SPARKLINE HISTOGRAMS FOR COMPARING EVOLUTIONARY OPTIMIZATION METHODS

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Abstract: Comparing evolutionary optimization methods is a difficult task. As more and more of articles are published in this field, the readers and reviewers are swamped with information that is hard to decipher. We propose the use of *sparkline histograms* that allow compact representation of test data in a way which is extremely fast to read and more informative than usually given metrics.

1 INTRODUCTION

The performance of evolutionary algorithms is generally evaluated by repeatedly running them against a set of test functions; this process generates a set of values for each algorithm/function pair, leading to a large amount of data which then needs to be interpreted. The common practice is to present tables containing average and standard deviations values, sometimes along with minima and maxima. When reading those tables however, one is not so much interested in the numbers as in the relationships between them: which algorithm is nearer to the global optimum? How far is one algorithm from another one? In this paper we present a new method for displaying the results in accordance to three constraints:

1. Convey more information than the usual tables of numbers.
2. Use no more space in print than the tables.
3. Be readable at first glance.

Table 1 shows an example of a traditional numerical table compared to a table containing stacked focused histograms. One histogram represent the distribution of the values in the set of results produced by one algorithm repeatedly applied to one test functions. Additionally, all the histograms in the same column are "focused" on the range which is considered interesting. One histogram therefore shows the reader all the result values, showing how spread or clustered they are; minimum and maximum values can be compared without needing to read actual numbers, and average values can be estimated and compared as well. This example moreover shows that the table requires no more space than the traditional one, and because it uses a graphical representation rather than a textual one, it is instantly readable.

2 SHORTCOMINGS OF CURRENT PRACTICES

As mentioned above, the performance of an evolutionary algorithm is generally evaluated by repeatedly running it against a set of test functions and statistically analysing the results and comparing them to those of reference algorithms. On a typical article of this nature, 3 to 5 algorithms are applied to a benchmark of 10 to 25 test functions, often in several dimensions.

Evolutionary algorithms being by design stochastic processes, one cannot compare the performance of two given algorithms A and B on a given test function f by applying each of them only once to the function and comparing the results: the result of this single run may not be representative of the actual performance of the algorithms. The common practice is therefore to run each algorithm multiple times, typically 25 to 100 times, which produces as a result a set of several thousands of numbers. This set obviously needs to be reduced in order to fit on the few pages allotted for that purpose in the article, which in turns poses the question of their presentation.

The current practice is to summarize the numbers in a double entry table containing, for each algorithm-function pair, the average end result reached by the

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Table 1: Stacked focused histograms convey more information regarding an algorithm's performance than a table filled with numbers and are readable at first glance.

	Function 4
Method 1	$-3.06e+02 \pm 5.68e+00$
Method 2	$-1.22e+02 \pm 2.59e+00$
Method 3	$-3.11e+02 \pm 1.49e+01$
Method 4	$\mathbf{-3.51e+02 \pm 3.58e+01}$

Function 4		
Method 1		...
Method 2		...
Method 3		...
Method 4		...
Min Max	-3.971e+02	-2.871e+02

Table 2: Example of stacked sparklines.

Laboratory environment	
	temperature **22.4° C**
	humidity **32%**
	light **412 lux**

algorithm when applied repeatedly to that function, along with the standard deviation. Although it arguably provides the reader with as much numerical data as it is possible in the given space, reading such a table is difficult: while the best results for each function can be highlighted (e.g., using a boldface font), the relative results of the various algorithms on on given function is visible only if the reader takes the time to read the numbers carefully and compare them.

3 THE SPARKLINE HISTOGRAMS

Histograms are a common data visualization technique that is often used for comparing dense sets of numbers. This visualization technique fits very well to task of comparing optimization algorithms, as it allows to present all the results of repeated experiments at the same time. This technique has been used in, for example, articles (García et al., 2009; Fan and Lampinen, 2003). The use of histograms conveys more data than a single statistical number, is easily readable, and easily satisfies the constraints 1 and 3 presented in the introduction.

Large histograms require a significant amount of space and often cannot be included due to the constraints on the article length. To solve this problem we look at the work of Edward Tufte (Tufte, 2006), which demonstrates that very small graphics can be as legible as large page filling graphics. In some cases, smaller graphic be a better choice due to more favourable aspect ratio. Tufte applies this idea in time-series visualisation and dubs his invention "sparklines", with the definition *"data-intense, design-simple, words-sized graphics"*. In essence, a sparkline is a font-high, one-word wide time series plot. Similarly to equations, sparklines can be set outside of the flow of the text, i.e.:

temperature **22.4° C**,

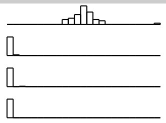
or be set inside it, , to visualise evidence on the spot. In a stacked, or a table form, such as shown in Table 2, they provide an easy way to compare several time series at a glance.

We propose to use similar sized graphics to represent the empirical distributions of the algorithm results. Our visualization consists of a table of normalized histograms (see Table 5). Each column of the table represents a given test case (e.g., a test function) and methods under comparison (e.g., algorithms) are stacked vertically. Histograms in one column all have the same range, which is the smallest interval containing the data from each of the methods presented in that column. This interval is then divided into a given number of subintervals usually called "bins"; the height of a column thus represents the number of data points falling into the corresponding bin. Reader can verify that we indeed use *less* print space than usual statistics table and thus meet the constraint 2 presented in the introduction.

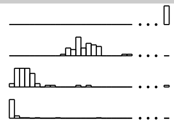
When the data produced by each method is tightly clustered, but the clusters are spread over a wide interval, the resulting histogram becomes quite uninformative, presenting only a few spikes in each cluster center. However, not all data needs to be shown: the reader, whose goal is to find the best algorithm for a given task, is likely to be more interested by the fact that one method is very much inferior to others rather than by how far behind it actually is. Thus we can greatly improve the value of the visualization by zooming in to the interesting region and by discarding the clearly inferior results. To do this, we augment the histogram by adding a "dump bin" at the far right of each histogram, separated from the other bins by an ellipsis ("..."). This "dump bin" contains all results crossing a given threshold and considered inferior and uninteresting. This addition has significant effect on the readability of the visualisation which can be seen in Table 3. The naive way of plotting the data, shown on the left, is dominated by the worst algorithm and effectively hides the variation between the other three. This is exacerbated by the single outlier run of

Table 3: Comparison between regular and focused histograms: the latter convey more information regarding the "better" algorithms.

Regular histograms

Function 3	
Method 1	
Method 2	
Method 3	
Method 4	
Min Max	1.05e-02 3.77e+01

Focused histograms, with a "dump bin"

Function 3	
Method 1	
Method 2	
Method 3	
Method 4	
Min Max	1.05e-02 1.68e+00

Method 1. The "dump bin" strategy effectively uncovers the important details between the three dominant algorithms.

In reading the graph the "dump bin" can then be read as the number of runs where the algorithm has failed to produce a meaningful result. The final and the most important component, the range, is depicted at the bottom row of each column. The range fixes the results in place and also documents the authors view on what is the interesting range of values. Table 5 presents a full-size example of stacked, focused sparklines and is compared against the traditional table of average values (in Table 4; see Section 5 for a detailed description of those tables). Reading a sparkline histogram is a three step procedure:

1. Verify the range, which gives a rough estimate of the scale of the values, serving the same function as the average value, or the y-axis of a convergence graph. If the range is outside of what the reader considers interesting, the rest of the graph can be discarded.
2. Focus on the dump bin. Those algorithms that are entirely, or almost entirely dumped can be discarded as uninteresting.
3. Draw conclusions from the remaining part, which is the one the author of the graphic has deemed interesting.

To create a stack of histograms as presented above, the following procedure has been applied. It must be noted that this procedure assumes that the optimization problem is a minimization, and that lower values are considered "better" than higher ones. Preliminary experiments have shown that histograms composed of $N = 25$ bins including the dump bin lead to dense, yet still legible graphics when set in a size equivalent to a 7pt font.

While binning the data is trivial, the choice of the threshold defining the "dump bin" is of first importance. However, additional preliminary experiments show that a good threshold can, in most cases, be derived automatically with a simple heuristic: ensure that 95% of the values of minimum two algorithms are displayed in the graphic and then find the threshold t so that total number of bins with data in the whole column of algorithms is maximised.

4 ON COMPARING OPTIMIZATION METHODS

The problem of comparing the performance of A and B on function f can be expressed as a comparison of two distributions based on an initial sampling. While conclusions are easy to draw when the two samples are not overlapping and are standing "far" form each other, those conditions are usually not met, making the comparison of A and B a difficult task. For samples which are normally distributed, one can apply Student's t-test to determine, with a given threshold probability, if the results of A and B are significantly different. This is however usually not the case, as explained in Section 2 as well as in a thorough study in (García et al., 2009). Even non-parametric test such as the *Mann-Whitney U test* (also known as the *Wilcoxon rank-sum test*) (Mann and Whitney, 1947) cannot be applied to the two samples since, although this test does not assume that the two samples are drawn from any particular distribution, it may be inappropriate if the distributions are too skewed or otherwise ill-behaved (Feltovich, 2003), which may confuses even this test. It is also worth noting that the use of this test is an implicit admission that the distribution of result samples is not necessarily normally distributed, which is in contradiction with the usage of average and standard deviation tables.

There is often a large difference between meaningful in statistical sense and meaningful in general: the difference between algorithms may in practice be insignificant (both perform in/adequately for the task) while statistically significant (samples have small difference in means and even smaller variance).

Moreover, the algorithms often have properties that are not evident in their mean, median, or similar statistic. Consider a simple case where two algorithms have the same mean statistic, but one has larger deviation. In the above perspective of comparing

Table 4: Average Fitness ± standard deviation at the end of the optimization.

	Method 1	Method 2	Method 3	Method 4
Function 1	$1.62e-01 \pm 1.67e-02$	$3.57e-02 \pm 3.47e-03$	$1.52e-02 \pm 7.50e-03$	$\mathbf{6.47e-03 \pm 4.88e-03}$
Function 2	$8.88e+01 \pm 1.26e+01$	$8.87e+01 \pm 2.39e+01$	$7.05e+00 \pm 3.95e+00$	$\mathbf{1.98e+00 \pm 2.36e+00}$
Function 3	$1.92e+01 \pm 3.57e+00$	$1.05e+00 \pm 1.77e-01$	$3.37e-01 \pm 5.80e-01$	$\mathbf{8.82e-02 \pm 1.95e-01}$
Function 4	$-3.06e+02 \pm 5.68e+00$	$-1.22e+02 \pm 2.59e+00$	$-3.11e+02 \pm 1.49e+01$	$\mathbf{-3.51e+02 \pm 3.58e+01}$
Function 5	$1.91e+03 \pm 9.94e+01$	$4.64e+03 \pm 1.18e+02$	$1.08e+03 \pm 1.42e+02$	$\mathbf{8.64e+02 \pm 1.38e+02}$
Function 6	$-1.30e+05 \pm 3.17e+03$	$-5.62e+04 \pm 1.47e+03$	$-1.33e+05 \pm 3.27e+03$	$\mathbf{-1.50e+05 \pm 1.14e+04}$
Function 7	$2.15e-01 \pm 2.50e-02$	$7.42e-02 \pm 6.98e-03$	$4.45e-02 \pm 9.44e-03$	$\mathbf{3.36e-02 \pm 1.03e-02}$
Function 8	$-1.76e+02 \pm 7.76e+00$	$-6.78e+01 \pm 2.35e+00$	$-1.56e+02 \pm 7.13e+00$	$\mathbf{-1.83e+02 \pm 2.58e+01}$
Function 9	$1.95e+03 \pm 1.51e+02$	$4.95e+03 \pm 1.24e+02$	$1.16e+03 \pm 1.55e+02$	$\mathbf{9.61e+02 \pm 1.65e+02}$
Function 10	$-1.65e+05 \pm 4.74e+03$	$-6.55e+04 \pm 1.99e+03$	$-1.54e+05 \pm 6.07e+03$	$\mathbf{-1.66e+05 \pm 8.11e+03}$

Table 5: Result Distributions.

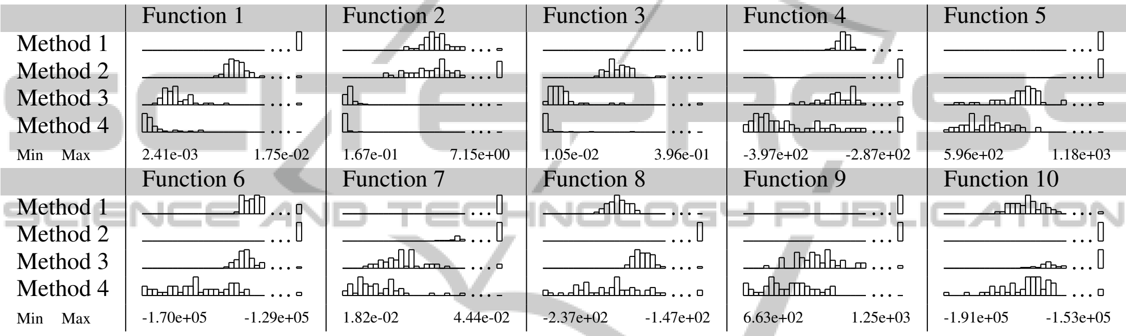


Table 6: Results of the Wilcoxon Rank-Sum test (Comparison with Method 4). A "+" symbol in the table means that Method 4 performs significantly better than the method it is compared to; an "=" symbol means that there is no statistically significant difference in performance between the two methods.

	Method 1	Method 2	Method 3
Function 1	+	+	+
Function 2	+	+	+
Function 3	+	+	+
Function 4	+	+	+
Function 5	+	+	+
Function 6	+	+	+
Function 7	+	+	+
Function 8	=	+	+
Function 9	+	+	+
Function 10	=	+	+

means, we would have to conclude that they are equal, in practise however, the difference can be crucial as we can run the highly varying algorithm several times to ensure significantly better results. Skewed distributions complicate the matter even further since we would need to observe more statistical parameters in addition to variance to decide which algorithm would be better at the case at hand.

The remarks above do not mean that statistical testing must be rejected: it is a powerful tool that does allow to draw conclusions from large and otherwise difficult to manage data sets. But their application requires planning and risk of misuse is high. In medical research, where statistics are of great importance, this situation has already been identified since the 1980s (Altman, 1991b; Strasak et al., 2007) and entire books on the field have been written (see e.g., (Altman, 1991a)) on how to do statistics correctly within this single field. With the increasing access to any type of information by the general public in the recent years, the awareness of the problems posed by improper use of statistical tools has moved out of the circles of scientific research and has reached the general public, as can be attested by an article on the subject in a popular science magazine (Siegfried, 2010), which is an enjoyable description of rampant misuse of statistics in fields of science.

When the hypotheses on which the statistical tests are based are not verified, no statistical test can be naively applied to the data in order to perform a quantitative analysis; one must then resort to a qualitative approach to the problem. The method for data presentation described in this paper is thus based on graphical representations of the data, especially in the forms of histograms. The readers are thus expected to exert their best judgement when comparing multiple, judiciously presented such figures in order to draw the appropriate conclusions regarding the performance of the algorithms. This method also aims at conveying as much information as both the usual average and standard deviation table and the statistical test table without making any assumption regarding the distribution of the data, while occupying about the same amount of space. Finally, it is believed to be readable at a glance.

5 COMPARISONS TO OTHER DATA PRESENTATIONS

To illustrate the effectiveness of our visualization method we present a comparison of four evolutionary algorithms that was computed for an earlier work (Weber et al., 2010). The data consists of four stochastic optimization methods, with two baseline algorithms (1 and 2) and two proposed improvements (3 and 4). The tests consists of a set of ten typical functions, commonly used in the field, in 500 dimensions. To evaluate our visualisation method, we present the same data in three formats: as the average and standard deviation in Table 4, as a statistical comparison in Table 6, and in our preferred format in Table 5. The first comparison is between the averages in Table 4 and the histograms in Table 5. A cursory comparison between Table 4 and Table 5 reveals that the required print areas needed for both tables more or less equal, leading to the conclusion that replacing the numerical table with a graphical one is feasible within the strict page limits imposed by many publishers. Moreover, the histograms table is composed of self-sufficient tiles and can, unlike the numerical table, be laid out more flexibly. The data can for example be presented as a square table, as a long column on the side of the page or even as separate blocks near the explanatory text of the article.

One claim could however be made in favor of average and standard deviation tables: they present the numerical data precisely and in an absolute way, which is not accomplished by the histogram representation. This is naturally true, but what is the importance of knowing the exact value of the average? Reasonably, this level of precision could be necessary only when making a comparative study but, as argued before, averages and standard deviations are not sufficient for this purpose. Since fitting all the numerical data in a printed article is infeasible and distracting, the only reasonable recourse to rely on the reproducibility of science and to re-compute the numbers for the tests. Alternatively, one can publish the gathered data in its entirety outside of the article.

To evaluate the work, the reader is instructed to first study the Table 4. Casual study reveals, mostly due the bold font, that Method 4 is likely to be the best candidate. At this point we make a claim: there are four functions for which this might not be the case. How long does it take to see which ones they are? This simple test clearly illustrates the fact that reading this table is difficult.

In contrast we observe Table 5. We instantly see that in many cases Method 4 has produced results closer to the optimum than other methods, with the closest competitor being Method 3. Method 2 seems to be in general not competitive compared to the other methods and Method 1 is in the competition but losing. In four cases (functions 4, 6, 8 and 10), we see significant overlap, which confirms the result of the Mann-Whitney U test in Table 6, indicating that for Functions 8 and 10, Method 4 is not performing significantly better than Method 1. The same test indicates however that on Function 4, Method 4 is outperforming Method 1 whereas the distributions are mostly overlapping. This might be caused be a long tailed and skewed histogram for Method 4 which causes Mann-Whitney-U test to give an counterintuitive result. These examples therefore illustrate the fact that our visualisation effectively conveys at least the same information as the Mann-Whitney U test, as well as information complementary to the test and its limits.

The visualisation shows several other points of interest, that are not evident in either standard deviation table or statistical test. Method 1 seems to have a rather robust behaviour. Although it rarely competes in the best solution quality, it seems to reliably achieve a certain level of fitness, which is most evident in Functions 2, 4, and 8. Method 4 works the opposite way, having a wide distribution and sometimes finding excellent results and yet at times failing badly. When considering repeated experiments, there is little use of running Method 1 again to improve the result, but running number 4 several times could be very beneficial. In some cases, some algorithms have their data entirely in the “dump bin”. This is the author’s way of visually claiming that those algorithms did not manage to produce any meaningful results.

6 CONCLUSIONS

In this text we have presented a novel visualization for comparing evolutionary optimization methods. We claim that this visualisation can convey more information than average/standard deviation tables and statistical test tables while retaining nearly the same usage of space and still improve on the readability of the paper. We also offer our opinion that reporting averages, standard deviations or any single statistical number in context of stochastic algorithms is not a useful practice and can be misleading. In our view, sparkline histograms completely supersede the use of average and standard deviation table.

We also present that histograms are a easier approach than statistical testing, which requires great care to do properly. We do not claim that statistical test are not a valid tool, but instead fear that, based on experience in other fields, they that can be easily misused. Sparkline histograms carry the same information in a form that is easily understood by a layman and offers far fewer places for mistakes and misinterpretations.

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