = 1$

Note that this definition does not necessarily imply that there exists an agent whose capacity is equal to 1.

In the following and to simplify notations, such a capacity is denoted υ_ϕ for a decisional power $\phi(B, gd, p)$. The decisional power of individuals a_j on which $\upsilon_\phi : 2^{\{a_1,..,a_N\}} \to [0,1]$ is based measures the cases where the final decision of a_j matches the group decision. An agent with a high decisional power is expected to bring several agents round and thus the decisional power is considered as an estimation of his "influence" in the group; although it is not an influence index in the sense of (Grabisch and Rusinowska, 2008).

3.2 Time-varying Probabilities

Note that this subsection is dedicated to the design of probability p as a time-varying function. It is thus supposed that convictions vectors $c(k)$ (the convictions vector of the agents w.r.t alternative $+1$ at time k) and $c'(k)$ (the convictions vector of the agents w.r.t alternative -1 at time k) are known at k. Their computation is provided in the next section.

The model proposed in this paper is based upon the extended decisional power in (Grabisch and Rusinowska, 2008) that allows the inclinations vectors to be unequally probable. The definition of the associated probability distribution $p : I \to [0,1]$ is now required (see section 2.1). This paper proposes to base the probability computation upon the convictions of agents with regard to the alternatives.

The conviction of an agent regarding an alternative is related to the probability this agent chooses this alternative, *i.e.*, the probability of his inclination as defined in (Grabisch and Rusinowska, 2008). As stated above, convictions evolve in time during the deliberation process.

$c(k) = \left(c_{a_1}(k),.. \;, c_{a_j}(k), \; ..., c_{a_N}(k)\right)$ where $c_{a_j}(k)$ is the conviction of agent a_j w.r.t alternative $+1$ at time k.

$c'(k) = (c'_{a_1}(k),... \;, c'_{a_j}(k), \; ..., c'_{a_N}(k))$ where $c'_{a_j}(k)$ is the conviction of agent a_j w.r.t alternative -1 at time k.

Let $i \in I$ be an inclinations vector, and let define $c^i(k) \in [0,1]$ as an "average" conviction at time k for i: this value summarizes the distributions of agents' convictions in i at k. $c^i(k)$ is an «aggregated conviction» of the group of agents for i. This aggregation should take into account relative importance of agents and their interactions.

The probability is built by recurrence on k.

<u>At time $k = 0$:</u>

$$c(0) = \left(c_{a_1}(0),... \;, c_{a_j}(0), \; ..., c_{a_N}(0)\right)$$

is the a priori convictions vector of agents.

$c_{a_j}(0) \in [0,1], j = 1..N$ is the a priori convictions of a_j, and it is also the probability of his conviction. Initially ($k = 0$), if i_{a_j} is the preference of a_j then the probabilities of the agent a_j regarding his preference and the other alternative are:

$$p_{a_j}(i_{a_j})[k = 0] = c_{a_j}(0)$$

and

$$p_{a_j}(-i_{a_j})[0] = 1 - c_{a_j}(0)$$

Before the debate starts, the inclination of each agent does not depend on the social network. Then, the probability distribution associated to a priori probabilities is the product of the individual probabilities p_{a_j} at $k=0$:

$\forall i \in I, p(i)[0] = \Pi_{j=1}^{N} p_{a_j}(i_{a_j})[0]$. It is thus possible:

- computing the decisional power for any agent a_j at $k=0$, $\phi_{a_j}(B, gd, p[0])$;

- computing the capacity $\upsilon_\phi[0]$ over $2^{\{a_1,..,a_N\}}$, for $k=1$, as proposed in subsection 3.1.

At time $k = 1$

Capacity $\upsilon_\phi[0]$ allows computing $c^i(1)$, the aggregated conviction at $k=1$ for the inclinations vector i. The relative importance of agents and of their coalitions is taken into account in the aggregation model of c^i through a Choquet integral.

Let $i \in I$ be an inclinations vector. Each coordinate i_{a_j} is one of the alternative -1 or $+1$. For agent a_j, $c_{a_j}(1)$ or $c'_{a_j}(1)$ is the conviction associated to his preference. A conviction vector $c(1) = \left(\overline{c}_{a_1}(1) \ \ ... \ \ \overline{c}_{a_n}(1)\right)$ is associated to each inclinations vector i, where for any j, $\overline{c}_{a_j}(1)$ is $c_{a_j}(1)$ or $c'_{a_j}(1)$. The synthetic conviction of the group for inclination i at time $k=1$ is computed with a Choquet integral defined upon $\upsilon_\phi[0]$.

$$c^i(1) = C_{\upsilon_\phi[0]}(\overline{c}_{a_1}(1), ..., \overline{c}_{a_N}(1)) \in [0,1]$$

A probability at $k=1$ can then be defined:

$$p(i)[1] = \frac{c^i(1)}{\sum_{t \in I} c^t(1)}$$

It is now possible:

- Computing the decisional power of any agent a_j at $k=1$, $\phi_{a_j}(B, gd, p[1])$;
- Computing the capacity $\upsilon_\phi[1]$ over $2^{\{a_1,..,a_N\}}$, for $k=2$, as proposed in subsection 3.1.

More generally, at time $k + 1$:

Capacity $\upsilon_\phi[k]$ computed at k allows computing $c^i(k+1)$, the aggregated conviction at $k+1$ for inclinations vector i with the Choquet integral:

$$c^i(k+1) = C_{\upsilon_\phi[k]}(\overline{c}_{a_1}(k+1), ..., \overline{c}_{a_N}(k+1)) \ .$$

Then, probability at time $k+1$ is defined as:

$$p(i)[k+1] = \frac{c^i(k+1)}{\sum_{t \in I} c^t(k+1)}$$

It is now possible computing $\phi_{a_j}(B, gd, p[k+1])$ and $\upsilon_\phi[k+1]$.

The probability required by the extended model of decisional power has been designed as a time varying variable because it evolves with the agents' convictions. Therefore, $\phi(B, gd, p[k])$ evolves in time too. This principle seems rather intuitive because it corresponds to the idea that the social influence of an agent depends on the more or less marked convictions of the other agents when he speaks.

3.3 Conviction State Equations

The aim of this section is to establish the state equations that model the dynamical relationship between convictions and influences. Let consider a pair of listener-agent, speaker-agent denoted a_l and a_s. Their convictions for the alternative $+1$ are $c_{a_l}(k)$ and $c_{a_s}(k)$, respectively, $c'_{a_l}(k)$ and $c'_{a_s}(k)$ for the alternative -1.

Two variables are necessary to model the rhetoric quantity that is exchanged between both agents a_l and a_s:

- The difference of convictions between both agents;

- The relative importances of agents a_l and a_s modeled by capacities $\upsilon_\phi[k](a_s)$ and $\upsilon_\phi[k](a_l)$.

Four rhetoric exchanges are distinguished. These four situations are presented in the case when a_l prefers alternative $+1$. Then, there exist two sub cases for agent a_s: his favorite alternative is the one of a_s or the opposite one. Each case can be divided again into two sub cases: a_s's conviction is greater than (respectively lower than) a_l's conviction.

When agent a_l prefers the alternative -1, convictions c' take the place of convictions c in the formula: the equations that appear in the computation of $c_{a_l}(k+1)$ when both agents have the same preference are the same ones to compute $c'_{a_l}(k+1)$ in case of opposite preferences and vice versa.

Synergic Exchange

It is the case when the preference of the agent a_l is reinforced by the intervention of the agent a_s who resolutely looks on the same alternative in favor.

The conviction of the agent a_l increases. The increase is proportional to the difference between both convictions and to the capacity of speaker a_s. This situation corresponds to the case when a_l and a_s have the same preference and moreover $c_{a_s} > c_{a_l}$. The intuitive difference equation is then (Figure 1):

$$c_{a_l}(k+1) - c_{a_l}(k) = (c_{a_s}(k) - c_{a_l}(k)).\upsilon_\phi[k](a_s)$$

or

$$c_{a_l}(k+1) = c_{a_l}(k) + (c_{a_s}(k) - c_{a_l}(k)).\upsilon_\phi[k](a_s)$$

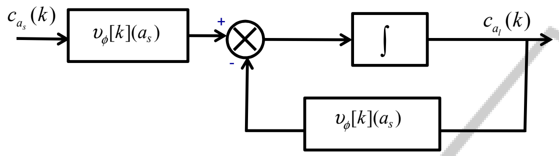

Figure 1: Synergic Exchange.

Revisionist Exchange

The agent a_l understands the argument of the agent a_s, who has the same preference but more moderately. a_s appears to speak with restraint from a_l point of view and a_l's doubt appears. a_l's conviction is thus mitigated by a_s intervention. This situation corresponds to the case when a_l and a_s have the same preference and moreover $c_{a_l} > c_{a_s}$. The intuitive difference equation is then (Figure 2):

$$c_{a_l}(k+1) - c_{a_l}(k) = -(c_{a_l}(k) - c_{a_s}(k)).(1 - \upsilon_\phi[k](a_l))$$

or

$$c_{a_l}(k+1) = c_{a_s}(k) + (c_{a_l}(k) - c_{a_s}(k)).\upsilon_\phi[k](a_l)$$

The agent a_l observes the indecision of agent a_s who nevertheless shares his opinion:
a_s contributes to a_l's doubt. The conviction decreases due to a_s's intervention that is proportional to $(1 - \upsilon_\phi[k](a_l))$ on one hand (lack of assurance of a_l related to his social position in the group) and to the difference between both convictions of agents a_s and a_l on the other hand.

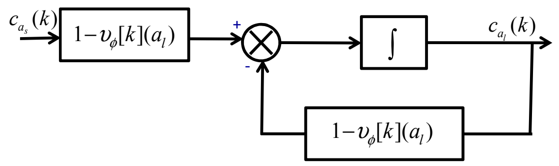

Figure 2: Revisionist exchange.

Antagonist Exchange

Both agents do not share the same preference; agent a_l nevertheless understands the advantages of a_s preference. A convincing intervention of a_s may contribute to make a_l doubtful whereas a non persuasive intervention may strengthen his preference on the contrary.

$(1 - c'_{a_s}(k))$ is a measure of a_s's hesitation and provides a_l with an estimation of the strength of a_s's opposition. According to the strength of this hesitation, the previous difference equations are usable with $(1 - c'_{a_s}(k))$ and two situations are to be distinguished (Figure 3).

A too weakly marked preference of a_s means a weak opposition from a_l point of view and reinforces a_l's opinion. a_l's conviction should then increase.

The intuitive difference equation is then (synergic exchange with $(1 - c'_{a_s}(k))$):

Case 1: $1 - c'_{a_s} \geq c_{a_l}$

$$c_{a_l}(k+1) - c_{a_l}(k) = ((1 - c'_{a_s}(k)) - c_{a_l}(k)).\upsilon_\phi[k](a_s)$$

or

$$c_{a_l}(k+1) = c_{a_l}(k) + (1 - c'_{a_s}(k) - c_{a_l}(k)).\upsilon_\phi[k](a_s)$$

In the second case, a_l's conviction decreases after a_s's intervention (revisionist exchange with $(1 - c'_{a_s}(k))$).

Case 2: $1 - c'_{a_s} < c_{a_l}$

$$c_{a_l}(k+1) - c_{a_l}(k) = -(c_{a_l}(k) - (1 - c'_{a_s}(k))).(1 - \upsilon_\phi[k](a_l))$$

or

$$c_{a_l}(k+1) = (1 - c'_{a_s}(k)) + (c_{a_l}(k) + c'_{a_s}(k) - 1).\upsilon_\phi[k](a_l)$$

All these different types of exchanges can be synthesized with a Sipos integral as follows:
The agents a_s and a_l have got the same preference:

$$c_{a_l}(k+1) = \breve{C}_{\upsilon_\phi}[k](c_{a_s}(k), c_{a_l}(k))$$

The agents a_s and a_l do not share the same preference:

$$c_{a_l}(k+1) = \breve{C}_{\upsilon_\phi}[k]((1 - c'_{a_s}(k)), c_{a_l}(k))$$

To conclude this part, the decisional power ϕ provides a semantic interpretation for the capacity υ in the recurrence equations in (Rico et al., 2004), conviction is here related to the probability an agent will choose an alternative rather than the other one (probability distribution over inclinations vectors).

Thus, the model in (Rico et al., 2004) becomes interpretable in games theory framework (Grabisch and Rusinowska, 2008). Revision equations of conviction appear as inputs-outputs balances according to alternatives assessment. Introducing time in the equations of (Rico et al., 2004) implies that revision equations of conviction are now seen as state equations of agents' mental perception. This new interpretation provides a semantics for the model of a debate in (Rico et al., 2004): it is related to the notions of influence and decisional power as proposed in (Grabisch and Rusinowska, 2008) with a formalism close to the one of dynamical models in control theory as suggested in (Imoussaten et al., 2009).

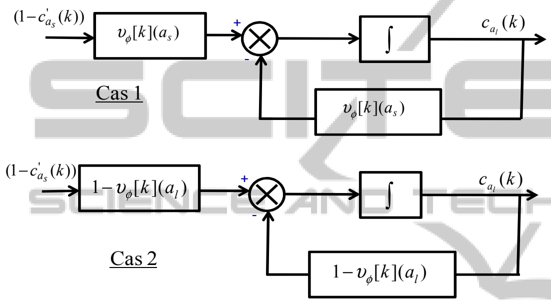

Figure 3: Antagonist exchange.

4 PREFERENCES CALCULUS

This section presents how to compute the preference during the debate.

Initially each agent a_j assesses both alternatives $+1$ and -1 with a score in $[0,1]$. These assessments are noted $n_{a_j}^{+1}$ and $n_{a_j}^{-1}$. It is then possible to build initial preferences and convictions:

- a_j prefers alternative a with the highest score, a_j's conviction related to alternative a is $n_{a_j}^{a} \big/ (n_{a_j}^{a} + n_{a_j}^{\bar{a}})$ and a_j's conviction related to the other alternative $\bar{a}$ is $n_{a_j}^{\bar{a}} \big/ (n_{a_j}^{a} + n_{a_j}^{\bar{a}})$.

Preferences changes depend on the way convictions evolve in time. For any agent a_j, it is supposed there exists a threshold $\varepsilon_{a_j} > 0$ such that when the difference between two convictions is below this threshold then the agent a_j cannot have a preference. The threshold value may be characteristic of each agent. To summarize:

- When $\left|c_{a_j} - c'_{a_j}\right| < \varepsilon_{a_j}$, then a_j has no preference;

- When $\left|c_{a_j} - c'_{a_j}\right| \geq \varepsilon_{a_j}$, a_j prefers the alternative with the highest conviction.

Finally, an agent without preference cannot intervene is stated as a rule of the debate.

5 ILLUSTRATION

5.1 Simulations of the Debate's Outcome

In order to illustrate the principle of the above dynamical representation of a debate, the four following elementary models for influence function B have been implemented:

- B is the *identity* that is to say for any inclination vector i we have $Bi = i$
- B is the *opposite of identity, for any inclination vector i we have* $Bi = -i$
- B is a mass psychology effect function. More precisely, if we denote $i^{\varepsilon} = \{k \in N / i_k = \varepsilon\}$, B satisfy : for each $i \in I$: $\left|i^{\varepsilon}\right| > t, then\ (Bi)^{\varepsilon} \supseteq i^{\varepsilon}$ where $t \in [1,n]$ and $\varepsilon = \pm 1$.
- B is a majority
- influence function models behaviors of type: if a majority of agents has an inclination +1, then all agents decide +1; if not, all agents decide −1

For the four cases, the group decision function gd is a mere majority and a basic capacity is designed as proposed in section 1.3.

Let consider a group of $N = 8$ agents. The initial convictions of agents relatively to both alternatives are considered as variates: 50 random drawings of these 8 initial probabilities are carried out (Figure 4). For each of these 50 initial convictions vectors the order the agents intervene in the debate is then considered: 200 permutations are randomly selected (among the $8!$ possible rankings) for each initial convictions vector.

Each of the four elementary illustrations is plotted in figure 4 (one for each B function). For each of the 50 initial convictions vectors randomly selected, a bar represents the number of outcomes ± 1 (light-grey for $+1$ and dark-grey for -1).

To each of these figures is associated the maximal number of rounds that have been necessary to achieve the ground decision for each initial convictions vector. In the proposed simulations this number does not exceed 8 rounds in any B-case.

The indifference threshold is $\varepsilon = 0.01$ for any agent. Agents speak in turns according to the order induced by the 200 permutations on condition they have a clear opinion: an agent a_j can speak if $\left|c_{a_j} - c'_{a_j}\right| \geq \varepsilon$.

For a same initial convictions vector it can be observed that for each function B, the outcome of the debate may depend on the order the agents intervene. This type of situation can be interpreted as weakly marked preferential contexts where any perturbation can change the debate's outcome. From this point of view, influence function B is a disturbance function in this dynamical model of a debate. As a consequence, simulations allow checking that the order the agents intervene in the debate and their influence are decisive variables with regard to the convergence of conviction state equations.

The social influence of an agent may thus be considered as a disturbance in the deliberation process except if it is relevantly used by the debate manager to govern the discussion. Indeed, in this later case, social influence can be envisaged as an actuator that enables controlling the outcome of the debate or at least accelerating its convergency. For example, when the outcome of the debate is quasi certain (the bar is almost completely light or dark grey), then the simplest control could consist in choosing the order the agents intervene that minimizes the maximal number of rounds. More complex control can be clearly envisaged but the aim of this paper was merely to propose a dynamical model of the debate in a framework close to control theory representations, then control techniques should be naturally implemented in the future.

5.2 Debate as a Decision Making Process

This part presents a potential application of the presented dynamical model. The aim is to use it as a vote system. More precisely, in this example, both alternatives -1 and $+1$ are not considered to be equivalent: +1 is the right decision while -1 is associated to an error. This situation may occur in classification problems when the agents are competitive classification algorithms.

The agents are expected to provide the right answer most of the time but they usually disagree on singular cases. A common solution is to use a voting process to achieve a group decision. For example, let the agents be 7 different classification algorithms whose success rates are respectively: 0.6, 0.7, 0.8, 0.8, 0.6, 0.7 and 0.6; then, the group success rate using normal vote is 0.86. Even when a weighted vote is introduced, the same rate is obtained because of the value of the Shapley–Shubik power index (Shapley, 1953) which is equal to 1/7 for any classification algorithms. Indeed as probability are hardly, bigger than 50% for each classification algorithms, for the normal vote as for the weighted vote the chosen value is the one which is chosen by at least four agents. This effect does not take place in the proposed method because the least agents are also the ones who change most easily his point of view. More precisely, this issue can also be tackled with our debate model with *identity* as B function, and *success rates* for convictions. It is supposed that 7 competitive classification algorithms are available and that the right solution is supposed to be alternative +1. The initial probability of the 7 algorithms to choose the alternative +1 are: 0.6, 0.7, 0.8, 0.8, 0.6, 0.7 and 0.6. Moreover it is supposed that 10 000 cases are studied by each agent. For each case, the answer of the agent is inferred according to his probability to be right (this is one method to model the aggregation function corresponding to the different classification algorithm. Then, for each of the 10000 cases, the choice with a majority vote procedure and the collective decision achieved with our model when convictions at the start are the initial probabilities are both computed. The program stops when all the classification algorithms do agree. While simple and weighted majorities obtain the right answer with a rate of 86 %, our method rate is 94 %.

For 7 agents, several values of probability to make the right decision are randomly generated and 3 rates are computed:

- the rate of success of the vote,
- the rate of success the weighted vote,
- the rate of success of our debate.

The rate of the weighted vote and of our debate according to the rate of simple vote are plotted in figure 5. Note that the same rate for the simple vote can be obtained with very different sets of probabilities. That is why the rate of success of the weighted vote is somewhat equal to the simple vote for very particular sets of probabilities where several agents (algorithms) are much better than the others. The debate always gives a better rate but its preferences change according to the profile of involved probabilities.

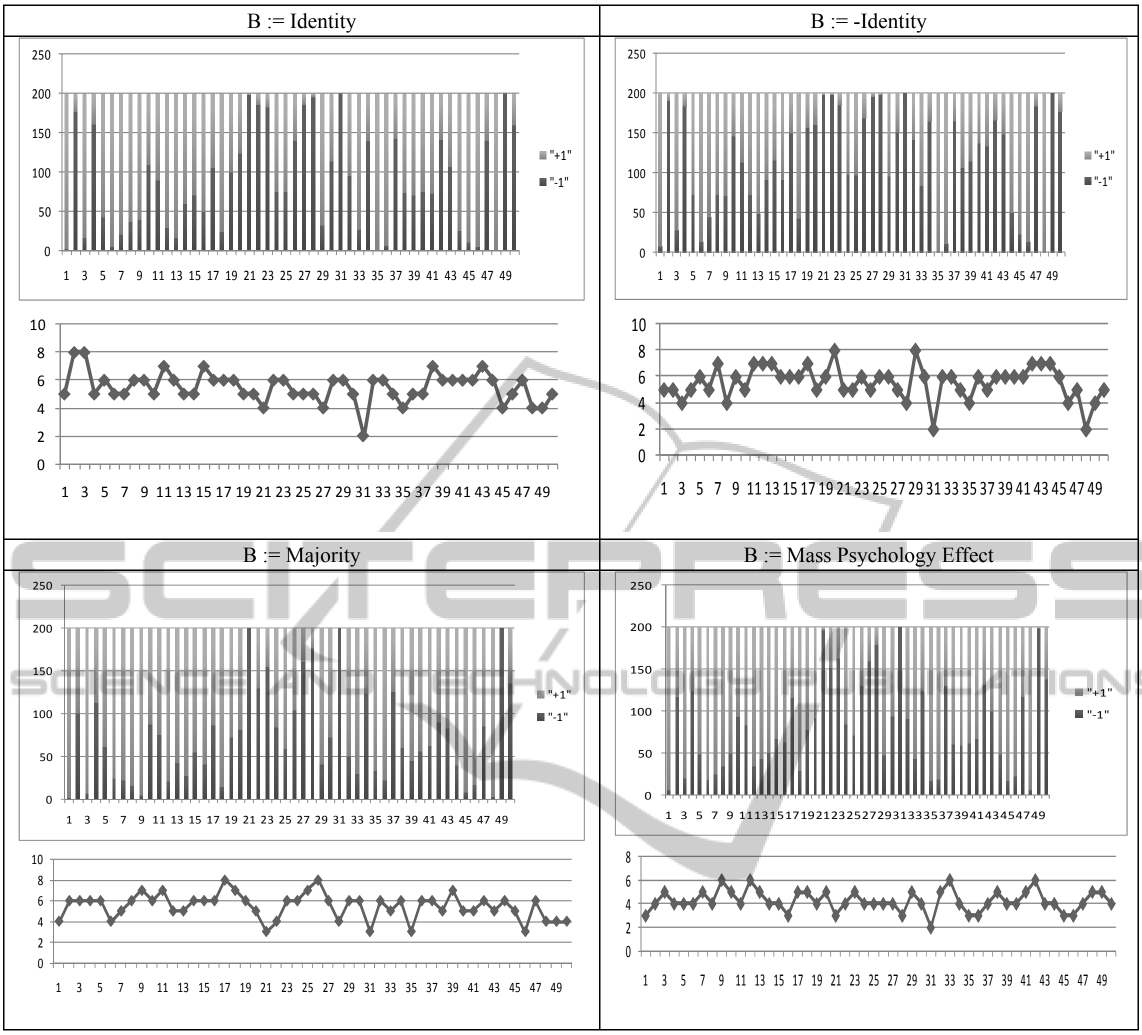

Figure 4: Simulations of the debate outcomes (50 initial convictions; 200 permutations).

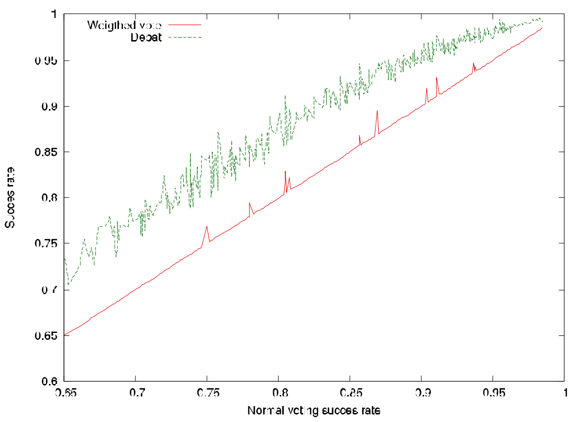

Figure 5: Simulations—weighted vote and debate.

6 CONCLUSIONS AND PERSPECTIVES

The state equations that have been established in this paper allow simulating macroscopically the outcome of a debate according to the initial inclinations of agents and the social influences in the group (the influence function is a priori known). The deliberation outcome depends on the order the agents intervene in the debate to explain their opinion and on the influence an agent may exert on a social network.

The formalism of the model that is proposed in this paper is close to the one used in control theory to model dynamical behaviors of technical systems. Governing a debate could then be seen as a control problem whose aim could be, for example, how to reach as quick as possible a consensus or how to reinforce one alternative rather than the other one, etc.

A debate is thus seen as continuous dynamical system: a state equations representation has been preferred to the muticriteria decision-making framework in (Rico et al., 2004) because time explicitly appears in revision of convictions. The model semantic is also inspired of games theory concepts proposed in (Grabisch and Rusinowska, 2008): influence and decisional power in a social network. In our dynamical extension, the decisional power is a time-varying variable itself and can be used as the actuator signal in the control loop of debate. The state equations system established in this paper allow stochastically simulating the outcome of a debate and effects of a control strategy on this issue.

One possible application of this model is obviously simulating a debate's outcome in order to obtain some indications about the final collective decision. When simulations are performed for a great number of initial agents' convictions and of speaker intervention rankings, the probability the outcome is ± 1 can be estimated. Hence, the dynamical influence model can be used to make the debate outcome more certain (it may appear as a dishonest method when agents are human beings but as a relevant technique when agents are artificial agents such as sensors or classifiers) or modify the convergence dynamics of the debate.

REFERENCES

Bonnevay, S., Kabachi, N., Lamure, M and Tounissoux, D., 2003, *A multiagent system to aggregate preferences*. In IEEE International Conference on Systems, Man and Cybernetics, pages 545-550, Washington, USA.

Grabisch, M. and Labreuche, C., 2002. *The symmetric and asymmetric choquet integrals on finite spaces for decision making*. Statistical paper, Springer Berlin / Heudelberg, vol. 43, 2002 pp 37-52.

Grabisch, M. and Rusinowska, A., 2008. *A model of influence in a social network*. Business and Economics. s.l.: Springer Netherlands.

Grabisch, M. and Rusinowska, A., 2009. *Measuring influence in command games*, Social Choice and Welfare 33: 177-209.

Hoede, C, Bakker, R., 1982. *A theory of decisional power*. Journal of Mathematical Sociology, 8:309-322.

Imoussaten, A., Montmain, J. et Rigaud, E., 2009. *Interactions in a Collaborative Decision Making Process: Disturbances or Control Variables?*. COGIS'2009, Paris.

Rico, A., Bonnevay, S., Lamure, M., Tounissoux, D. A, 2004. *Debat modelisation with the Sipos integral*. LFA'2004, Nantes.

Rusinowska, A. and De Swart, 2007. *On some properties of the Hoede-Bakker index*. Journal of Mathematical Sociology 31: 267-293.

Shapley, L. S., 1953. *A Value for n-Person Games*. in H. Kuhn and A. Tucker (Eds.), Contribution to the Theory of Games, vol. II, number 28 in Annals of Mathematics Studies Princeton, pp. 303-317.