IMPLICIT SEQUENCE LEARNING
A Case Study with a 4–2–4 Encoder Simple Recurrent Network

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Abstract: Without any doubt the temporal order inherent in a task is an important issue during human learning. Recurrent neural networks are known to be a useful tool to model implicit sequence learning. In terms of the psychology of learning, recurrent networks might be suitable to build a model to reproduce the data obtained from experiments with human subjects. Such model should not just reproduce the data but also explain it and further make verifiable predictions. Therefore, one basic requirement is an understanding of the processes in the network during learning. In this paper, we investigate how (implicitly learned) temporal information is stored/represented in a simple recurrent network. To be able to study detailed effects we use a small network and a standard encoding task for this study.

1 INTRODUCTION

Temporal order plays an important role in human learning, e.g. humans tend to use the serial order in free recall (Mandler and Dean, 1969). Even if the temporal component is learned implicitly it helps us to solve the task. Arthur Reber investigated the phenomenon of implicit learning using finite state grammars. In sequential reaction time tasks he could show that test persons learn the rules of an underlying grammar implicitly (Reber, 1989).

Axel Cleeremans provides a formal model for implicit sequence learning based on connectionist theory. The model applies a simple recurrent network (SRN), also known as Elman network, and fits the network's behaviour to the experimental data of Reber (Cleeremans, 1993).

A SRN is very similar to a common multilayer feedforward network. It involves additional recurrent links to provide the network with a dynamic memory. Therefore it is able to recognise the temporal properties of a sequential input (Elman, 1990).

The prediction capabilities of the SRN were evaluated by Lalit Gupta and Mark McAvoy (Gupta and McAvoy, 2000). They focus on the prediction of non–orthogonal vector components of real temporal sequences. This leads to a number of applications in the field of signal classification (Übeyli and Übeyli, 2008). Another area for the application of SRNs is autonomous robot control (Slušný et al., 2007).

Up to now, it is little known about the mechanisms of implicit sequence learning in SRNs. Using the example of a biologically inspired classification task we showed that the network's learning performance depends on the presence of a temporal order in the input sequence (Glüge et al., 2010).

In terms of the psychology of learning and biological plausibility there is no doubt about the need for models based on recurrent networks since biological neural networks are recurrent. SRNs might be suitable to reproduce the data obtained from experiments to study the effect of temporal order on associative learning by humans (Hamid et al., 2010). Such model should not just reproduce the data but also explain it and further make verifiable predictions. Therefore, one basic requirement is an understanding of the processes in the network during learning/training.

In (Heskes and Kappen, 1991; Heskes and Kappen, 1993) the learning process and the learning dynamics of neural networks were investigated from a general point of view.

In this paper, the focus is not on the learning process in the SRN. But rather we investigate how (implicitly learned) temporal information is stored/represented in a SRN. Further, the influence of the sequential input during training and testing is examined.

After training the network is tested on different

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input sequences under different conditions, e.g. with and without working memory. To be able to study detailed effects we use a small network and a standard encoding task for this study.

2 THE 4–2–4 ENCODER SRN

Figure 1 shows a conceptual diagram of a SRN. Compared to a feedforward network the SRN has an additional context layer. Each hidden unit is connected to *one* corresponding context unit. The weights of these recurrent connections are fixed to 1. Thereby, each context unit stores a copy of the output of the corresponding hidden unit. At the next time step each context unit feeds its copy back to all hidden layer units. Due to the time delay and the feedback loop in the information processing flow the network is able to memorise earlier internal states. Therefore, the output of the hidden layer depends on the actual input *and* implicitly on input from all previous time steps.

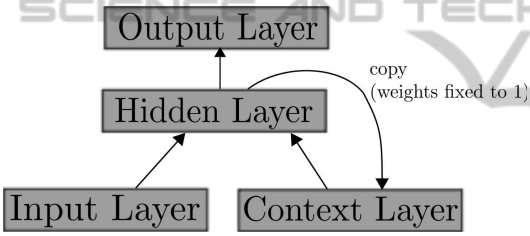


Figure 1: Conceptual Diagram of a SRN.

2.1 Learning Task

The encoding task is the conversion of four $1-$of-4 coded input vectors into a binary representation and vice versa. For the network, the task is first to find a mapping for the $1-$of-4 coded input into a binary code. The second part is the retrieval of the binary coded input into a $1-$of-4 coded output. Table 1 shows one possible solution for the problem.

Table 1: 4–2–4 encoding.

Input	Binary Code	Output
0001	00	0001
0010	01	0010
0100	10	0100
1000	11	1000

Attention should be paid to the fact that the encoding is independent of the sequence of input vectors. The network learns a direct mapping between input and output. Nevertheless, we will see that the SRN implicitly learns a temporal relation between inputs and uses this as an advantage for memorising.

2.2 Network Configuration

If we denote $y^{(0)}(t)$, $y^{(1)}(t)$ and $y^{(2)}(t)$ as output vectors of the input, hidden, and output layer at time t, $a^{(l)}(t)$ with $l = 1,2$ as network activation vectors of the hidden and output layer, and $w^{(l,l')}$ as weight matrices between layer l and l', the forward pass of the 4–2–4 encoder SRN (Fig. 2) with activation function f can be written as:

$$a_i^{(1)}(t) = \sum_j w_{ij}^{(1,0)} y_j^{(0)}(t) + \sum_j w_{ij}^{(1,1)} y_j^{(1)}(t-1), \quad (1)$$

$$y_i^{(1)}(t) = f(a_i^{(1)}(t)), \quad (2)$$

$$a_i^{(2)}(t) = \sum_j w_{ij}^{(2,1)} y_j^{(1)}(t), \quad (3)$$

$$y_i^{(2)}(t) = f(a_i^{(2)}(t)) \; . \quad (4)$$

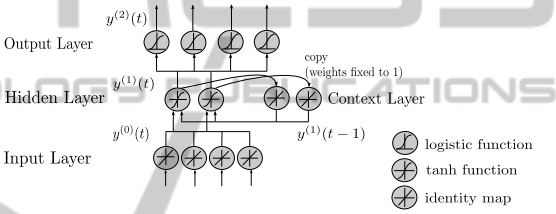


Figure 2: 4–2–4 SRN.

The input layer consists of 4 input units which simply excite or not, given the coding of the input vector. The hidden layer has the *hyperbolic tangent* as activation function. Hence, the output of the hidden layer is

$$y_i{}^{(1)}(a_i{}^{(1)}) = \tanh(a_i{}^{(1)}) \; . \quad (5)$$

Each hidden layer unit is connected to one corresponding unit in the context layer. The connection weights are fixed to 1. The context units are fully connected to the hidden layer providing it with the hidden layer output from the previous time step. Four different input vectors can be represented by two bit. Since we want the network to code the input vectors into a binary representation the number of units in the hidden layer is set to two. Furthermore, the two hidden units are fully connected to four output units. This corresponds to the retrieval of the binary coded input into a $1-$of-4 coded output. As the output units shall generate values between 0 and 1 their activation function is the *logistic* function. The network output is

$$y_i{}^{(2)}(a_i{}^{(2)}) = \frac{1}{1+e^{-a_i{}^{(2)}}} \; . \quad (6)$$

Note that the hyperbolic tangent at the hidden layer produces values between -1 and 1. The internal representation for the input will consist of values in the

interval $(-1,1)$ and not of 0 and 1 as in Table 1. It is still required that the internal representation be 'binary' in the sense that the hidden layer has to produce coding values for a uniquely distinguishable mapping at the output layer. The input and output is presented to the network as shown in Table 1.

The SRN can be seen as a feedforward network with additional inputs from the context layer and any algorithm for feedforward networks can be used to train it (Elman, 1990).

2.3 Network Training

We use the backpropagation algorithm to train the SRN. To do so, two constraints have to be fulfilled.

1. The context units must be initialised with some activation for the forward propagation of the first training vector. Commonly, these initial values are zero.
2. The activation levels of the hidden layer must be stored in the context layer after each back propagation phase. Hence, the context layer shows the state of the hidden layer delayed by one time step.

After each input the network output is compared to the desired output and the mean square error is propagated back through the network. The weights are updated with the constant learning rate $\varepsilon = 0.1$. Since each output is evaluated right away, the process corresponds to *online* learning.

The weights are initialised with uniformly distributed random values in the interval $[-0.3, 0.3]$ (apart from the fixed hidden–to–context layer weights). The learning rate and weight initialisation interval are chosen according to preliminary tests. We use the combination that yielded best training results after 1000 training cycles.

3 SIMULATIONS

The above described network was implemented in Matlab. Since we are interested in the network's ability in terms of implicit sequence learning we present the training input in two different ways, a sequential and a random one.

One training cycle consists of a presentation of all four input vectors that are shown to the network one after another. For the case of a deterministic order the first cycle is repeated for the whole training. This implies a strong temporal relationship between the input vectors since each one has a fixed successor. For the case of a random order in each cycle the temporal correlation between the input vectors is very weak.

If we denote the input vectors with the numbers from one to four we can describe the two types of sequences as follows:

det. sequence $\ldots | \overbrace{1\ 2\ 3\ 4}^{\text{cycle n}} | \overbrace{1\ 2\ 3\ 4}^{\text{cycle n+1}} | \ldots$

random sequence $\ldots | \overbrace{4\ 3\ 1\ 2}^{\text{cycle n}} | \overbrace{2\ 1\ 3\ 4}^{\text{cycle n+1}} | \ldots$

The network is trained for 1000 cycles. Hence, every input vector is shown 1000 times to the network. This results in 4000 training steps or rather 4000 weight updates.

3.1 Training

3.1.1 Success of the Training

To measure the success of the network we evaluate the output according to the winner–take–all principle. The unit with the highest activation is counted as 1 the remaining as 0. Thus, the network's output is always mapped onto a corresponding target vector.

We evaluate the network output in terms of the probability of success (P_S) for each training cycle. When training starts the probability to excite the correct output is one out of four ($P_S = 0.25$). At the end of training the network should have learned the coding and always deliver the target vector, therefore we expect $P_S = 1$.

Since the weights are initialised randomly the learning curves for single networks may differ considerably. As we want to compare the general behaviour of the network we train 100 networks on each type of input sequence. Afterwards we calculate the mean probability of success over the 100 networks ($\overline{P_S}$) for the two test cases. In Figure 3 $\overline{P_S}$ is plotted against the number of training cycles. In general, the networks perform better on a deterministic input sequence than on a random one. In both cases, however, the expected $\overline{P_S} = 1$ is not reached, which we will explain in the following.

Figure 4 shows the distribution of P_S for the 100 networks after training. We plot the number of networks n against the final probability of success P_S. Trained with a deterministic sequence one half of the networks ($n = 51$) learned the encoding completely ($P_S = 1$ by the end of the training). On the other hand, only 23 networks could succeed if trained with a random sequence.

In summary it is more likely that a network trained with a deterministic sequence is able to learn the task

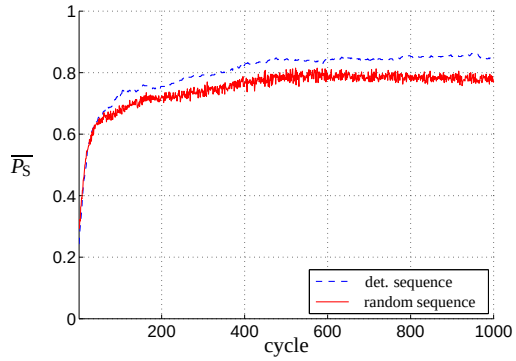


Figure 3: Success of training for the deterministic and random sequence.

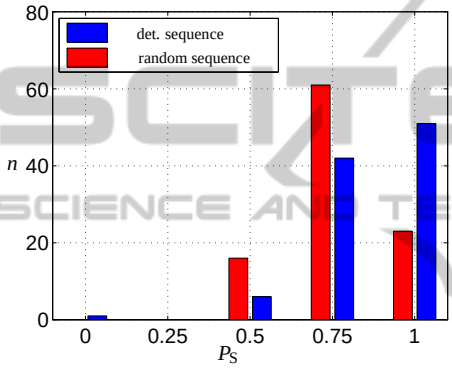


Figure 4: Distribution of final probability of success after training of 100 nets.

in the training. The reason is the temporal correlation between the input vectors. This extra information, which is only provided in the deterministic sequence, raises the probability of the SRN to learn the task.

3.1.2 Unstable Solutions and Training Algorithm

Apparently, the learning task is quite demanding, since a high percentage of the networks could not find an optimal solution. Those networks that did not succeed in training learned an unstable solution.

Figure 5 exemplarily shows the activation of the two hidden units during training for a network that did not find a distinct encoding. Each input vector is presented once per training cycle. We plotted the activation of each hidden unit for one specific input. The combination for the units' activation represents the network state and therefore, the coding of the input vector. At the beginning of the training all inputs are represented by activations around zero. At the end of the training one can see that input vector 2 and 4 are well distinguishable while the inputs 1 and 3 re-

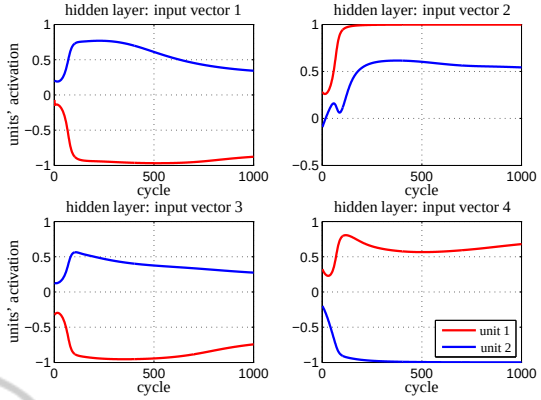


Figure 5: Hidden layer output during training for the four inputs of a network that did not succeed in training.

sult in a nearly identical activation. The network did not learn to distinguish these input vectors.
Table 2 shows the exact numerical values for the activations caused by each input vector at the end of the training.

Table 2: Final hidden layer activations of a network that did not succeed in training (input 1 and 3 hardly distinguishable).

Input Vector	Unit 1	Unit 2
1	−0.8770	0.3435
2	0.9999	0.5431
3	−0.7429	0.2746
4	0.6793	−0.9983

It might be possible to avoid such unsuccessful training results by a more sophisticated training algorithm (e.g. a variable learning rate, a better measure for the network error etc.). Nevertheless, some networks learned to solve the task with the simple learning algorithm we used. Figure 6 shows the activation of the two hidden units during training for a network that found a distinct encoding. Again, we plotted the activation of each hidden unit for one specific input vector.
Table 3 shows the exact numerical values for the activations caused by each input at the end of the training.

Table 3: Final hidden layer activations of a network that succeeded in training (all inputs distinguishable).

Input Vector	Unit 1	Unit 2
1	0.2384	0.9794
2	−0.9999	0.9876
3	−0.9988	−0.9872
4	0.9993	−0.9068

Since we are only interested in the influence of the temporal context in the input sequence and the role of

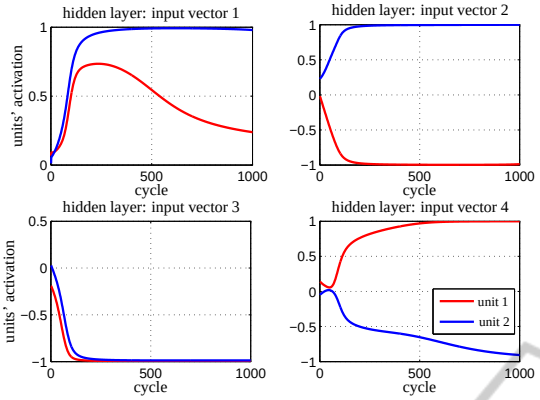


Figure 6: Hidden layer output during training for the four inputs of a network that succeeded in training.

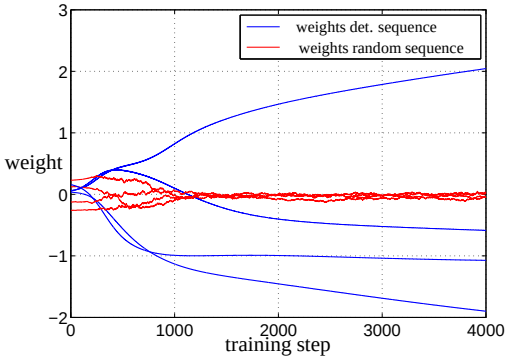


Figure 7: Development of weights in context layer.

the context layer we did not optimise the learning algorithm. By that, we have the opportunity to compare the results of the training while the *only* difference is the sequence of training inputs. An optimal learning algorithm for the given task is not in the focus of our study.

3.1.3 Weights of the Context Layer

The success of the training shows that the networks use the temporal relations of the input vectors. The question is, how is this represented in the network? The answer lies in the weights of the network since this is the only parameter that is changed during training. Further, the key component to sequence learning is the context layer, thus we investigate the development of the weights of this layer.

Two context units are fully connected to two hidden units. This results in four weights in the context weight matrix, $w_{ij}^{(1,1)}$ with $i,j = \{1,2\}$ (cf. Eq. 1). Figure 7 exemplarily shows the weights of the context layer during training. We plotted the four context weights of two networks that solved the task ($P_S = 1$ at the end of the training). Each training cycle consists of four input vectors. A training of 1000 cycles results in 4000 weight updates. The red lines show the weights of a network trained with a random sequence. The blue lines those of a network trained with a deterministic sequence. Each line shows one weight of the weight matrix.

For a deterministic sequence the weights take a value between -2.5 and 2.5. Longer training would result in higher weights, since the network tries to generate exactly 1 at the output. This value is never reached by the activation functions of the hidden and output units.

The random sequence leads to vanishing context weights. The network learns that there is no temporal dependency in the input. This leads to the remarkable result that the SRN turns into a standard feedforward network and omits all context information.

3.2 Testing

For testing, we chose the networks that solved the task at least in the last training cycle ($P_S = 1$). As we will see this criterion does not ensure that the solution the network found is stable. We got 51 networks trained with the deterministic sequence and 23 trained with the random sequence for the test. The two sequence types were presented to this networks again for 1000 cycles.

3.2.1 Test with the Deterministic Sequence

Networks Trained with the Deterministic Sequence should also produce comparable results during testing. In our case 3 of the 51 networks performed worse in the test run ($P_S = 0.25$ or 0.5). An analysis of these networks showed that two of them did not learn a distinct encoding in the hidden layer. In fact, the two networks did not provide a binary coding but a mixed coding or fuzzy coding which was unstable.

The third had learned a clear encoding in the hidden layer but produced poor results in the test run. Further investigation of the network showed that the strange behaviour in the test run results from the missing context information at the initial test input. When testing starts, the output of the context layer is set to zero. Therefore, the first test input is processed at the hidden layer without contextual information. This gap in the flow of information from the context to the hidden layer was sufficient to cause "confusion" in the network which lead to the poor results. In other words, the network based the decision, which input vector to code, primarily on the context of an input then on the input itself.

The mean probability of success of the 51 networks during testing is constant $\overline{P_S} = 0.9657$.

Networks Trained with the Random Sequence turned into feedforward networks by setting the context weights to zero. The encoding is based solely on the current input. Therefore, we expect that it does not matter whether the networks are tested with a random or deterministic sequence. In case of the random sequence 5 of the 23 networks showed an oscillating behaviour with a success rate between 75% and 100%. Apparently the 5 outliers learned an unstable solution during training. The remaining networks solved the task in testing as in training to 100%.

The mean probability of success of the 23 networks during testing is constant $\overline{P_S} = 0.9457$.

3.2.2 Test with the Random Sequence

Networks Trained with the Deterministic Sequence did not learn just the input itself but also the temporal correlation between inputs. Tested on a random sequence this temporal information is missing. Further the temporal correlation that was learned during training can be misleading. For instance if the network learned that input vector 1 is followed by 2 but in the next step vector 3 is presented. Then the network has to process two conflicting informations. The input layer indicates vector 3 to the hidden layer but the *context* at the present time–step indicats vector 2 to the hidden layer.

This results in a poor overall performance in the test run. Figure 8 shows the mean probability of success of the 51 networks during testing. The spiky character of the curve can be explained with the fact that parts of the random sequence may be equal to the deterministic sequence.

Networks Trained with the Random Sequence keep the high performance during testing. The mean probability of success of the 23 networks during testing is about $\overline{P_S} \approx 0.95$ (Fig. 8). This can be explained by the pure feedforward processing of these networks. The training with the random sequence turned them into feedforward networks, thus the learned mapping between input and output is independent of the temporal context of the input.

3.2.3 Summary of the Test Results

Table 4 shows the mean probability of success of the tested networks for the four combinations of training and testing.

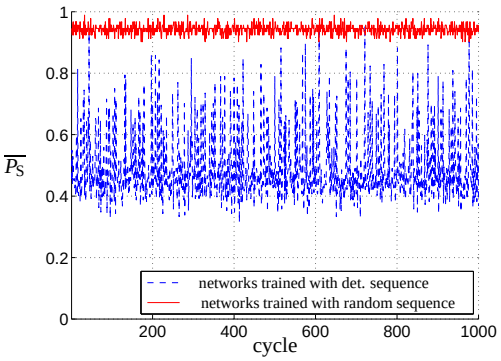


Figure 8: Probability of success during testing with the random sequence.

Table 4: Results of Testing.

	$\overline{P_S}$	training	
		det. seq.	random seq.
testing	det. seq.	0.9657	0.9457
	random seq.	≈ 0.5	≈ 0.95

Networks tested with their training sequence unsurprisingly perform very well in the test run. Those who did not succeed in the test learned an unstable solution during training.

We saw that networks trained with the deterministic input sequence performed very poor in the test with a random sequence. The networks learned the coding based on the actual input plus contextual information. The test result shows that this contextual information is not just some add on but absolutely necessary for the networks to solve the new task. Since the networks saw just one type of input sequence they do not generalise to other types of sequences.

The networks that were trained with a random sequence turned into feedforward networks during training (cf. Sec. 3.1.3). Therefore, they could deal with any kind of input sequence since the encoding of an input is independent of the input sequence.

3.3 Test Without Context Layer

The result of the test run with the random sequence shows that networks trained with the deterministic sequence heavily rely on the temporal structure of the input. To investigate the influence of the context layer we tested the networks again. This time we take the trained networks and set their context weights to zero. Thereby networks trained with a deterministic sequence lose the previously learned temporal correlation between input vectors.

Those networks trained with a random sequence have zero context weights already thus, the replacement by zero should have no effect to them.

We test the modified networks with the deterministic sequence for 1000 cycles. Table 5 shows the mean probability of success in this test. The performance of the networks trained with a deterministic sequence drops dramatically while those trained with a random sequence still perform very well (cf. Tab. 4).

Table 5: Results of Testing without context layer.

		training	
	$\overline{P_S}$	det. seq.	random seq.
testing	det. seq.	0.5196	0.9257

Figure 9 shows the distribution of P_S for the 51 networks trained with the deterministic sequence. Not all networks rely on the context layer to the same extend. The majority of the networks ($n = 29$) achieve $P_S = 0.5$ in the test run. For 13 networks the context layer was less significant. They obtain a probability of success above average, $P_S = 0.75$. Further, 9 networks rely on the context layer to a greater extent and therefore perform below average, $P_S = 0.25$.

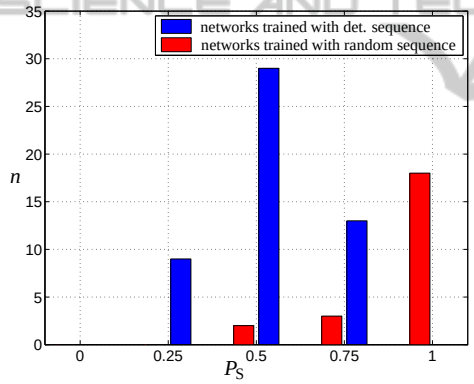


Figure 9: Distribution of success for testing without context layer.

The 23 networks trained with a random sequence have zero context weights already. Their probability of success remains high, $\overline{P_S} = 0.9239$. Figure 9 shows that 18 networks succeeded the test run. Another 5 networks achieved only a probability of success of 50% or 75%. This does not imply an importance of the context layer to these networks. The mean probability of success $\overline{P_S} = 0.9239$ is comparable to those achieved in the test with context layer in Section 3.2.1 ($\overline{P_S} = 0.9457$).

All in all, the test shows the relevance of the context layer for networks trained with the deterministic sequence. After removal of this layer the performance in this test drops from $\overline{P_S} = 0.9657$ (with context layer) to $\overline{P_S} = 0.5196$ (without context layer). Further, Figure 9 points out that the influence of the context layer may differ from network to network even if the networks are trained in the same manner.

For networks trained with a random sequence the context layer is of little importance as expected.

3.4 Output Sequences and their Characteristics

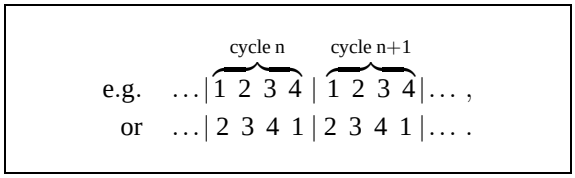
3.4.1 Sequence Generation

Recurrent networks tend to oscillate, even if the input is zero the networks produce an activation in the output layer. Networks trained with a random sequence do not oscillate since the feedback connections are zero. We presented one initial input followed by zeros for the duration of 1000 cycles and observed the sequences that are generated. The networks trained with a deterministic sequence produce a variety of sequences. We observed four classes of sequences after evaluation of the behaviour of the 51 networks that succeeded the training with the deterministic sequence. The types of sequences are:

1. full cycle oscillation,
2. half cycle oscillation,
3. constant after transient oscillation,
4. others.

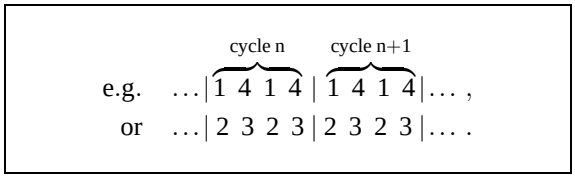
This distinction is not universally valid but seems to be most adequate in terms of evaluation of the observed data.

Full Cycle Oscillation (FCO): is a sequence that reproduces the trained input sequence completely,



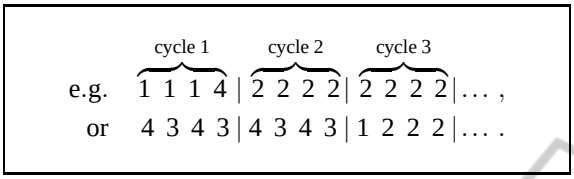
In every cycle all four training inputs appear in the order of the deterministic sequence (1, 2, 3, 4) but the cycle does not necessarily start with “1”.

Half Cycle Oscillation (HCO): is a sequence that reproduces some part of the trained input sequence with the period of a half cycle,



Two input pattern appear alternating two times per cycle.

Constant after Transient Oscillation (CTO): is a sequence that takes a constant value after a transient oscillation of about two cycles,



Others: are the sequences that do not fit into the three classes above. For instance, sequences that reproduce the trained input with a blemish or sequences that produce an oscillation with a period that spans over several cycles,

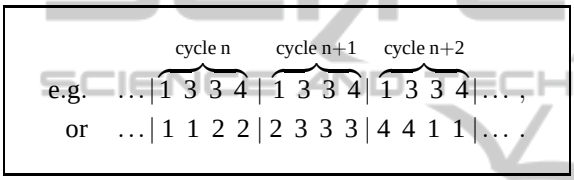


Figure 10 shows the distribution of the generated sequences over the classes of sequences. Most of the networks ($n = 23$) generate the sequence presented during training after activation with one *single* input pattern. In terms of sequence learning these networks performed best. Another group of networks ($n = 17$) has an oscillating behaviour with the period of a half cycle. This can be interpreted as a clock signal with two pulses/beats per cycle. A constant output is produced by 5 networks after a short transient oscillation. The remaining networks ($n = 6$) produced some oscillation that does not fit into the aforementioned class.

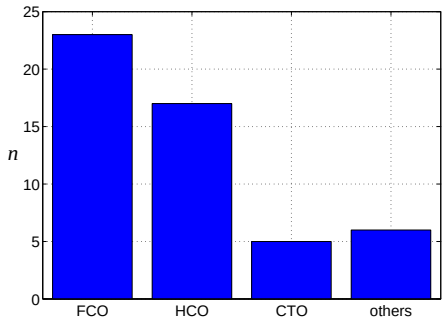


Figure 10: Distribution of sequences generated by 51 networks trained with deterministic sequence over classes of sequences.

The key component to the generation of an oscillation by the network is the weight matrix of the context layer. After one initial input, the input layer makes no further contribution to the processing in the network. The output layer provides the encoding from the binary representation of the network state into the 1–of–4 coded representation at the output. The sequence of network states is solely generated by the interplay of the hidden and context layer. Figure 11 shows the mentioned part of the SRN.

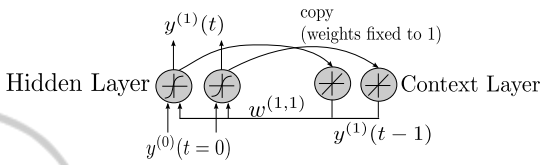


Figure 11: Interaction of Hidden and Context Layer.

The process of sequence generation can be described by

$$y_1^{(1)}(t) = \tanh(w_{11}^{(1,1)} y_1^{(1)}(t-1) + w_{12}^{(1,1)} y_2^{(1)}(t-1)), \quad (7)$$

$$y_2^{(1)}(t) = \tanh(w_{21}^{(1,1)} y_1^{(1)}(t-1) + w_{22}^{(1,1)} y_2^{(1)}(t-1)),$$

with initialisation

$$y_i^{(1)}(t=0) = \tanh(\sum_j w_{ij}^{(1,0)} y_j^{(0)}(t=0)). \quad (8)$$

After the initial input, the network state $y^{(1)}(t)$ depends only on the last state $y^{(1)}(t-1)$. The transition of one state to another is controlled by the context weight matrix $w^{(1,1)}$. From this follows, that the properties of $w^{(1,1)}$ determine which sequence the network generates or in other words, which sequence the network learned during training.

3.4.2 Properties of the Context Matrix

By polar decomposition of a matrix it is possible to separate the matrix into a component that *stretches* the space along a set of orthogonal axes and a *rotation* (Conway, 1990). The polar decomposition of a complex matrix A has the form

$$A = RS \quad (9)$$

where R is a unitary matrix and S is positive-semidefinite. The matrix S represents the component that stretches the space while R represents the rotation. The matrix S is given by

$$S = \sqrt{A^* A} \quad (10)$$

where A^* denotes the conjugate transpose of A. If A is invertible, then the matrix R is given by

$$R = AS^{-1}. \quad (11)$$

We used the polar decomposition to extract some properties of the context matrix that are related to the generation of the specific types of sequences.

Full Cycle Oscillation. The component of the context matrix $w^{(1,1)}$ that represents the rotation, takes the form of a rotation matrix that acts as a rotation in Euclidean space. For example

$$R^{(1,1)} = \begin{pmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{pmatrix}. \quad (12)$$

By that the generation of a sequence (7) turns into

$$y_1^{(1)}(t) = \tanh(R_{11}^{(1,1)} y_1^{(1)}(t-1) + R_{12}^{(1,1)} y_2^{(1)}(t-1)), \quad (13)$$
$$y_2^{(1)}(t) = \tanh(R_{21}^{(1,1)} y_1^{(1)}(t-1) + R_{22}^{(1,1)} y_2^{(1)}(t-1)),$$

which is a rotation of vector $y^{(1)}$ in the y_1y_2–plane (counter)clockwise by an angle of Θ. This view neglects the effect of the component of $w^{(1,1)}$ that stretches the space. This is possible since the hyperbolic tangent always maps $y^{(1)}$ on values between -1 and 1. For the observed context matrices the angle Θ lies between 78 and 100 degree. Figure 12 illustrates the process of the sequence generation. At every time

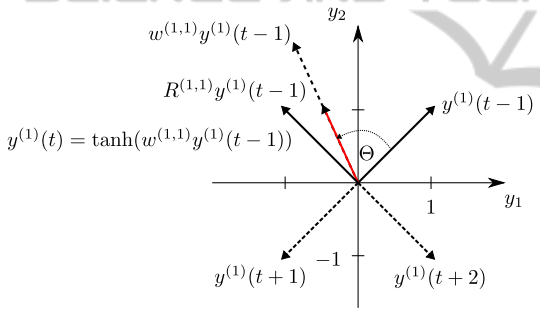


Figure 12: Rotation of $y^{(1)}$ by the context matrix $w^{(1,1)}$.

step the context matrix rotates $y^{(1)}$ into a new quadrant where the hyperbolic tangent maps the single components of the rotated vector onto the nearest 1 or -1. By that $y^{(1)}$ passes all four quadrants and therefore we observe all four training inputs at the output of the network.

Half Cycle Oscillation. For networks that generate a HCO the component of $w^{(1,1)}$ that represents the rotation takes the form of a reflection matrix. A reflection matrix is orthogonal with determinant -1. The eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = -1$. In terms of a geometric interpretation $w^{(1,1)}$ reflects $y^{(1)}$ from one quadrant into another and reverse. Hence, $y^{(1)}$ passes two quadrants and therefore, we observe only two of the training inputs at the output of the network.

Constant after Transient Oscillation and Other. For these types we found no property in the context matrix that is shared by all observed matrices by now. A deeper investigation of the relation between properties of the context matrix and these sequences remains future work. Further the influence of the initial input on the whole process needs to be analysed.

4 CONCLUSIONS

The learning task was constructed in a way such that the sequential order of the input was not needed for the solution. In this sense one can speak about *implicit* sequence learning, since the network was never explicitly trained to reproduce or predict the sequence.

During training (cf. Section 3.1) the sequential order results in a higher learning performance. Nearly twice as many networks learned the coding if trained with a deterministic sequence than with a random one (cf. Figure 4).

The observation of the weights in the context layer shows the influence of this network layer in the process of implicit sequence learning. A random input sequence provides no sequential information and a network trained with such input learns to reject the information provided by the context layer. This results in vanishing context weights (cf. Figure 7). By that the network turns into a standard feedforward network.

The testing of the trained networks in Section 3.2 shows the importance of a previously learned sequential correlation between single inputs. The performance of the networks trained with a deterministic sequence heavily depends on the presence of the temporal context in the input (cf. Figure 8 and Table 4).

The relevance of the context layer for the networks trained with the deterministic sequence was investigated by a test without this layer in Section 3.3. The overall performance drops dramatically after this change (cf. Table 4 and Table 5) but not all networks rely on the context layer to the same extent (cf. Figure 9).

There is no sequence learning without a context layer that provides the network with some memory. On the other hand, there is no guarantee that the network learns exactly the presented sequence in this layer. In fact, the input sequence often can only be reproduced in combination with an activation from the input layer. Section 3.4 clearly shows this fact. The variety of generated sequences (cf. Figure 10) points out that the networks find different representations of the sequential information. The context weight matrix is the determining factor in this process.

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