

# GENETIC ALGORITHM BASED ON DIFFERENTIAL EVOLUTION WITH VARIABLE LENGTH

## *Runoff Prediction on an Artificial Basin*

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**Keywords:** Differential evolution, Hydrology, Evolutionary computation.

**Abstract:** Differential evolution is a successful approach to solve optimization problems. The way it performs the creation of the individual allows a spontaneous self-adaptability to the function. In this paper, a new method based on the differential evolution paradigm has been developed. An innovative feature has been added: the variable length of the genotype, so this approach can be applied to predict special time series. This approach has been tested over rainfall data for real-time prediction of changing water levels on an artificial basin. This way, a flood prediction system can be obtained.

## 1 INTRODUCTION

Differential evolution was firstly named in 1998 by Kenneth Price and Rainer Storn (Storn and Price, 1997). This is a powerful evolutionary algorithm for solving optimization problems over continuous spaces. DE has been applied to several fields and its good behaviour has become clear. The secret lies in the way DE generates the population after each generation (Storn and Price, 1997), (Feoktistov, 2006). This process uses three operators: mutation, crossover and selection. The first two operators generate the candidate vectors and the last one decides which one enters the next generation.

### 1.1 Mutation

The population is initialized by a set of NP randomly generated individuals ( $x_{i,G}$ ,  $i=1..NP$ ). At each generation, three mutually different individuals

(random vectors) are randomly chosen from the population below (general mutation strategy). Then, a mutant vector ( $v$ ) is generated in the way written:

$$v_{i,G+1} = x_{r1,G} + F \cdot (x_{r2,G} - x_{r3,G}) \quad (1)$$

$i=1..NP$

The parameter  $F$  (scaling factor) tries to manage the trade-off between exploitation and exploration of the search space.

Although only one mutation scheme was presented here, there are several ways of implementing this operator. Depending on the problem, one strategy will be more suitable than others. Some research works (Qin et al., 2009) alternate different strategies and parameter values on different generations depending on the cost function value.

In (Feoktistov, 2006) several strategies are divided in four groups:

1. RAND group: the trial individual is generated

without any information about the values of the objective function.

2. RAND/DIR group: strategies that use values of the objective function to determine the good direction.
3. RAND/BEST group: the best individual is used to form the trial one.
4. RAND/BEST/DIR group: combines the advantages of the last two groups.

The strategy to choose is always defined by the problem and the concrete objective function.

## 1.2 Crossover

Once the mutant vectors have been generated, the crossover is performed in order to intensify the search inside a region. This step is performed based on the crossover constant ( $CR$ ), which is given a random value between 0 and 1. Then, the trial vectors are generated as follows:

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1}, & randb(j) \leq CR \text{ or } j = rnbr(i) \\ x_{ji,G}, & randb(j) > CR \text{ or } j \neq rnbr(i) \end{cases} \quad j=1...D \quad (2)$$

1.  $D$  represents the dimensionality of the problem.
2.  $randb(j)$  generates a random number between 0 and 1 for the  $j$  component.
3.  $rnbr(i)$  generates a number between 1 and  $D$  which ensures that  $u_{i,G+1}$  gets at least one parameter from  $v_{i,G+1}$ .

## 1.3 Selection

Once these two operations have been performed, the trial vector is compared to the target vector; if the trial vector achieves a smaller cost function value, it will be included in the next generation; otherwise, the target vector will be kept.

The main idea of this approach is to adapt the step length intrinsically along the evolutionary process. As the evolution goes on, the population converges and the step length becomes smaller.

## 2 MOTIVATION

Many problems need to calculate the parameters that fit a function in order to establish a prediction based on correlations. Therefore, a useful technique is to consider a sliding window which includes some prior values to the current one ( $t$ ) in order to

calculate the next one ( $t+1$ ). Time series prediction is an example of this kind of problems.

If we want to apply the differential evolution approach to this kind of problem, we must perform an adaptation of the basic algorithm.

Therefore, this paper proposes a differential evolution approach where the individuals can have different lengths. On one hand, the length of the best individual offers the size of the time window; on the other hand, each gene corresponds to the coefficients which weight each time  $t$ .

## 3 METHOD

Due to the variable genotype length approach, several features were introduced apart from the general search strategies described before.

Firstly, each individual can have different lengths. This feature determines the mutation and crossover operators, so the generic DE operators must be changed as it is explained below.

With respect to the mutation operator, it includes arithmetic operations to be executed over individuals with different lengths. So, the following considerations must be taken into account:

- The smallest individuals will be completed with zeros until they reach the length of the corresponding individual which will be added, subtracted or multiplied to. By completing with zeros, the resultant vector does not differ so much with respect to its predecessors.
- Once the mutant vector is obtained, a mutation length operator is applied with the aim of avoiding the length increase resulting from the previous step. This way, the mutant vector can keep or increase or decrease its length in one unit. This new cell will contain a random value. In future experiments, the number of cells increased/decreased will be introduced as a parameter in order to find the best value.

A general scheme of the proposed mutation scheme can be seen in Figure 1.

The crossover operator completes the length of the target vector until the length of the target vector is reached.

The selection operator is performed as it has been explained before. The fitness function penalizes the longest individuals adding the length of the individual (weighted by an experimental value) to the MSE. Longer individuals make the equation more complex. Thus, small individuals are more.

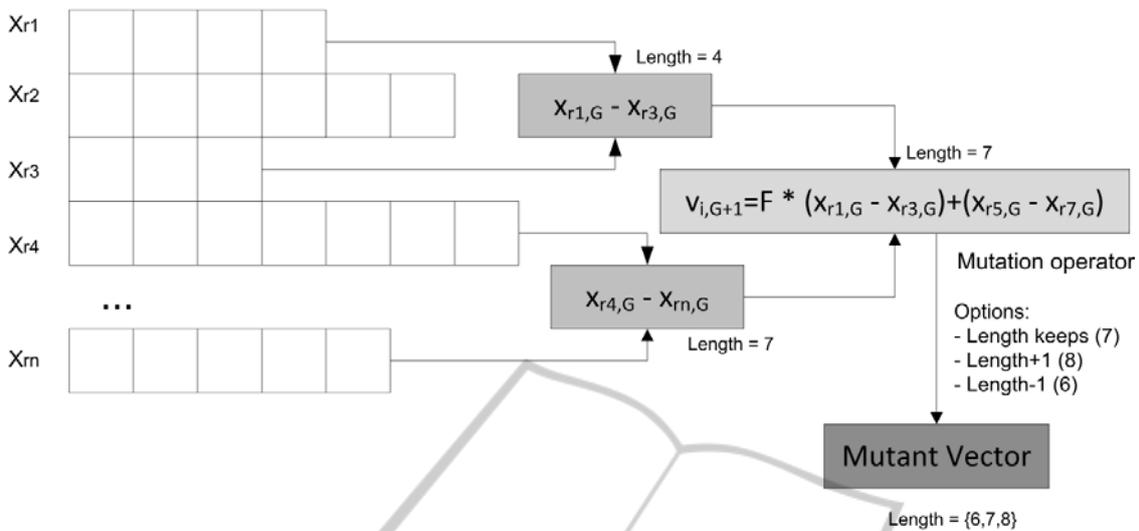


Figure 1: General scheme of the mutation process.

## 4 RESULTS

### 4.1 Problem Description

The method developed in this work was applied to the field of Hydrology, more specifically, to the prediction of the flow rate resulting from the rain. Before explaining the results, several concepts must be introduced (Dorado et al., 2003):

- A river basin is an area drained by rivers and tributaries. In the case of an urban basin, the streams and rivers are replaced by a sewage system.
- Run-off is the amount of rainfall that is carried away from the river basin by streams and rivers.
- The modelling of the run-off flow in a typical urban basin is that part of hydrology which aims to model sewage networks. Its objective is to predict the risk of rain conditions for the basin and to sound an alarm to protect from flooding or from subsidence.

In general, the goal of this type of problem is to predict and model the flow of a typical urban basin from the rain collected by a sensor. The transference function between the rainfall and the runoff has many different conditions, as street slopes or roof types. The system is of such variability that it becomes impossible to define an equation capable of modelling the course of a drop of water from the moment it falls to the instant in which it enters the drain network. The course of water through the network is quite complex and, although there are equations for modelling it, they are subject to the use

of adjustment parameters which make them depend on a calibration process.

There are several methods for calculating the rainfall-runoff process (Viessmann et al., 1989).

One family is based on the use of transfer functions, usually called “unit hydrographs” (Hydroworks, 1995).

Another approach is based on hydraulic equations, whose parameters are fixed by the morphologic characteristics of the study area (kinematic wave). Commercial packs for calculating sewage networks usually provide both “unit hydrographs” and “kinematic wave” modules.

The use of calculation methods not based on physics equations, such as Artificial Neural Networks and Genetic Programming (Drecourt, 1999), is becoming widespread in various civil and hydraulic engineering fields.

In order to make a solution for this problem, we use a variant of an Autoregressive Moving Average Model.

$$y_t = \sum_{i=0}^p \theta y_{t-i} + \sum_{i=1}^q \varphi \varepsilon_{t-i} \quad (3)$$

As we can see in (3) we need to search for the optimal values of  $p$  (size of the input time window) and  $q$  (size of the output time window) and for the values of  $\varphi$  and  $\theta$  coefficients. For this purpose, we need a variable length codification of this problem.

### 4.2 Physical Model and Parameter Configuration

In this work, synthetic data has been generated, by simulating a rainfall scenario in a physic model done

to scale. It is placed in the Centre of Technological Innovation in Civil Engineering (Cea et al., 2009). This is an experimental model which simulates the rainfall effect over a metallic structure like a basin. This way, a superficial runoff is generated. This experiment has been constructed in order to get an equation for modelling the rainfall-runoff transformation and then construct a virtual lab for runoff predictions. Thereby, real experiments can be avoided.

The constructed system is composed of these two parts (Figure 2):

- Metallic structure: with a 2.0x2.5m rectangular plant. It is composed of three plans with a 5% slope each (Figure 2).
- Hydraulic system: metallic grid where the system of rainfall simulation is fixed. It is composed of polyethylene tubes connected to another one which supplies the water.
- Equipment for runoff register: test tube for more than 30 liters which collects the outgoing water.



Figure 2: The experimental design in the laboratory.

The process consists in opening the dissemination system for a specified period and then measuring the runoff.

As a result, the two graphics in Figure 3 have been obtained. The first one represents the rainfall over the basin and the second one represents the

runoff.

The proposed algorithm has been adapted according to the problem described before.

Firstly, the individuals were divided in two parts: one representing the coefficients  $\theta_i$  ( $i=0..p$ ) and another one representing the coefficients  $\phi_j$  ( $j=1..q$ ). Due to the nature of the problem, it would be meaningless to combine both parts in the same evolution process. This way, each individual is composed of two parts which evolve independently, but the fitness takes into account both parts together.

Secondly, following the conclusions achieved in (Qin et al., 2009) and (Mayer et al., 2004) and our own experiments, the configuration parameters were set as follows:

- Population size ( $NP$ ) = 500
- Crossover rate ( $CR$ ) = 0.3
- Scaling Factor ( $F$ ) = 0.5

The following strategy belongs to the RAND/DIR group. Following the nomenclature used in (Storn and Price, 1997), the chosen strategy was DE/rand/2/bin. It consists in choosing five random individuals and operating as follows (4):

$$v_{i,G+1} = x_{r1,G} + F \cdot \begin{bmatrix} (x_{r2,G} - x_{r3,G}) \\ +(x_{r4,G} - x_{r5,G}) \end{bmatrix} \quad (4)$$

### 4.3 Discussion

Three different approaches were used in order to establish a comparison with the method presented in this paper:

Classical genetic programming (GP): a search technique proposed by (Koza, 1992). This technique generates algorithms and expressions represented as a tree structure. GP has been applied to problems of rainfall-runoff transformation (Drecourt, 1999), and it generated complex expressions difficult to understand.

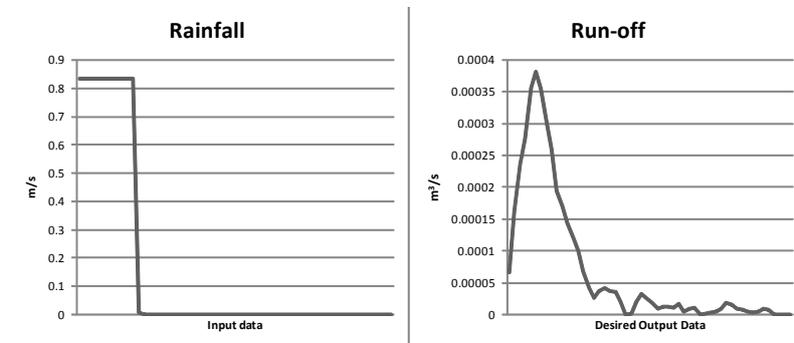


Figure 3: Input and output of the simulation.

- **Clonal Selection Algorithms (CSAs):** these algorithms are a type of Artificial Immune Systems (AIS) (Bownlee, 2007). They are based on Burnet's clonal selection theory (Burnet, 1959; Burnet, 1976; Burnet, 1978), which is inspired by Darwin's theory of natural selection to explain the diversity and adaptability of life. This type of algorithm is primarily focused on mimicking the clonal selection principle, which is composed of three mechanisms: clonal selection, clonal expansion and affinity maturation via somatic hypermutation. The implementation used was the Optimization Immunological Algorithm (opt-IA) proposed in (Cutello et al., 2004), as it has proved to have a good performance in optimization problems (Cutello et al., 2005). To the authors' knowledge, this technique has not been applied to rainfall-runoff transformation problems before. This technique returns a list of coefficients part of a formula, which can be easily used to predict outputs.
- **Hydrographs:** this technique was explained before, as a typical approach used for calculating rainfall prediction.

As it can be seen in Figure 4, the proposed method fits the desired signal better than the other approaches. In fact, if the Mean Square Error obtained by these techniques is compared, the lowest value is reached with the DE approach proposed in this work.

The HEC-HMS software (HEC-HMS, 2010) has been used to calculate the hydrographs' MSE. This

software calculates the hydrograph produced by a basin. With the rainfall and basin data as input, it generates the output hydrograph in a graph or table.

Table 1: MSE obtained.

Method	MSE
Genetic Programming	2.50E-05
Clonal Selection Algorithm	1.89E-05
Hydrographs	1.79E-05
Differential Evolution	1.58E-05

The results corresponding to genetic programming have been calculated with the algorithm proposed in (Rabuñal et al., 2007).

The following equation (5) has been obtained by applying the proposed algorithm to the described data. Table 1 and Figure 4 show the results.

$$y_t = 1.025 \cdot \varepsilon_t + 0.463 \cdot \varepsilon_{t-1} + 0.579 \cdot y_{t-1} + 0.119 \cdot y_{t-2} \quad (5)$$

## 5 CONCLUSIONS

The main objective of this work has been the construction of a virtual laboratory for calculating the rainfall-runoff transformation without building a physical model.

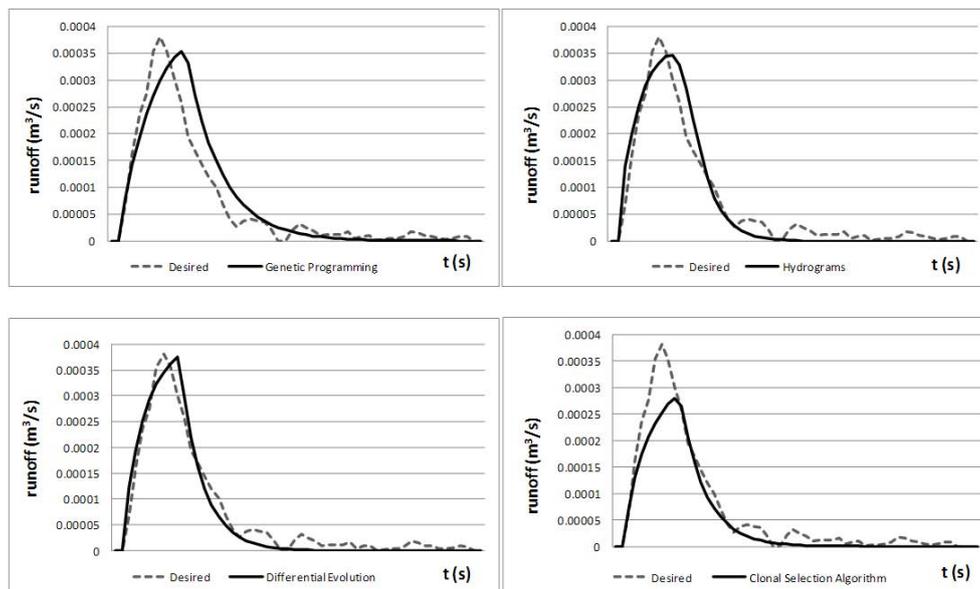


Figure 4: Rainfall-Runoff transformation predicted with the different techniques.

This way, an equation will be calculated in order to predict a run-off value using previous rainfall values.

Several approaches try to solve this problem in different ways. In this article, a Differential Evolution technique is proposed. The main included feature is the variable length of the individuals in the genetic population.

The results obtained have been compared with three different techniques used for predicting the rainfall-runoff transformation. The presented approach gets good results.

## ACKNOWLEDGEMENTS

This work was supported by the General Directorate of Research, Development and Innovation (Dirección Xeral de Investigación, Desenvolvemento e Innovación) of the Xunta de Galicia (Ref. 08MDS003CT). The work of Vanessa Aguiar is supported by a grant from the General Directorate of Research, Development and Innovation of the Xunta de Galicia.

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