CONTEXTUALIZING ONTOLOGIES FOR AGENTS

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Abstract: It is well accepted that the usefulness of agents is enhanced by ontologies, but a common problem encountered by agents is the difficulty of accessing heterogenous ontologies. This problem is addressed by contextualizing ontologies, but how? We show how agents can contextualize ontologies that are represented using description logics. Several attempts have been made in addressing this contextualization problem, but we use the technique of the Tiered Logic Method (TLM) to build a system that is much simpler, more elegant, and easier to implement than existing technologies. Moreover, since TLM is a methodology it also has applications in other types of system. We sketch proofs of soundness, completeness and decidability for such a system, subject only to simple finiteness constraints, which would be satisfied in any practical case. Finally this method solves the problem of transitive subsumption propagation,which is still unaddressed by other well known proposals.

1 INTRODUCTION

Ontologies play a vital role in applications that involve the cooperation of multiple agents (Obitko and Mařík, 2003). How can agents deal with heterogenous ontologies? The problem is solved if the agent has a map of its ontology to the one that it is about to read. Like databases, ontologies are described using a computer language, so how do we contextualize ontology languages (Bouquet et al., 2004).

Ontology languages such as OWL2 are founded on Description Logic (DL) (Baader et al., 2003), so the problem becomes an issue of how DLs may be contextualized. Most researchers start with the Local Model Semantics of (Giunchiglia and Ghidini, 2000) and DDL (Borgida and Serafini, 2002). Here, concepts in one ontology are linked to concepts found in another ontology using *bridge rules*. This was extended in $\mathscr{E}$-Connections (Kutz et al., 2004), using a relation E, which relates individuals in different ontologies to capture the meaning of bridge rules. Further investigation has led to Packaged-DL (P-DL) (Bao et al., 2006), akin to the encapsulation found in Object Oriented programming, allowing one ontology to import or reuse concepts from another ontology. The drawback is that P-DL syntax requires a revamp of existing DLs and existing ontology reasoners need to be re-engineered, and at present there is no reasoner that implements P-DL. The system IDDL of (Zimmermann and Duc, 2008) uses local and global systems related to ours, but has to introduce special reasoning rules rather than using a standard (propositional) logic as we do. Except for P-DL and IDDL, all of the above suffer from one crucial weakness: they do not support transitive subsumption propagation, that is to say, the rule: $i : A \overset{\sqsubseteq}{\rightarrow} j : B$ and $j : B \overset{\sqsubseteq}{\rightarrow} k : C$, imply $i : A \overset{\sqsubseteq}{\rightarrow} k : C$.

In this paper we contextualize DLs preserving transitive subsumption by using the Tiered Logic Method (TLM) of (Cruz and Crossley, 2009), and we present a solution which, we believe, is simpler than others. Moreover, the ideas have been tested in existing reasoning engines without re-engineering being required. We apply TLM to contextualize a typical DL called $\mathcal{ALC}$, which is a a subset of $\mathcal{SHIQ}(\mathcal{D})^-$ the present foundation for ontology languages such as OWL. However, the principles found here are also applicable to contextualizing $\mathcal{SHIQ}(\mathcal{D})^-$.

DLs - A Review. DLs use the fundamental notion of *concepts* which are sets of individuals.[2] $\mathcal{ALC}$ syntax is grouped into statements that are terminological

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[2]For simplicity of exposition we do not use *roles*, *attributes* or *features* although the extension required to include them is routine. For full details see (Cruz, 2010).

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$$\frac{\Gamma \models_\gamma (A^l)^k}{\Gamma \models_\gamma A^l} \text{ (Flat)} \qquad \frac{\Gamma \models_l A^l}{\Gamma \models_l A} \text{ (Flat-0)}$$

Figure 1: The transfer rules.

ones which comprise the *TBox* and those that are assertional ones which comprise the *ABox*. We follow the standard terminology of (Baader and Nutt, 2003). We shall write $C = \{a, b\}$, etc. as an abbreviation for $a : C,\ b : C$, etc.

Traditionally DL literature defines an *interpretation structure* for a system such as ours as a pair $\mathscr{I} = \langle \Delta_\mathscr{I}, \bullet^\mathscr{I} \rangle$. Here the set, $\Delta_\mathscr{I}$, is the *domain* of the interpretation (also known as the *abstract domain*) and $\bullet^\mathscr{I}$ is the *interpretation function*. However we shall switch to a slightly different notation so that we can keep the superscript to indicate the 'context' to which a statement belongs, in line with (Cruz and Crossley, 2009):

instead of a superscript for the interpretation function we shall instead use the Greek iota ι. Thus instead of $C^\mathscr{I}$ we shall write $\iota(C)$. Hence our interpretation $\mathscr{I}$ will be written as $\langle \Delta_\mathscr{I}, \iota \rangle$.

2 TIERED DESCRIPTION LOGIC

Our underlying methodology, TLM, follows the general pattern of (Cruz and Crossley, 2009), but our approach here is that of description logic and therefore we shall be principally concerned with semantic equivalence rather than the syntactic (viz. provable) equivalence of that paper.

We start with a number of *ontologies* or *localities*, which we denote by superscripts $i, j, k, \ldots$. Each of these localities will have its own description logic.[3] Such a language constitutes the *strictly local language* at each locality in *tier-0*: the local level. We then move up to what we call *tier-1* which is the *global* or *system level* and combine them using positive propositional logic. Finally we take the global statements back down into tier-0 and form the local language.

In order to move statements between the two tiers we use the idea of *flattening* (Buvač et al., 1995). This entails that once a statement has been made (and its semantics determined for its own locality) then the truth or falsehood of the statement is unaffected by reporting it in another locality, see Figure 1.

In our earlier work, (Cruz and Crossley, 2009), we used predicate logic at tier-0 and full classical propositional calculus at tier-1. We can use full propositional calculus if we do not wish to have 'Bridge rules' or 'Bridge declarations' but these are what gives the tiered logic method its strength. A bridge declaration is a statement that says that a concept in one locality is subsumed by a concept in another. For example, the concept HOUSE in an English-speaking locality can be subsumed under the concept CASA in a Spanish-speaking one. So a real estate agent selling in both Britain and Spain may 'bridge' between British and Spanish contexts by a bridge declaration $\mathtt{HOUSE}^{Britain} \sqsubseteq \mathtt{CASA}^{Spain}$.

[3] These languages may differ, see Remark 2.

Remark 1. *When we wish to combine bridge statements involving different localities there is a problem about negation. Consider a bridge statement such as* $A^i \sqsubseteq B^j$. *We can certainly consider this as a global statement as we did in the house/casa example. But what would* $\neg(A^i \sqsubseteq B^j)$ *mean in our system? Viewed from a traditional logic point of view it certainly means that some members of A are not members of B, but we cannot answer the question: Where are these elements? Elements in A are in the locality i, but may or may not be in locality j (cf. Remark 2). So the problem is how to say, in our language: x is not in j.*

We therefore restrict our propositional calculus to positive *propositional calculus (+PL), that is to say, we only employ the connectives* $\vee$ *and* $\wedge$*, and do not use* $\neg$ *or* $\rightarrow$*. We claim this is still in the spirit of description logic since, in a DL, it is not possible to negate a terminological statement.*

Note that (Zimmermann and Duc, 2008) deal with this in a different way, and the price they have to pay is that, in their system, 'reasoning on IDDL systems is not trivial'.

⟨DL, +PL⟩ Syntax. By *Strictly Local Syntax*, we mean the DL syntax as described in Section 1, for a given locality k. Note that all DL statements, which we shall call *Strictly Local Statements*, are sentences.

Remark 2 (Overlap Requirements.)**.** *It is possible to have overlaps of individuals or atomic concepts in the languages at the different localities. In such cases we shall impose the requirement that if two atomic statements, from different localities, are syntactically identical, then they are also semantically identical, and vice versa. This carries over to more complicated statements in a straightforward way. We shall also assume that, apart from such overlaps, there are no symbols in common between the DLs in different localities.*

Definition 1. Bridge Declarations: Syntax. *When we relate a concept term in one ontology such as A in i to a concept term in another ontology such as B* *in j then we call such a declaration a* Bridge Declaration *and we express this as a terminological statement* $A^i \sqsubseteq B^j$*, or* $A^i \equiv B^j$ *as the case may be.*

Definition 2. Basic Global Formulae. *If* φ *is a strictly local statement in locality k, then* φ^k *is a* basic global statement, *with the intended meaning that* φ *is true in k. If* $A^i \sqsubseteq B^j$ *is a bridge declaration, then it is also a* basic global statement.

Global Statements. *1. Basic global statements are* global statements *(henceforth designated by bold letters:* $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ *etc.). 2. If* $\mathbf{X}$ *and* $\mathbf{Y}$ *are global statements, then* $(\mathbf{X} \vee \mathbf{Y})$ *is a* global statement. *3. If* $\mathbf{X}$ *and* $\mathbf{Y}$ *are global statements, then* $(\mathbf{X} \wedge \mathbf{Y})$ *is a* global statement. *4. If* $\mathbf{X}$ *is a global statement and k is any locality, then* $\mathbf{X}^k$ *is a* global statement.

We now extend strictly local syntax and semantics, referring to the ordinary $\mathcal{ALC}$, with the admixture of global statements, to give our *local syntax*.

Definition 3. Local Syntax. *For locality k, this is the inductive closure of: 1. All strictly local statements are local statements of locality k, 2. If* $\mathbf{X}$ *is a global statement then it is a local statement in k, 3. If* A, B *are local statements, then local statements formed using positive propositional connectives are also local statements of k, e.g.* $(A \wedge B), (A \vee B)$*. 4. If A is a local statement and k is a locality, then* A^k *is also a local statement of k.*

⟨DL, +PL⟩ Semantics. Strictly local semantics has been covered in Section 1. A *local model* for a locality k is therefore of the form $\mathscr{I}^k = \langle \Delta^k, \Delta^k, \iota^k \rangle$.

Definition 4. Global Models. *If* $\mathbb{K}$ *is the set of localities, we define a* global model *to be a structure* $\mathfrak{M} = \{\mathscr{I}^k : k \in \mathbb{K}\}$ *such that* $\mathscr{I}^k$ *is a local model for the locality k.*

We write $\mathscr{I}^k \vDash_k \varphi$, where φ is either a terminological or assertion statement in locality k, to mean that $\mathscr{I}^k$ is a model for locality k.

Definition 5. Semantics of Basic Global Statements. *If* $\mathfrak{M} = \{\mathscr{I}^k : k \text{ is a locality}\}$*, then we shall write* $\mathfrak{M} \models_k A^j[a]$ *to mean that* $x : A^j$ *is true in* $\mathfrak{M}$ *iff A is a concept term in locality j such that, when* $x \in O^k$ *(the set of individuals in locality k) is assigned the value a i.e.* $\iota^k(x) \in \Delta_{\mathscr{I}^k}$*, then* $\mathscr{I}^j \models_j a : A$.

Bridge Declaration Semantics. The basic step in defining global satisfaction is as in (Cruz and Crossley, 2009): a basic global statement of the form φ^k is true globally if, and only if, the local interpretation $\mathscr{I}^k$ makes φ true. Then use the usual rules of propositional calculus for combinations of global statements. What is our corresponding semantic definition for bridge declarations? Intuitively $A^i \sqsubseteq B^j$ translates as $\forall x(x : A^i \rightarrow x : B^j)$, which in turn may be rendered as $\forall x((x \in i \wedge x \in A) \rightarrow (x \in j \wedge x \in B))$. We take the natural semantics for this, which may be found in clause 2 in the next definition.

Remark 3. *Note that our natural interpretation of* $A^i \sqsubseteq B^j$ *means that if* $c \in O^i$ *and* $c \in A$*, then c must be in* O^j*. An important consequence of this is that when we have a bridge declaration* $A^i \sqsubseteq B^j$*, which is true for a given global model, then* $A^i \sqsubseteq \top^j$ *will also be true, where* $\top^j$ *is the universal concept for locality j.*

Definition 6. Global Semantics. *If* $\mathfrak{M}$ *is a global model, we define* global satisfaction *inductively on the complexity of a global statement* $\mathbf{X}$.

1. *If* $\mathbf{X}$ *is a basic global statement* φ^k*, where* φ *a strictly local statement in k, then* $\mathbf{X} = \varphi^k$ *is* globally satisfied *in* $\mathfrak{M}$*, written* $\mathfrak{M} \models_\gamma \varphi^k$*, iff* $\mathscr{I}^k \models_k \varphi$*. In this case we also say that* φ *is* locally satisfied *at k, and write* $\mathfrak{M} \models_k \varphi^k$.
2. *if* $\mathbf{X} = (A^i \sqsubseteq B^j)$*, then* $\mathfrak{M} \models_\gamma \mathbf{X}$ *iff it is the case that for all objects* $c \in O^i$*,* $\mathscr{I}^i \models_i A[c]$ *implies* $c \in O^j$ *and* $\mathscr{I}^j \models_j B[c]$*. The analogous definition applies for the case of the* $\equiv$ *connective.*
3. *If* $\mathbf{X} = (\mathbf{Y} \vee \mathbf{Z})$*, then* $\mathfrak{M} \models_\gamma \mathbf{X}$ *iff* $\mathfrak{M} \models_\gamma \mathbf{Y}$ *or* $\mathfrak{M} \models_\gamma \mathbf{Z}$*, and analogously for* $\mathbf{X} = (\mathbf{Y} \wedge \mathbf{Z})$.
4. *if* $\mathbf{X} = \mathbf{Y}^k$*,* $\mathfrak{M} \models_\gamma \mathbf{X}$ *iff* $\mathfrak{M} \models_k \mathbf{Y}$.

Local Semantics.

1. *If* φ *is a strictly local statement in k, then* $\mathfrak{M} \models_k \varphi$ *iff* $\mathscr{I}^k \models_k \varphi$.
2. *If a basic global statement* $\Phi = \varphi^i$ *(which is by definition a local statement too), then* $\mathfrak{M} \models_k \Phi$ *iff* $\mathscr{I}^i \models_i \varphi$.
3. *If* Φ *is a bridge declaration,* $A^i \sqsubseteq B^j$*, then we define* $\mathfrak{M} \models_k \Phi$ *iff* $\mathfrak{M} \models_\gamma \Phi$ *as in Definition 6, clause 2. Similarly for* $A^i \equiv B^j$.
4. *If* $\Phi = \Psi \vee \Theta$*, then* $\mathfrak{M} \models_k \Phi$ *iff* $\mathfrak{M} \models_k \Psi$ *or* $\mathfrak{M} \models_k \Theta$*, and analogously for* $\Phi = \Psi \wedge \Theta$.
5. *If* $\Phi = \Psi^i$*, then* $\mathfrak{M} \models_k \Phi$ *and* $\mathfrak{M} \models_\gamma \Phi$ *iff* $\mathfrak{M} \models_i \Psi$*. Note here the change from* γ *to i.*
   If $\mathscr{I}^k \models_k A$ *we say* $\mathfrak{M}$ locally satisfies *A in locality k.*

For the simplification of statements we have semantic equivalents, as opposed to the syntactic equivalents of (Cruz and Crossley, 2009). We use the ordinary rules of (positive) propositional calculus for tier-1 and the usual DL rules for tier-0. Between the tiers we use the semantic version of the flattening rules of (Buvač et al., 1995). ⟨DL, +PL⟩ is a system with local ontologies which we treat as localities. These localities are assumed to be consistent with each other.

For soundness (and consistency) of a set Γ of statements we may assume Γ contains only global statements initially since any statement A local to k is semantically equivalent to the global statement A^k.

Definition 7. *A global set of statements, Γ, is said to be (a)* globally consistent, *(b)* locally consistent *or* strict locally consistent *to mean that there is, respectively, (a) a global, (b) a local model for Γ, respectively.*

Layered Tabelaux. In the DL world, logic is treated not as a system of axioms and rules of deduction but in terms of tableaux in the same style as modal logics are often treated. Our aim is to show that, given a consistent set of global statements, we can extend these to a complete tableau.

We now introduce thetableau construction method for $\langle$DL, +PL$\rangle$ which, *inter alia* builds an ordinary DL tableau for each locality using the *Tableau Completion rules*, TCR, as found in (Haarslev et al., 2000) and earlier in (Buchheit et al., 1993). From now on, an *ABox* $\mathscr{A}$ will be superscripted with its locality: if $\mathscr{O}(k)$ is the ontology in locality k, then its *ABox* is $\mathscr{A}^k$; likewise for *TBoxes*. The necessary additional rules for our tabelaux are in Section 2. We call the tableaux of $\langle$DL, +PL$\rangle$ *layered tableaux* because the tableaux for $\langle$DL, +PL$\rangle$ are trees of forests of trees.

The main tree deals with global statements but the global statements are evaluated to eventually wind down to the local tableaux they influence. So our tableau expansion rules are of two kinds: a) those that govern global statements and b) those that govern local statements.

We recall the relevant results from (Horrocks et al., 2000) and (Cruz and Crossley, 2009) for local tableaux.

Definition 8. *An ABox $\mathscr{A}$, is consistent iff there is an interpretation $\mathscr{I}$ of its TBox such that it is an interpretation of all the assertions in $\mathscr{A}$.*

Theorem 1. *A concept C is satisfiable in $\mathscr{A}$ iff $\mathscr{A} \cup \{a : C\}$ is consistent with $\mathscr{A}$.* □

Definition 9. *A tableau is* clash free *iff none of its nodes contains a clash. It is* complete *if no tableau completion rules can be applied to it.*

Theorem 2. *Satisfiability and subsumption of concepts is reducible to testing consistency of ABoxes (cf. (Horrocks et al., 2000), Theorem 1).* □

Theorem 3. *An ABox, $\mathscr{A}$, is consistent iff it has a complete tableau. This also implies that because it has a complete tableau, $\mathscr{A}$ has a model, cf. (Haarslev et al., 2000).* □

Theorem 4. *Every global statement is semantically equivalent to a propositional combination (using only $\vee$ and $\wedge$) of basic global statements (including bridge declarations).* □

The proof is easily established as a semantic version of Theorem 1 of (Cruz and Crossley, 2009). □

Because we are starting off with a consistent set of formulae, conflicts that may occur due to bridge rule declarations will be detected. Furthermore, the issue of blocking is handled by the local tableau completion rules, following (Haarslev et al., 2000).

The Completion Procedure. At the top we have a propositional tableau, but we then extract each local component and throw it to the appropriate local DL tableau for its local handling. Conversely, without simplification techniques, some of these components will affect other localities and need to be put back into the collection of global statements. This process may be circumvented by first *flattening* the global statements, and then expressing them as propositional combinations of basic global statements.

We now give a formal description.

We write $\mathfrak{T}$ for the tree whose root is Γ and whose nodes are sets of statements produced by the construction operation on global statements. After a finite number of steps the global statements will all have been 'reduced' to propositional combinations of basic global statements. The basic global statements with superscript k are then treated *locally*, in their relevant localities, using the TCR in the usual way. For each k this will create a standard DL tableau. The process terminates when no more 'reductions' can be made.

The first two steps, which are always to be rechecked, ensure that we do not generate inconsistent tableaux.

1. If at any stage we would be adding a contradictory atomic statement, e.g. $a : \neg A$ to a branch in a local tableau containing $a : A$, then we abort that branch.
2. If all the branches in a local tableau are aborted, then remove the branch above that. Note that this will remove all the tableaux coming from that branch. This is equivalent to a local contradiction translating into a global one.
3. If we split at a node then we replicate the tableau that we have at that node before adding the new items to the two emanating branches.
4. Reduce the number of superscripts in a global statement to a minimum by flattening.
5. By Theorem 4 we can reduce every global statement to a propositional combination of basic global statements and bridge declarations.

Figure 2: Our tableau rules for layered tableau $\mathfrak{T}$.

Rule Name and Formula in Γ	**Operation**
Flat rule $(\mathbf{X}^j)^k$	Add $\mathbf{X}^j$ to $\mathfrak{T}$.
Flat-0 rule for a basic global statement φ^j	Send φ to the tableau for locality j (see Step 8).
α rule $(\mathbf{X}^i \wedge \mathbf{Y})^j$	Add $\mathbf{X}^i, \mathbf{Y}^j$ to $\mathfrak{T}$.
β rule $(\mathbf{X}^i \vee \mathbf{Y})^j$	Add $\mathbf{X}^i$ as a left node to $\mathfrak{T}$, $\mathbf{Y}^j$ as a right node to $\mathfrak{T}$. *Note. This will form a split in our layered tableau.*
Bridge rule for $A^i \sqsubseteq B^j$	For each c in locality i, if $c \in O^i$ and $c : A$ is in the tableau for i then add $c : B$ to the tableau for j. Conversely, if $c : \neg B$ is in the tableau for j and $c \in O^i$, then add $c : \neg A$ to the tableau for i. *Only do these steps if consistent.*

6. Apply the rules in Figure 2 to develop the tableau.
   (a) Peculiar to our case are Flat and Flat-0. These rules allow us to 'enter' a locality. Furthermore, what is of critical importance in these rules are the β and Flat-0 rules. β will cause a branch to split in the layered tableau. Flat-0 will 'send' a statement into a local (sub-)tableau (see Step 8).
   (b) The next two are the usual propositional tableau rules, see e.g. (Smullyan, 1968).
   (c) The bridge rule 'sends' a basic global statement to its appropriate locality after stripping the superscript.
   If we have $c : A$ for c in O^i in the (local) tableau for i, then we add $c : B$ to the tableau for j. Note that if c was not already in O^j then we add it to that locality as a new name. On the other hand if $c : \neg B$ is in that tableau for j, *and* $c \in O^i$, then we add $c : \neg A$ to the tableau for i. Of course either of these procedures might produce a clash. In this case we abort this branch (see Step 1 above).
7. Next we put each concept term into negation normal form, see (Haarslev et al., 2000), i.e. where all negation signs are only applied to atomic concepts.
8. At this stage we use the TCR in each locality, to develop the individual tableaux. However, each time we add a new constant to a tableau we must then use any applicable bridge declaration (see Figure 2 and Step 6c) again because such an addition may add new statements to a different local tableau.The TCR will add new atomic assertions to a local tableau.
9. Finally we repeat all of the above steps fairly until only atomic assertions and bridge declarations remain.

Remark 4. *We cannot eliminate the bridge declarations since, whenever another rule introduces a statement such as $c : A^i$, we must check whether any bridge declaration for $A^i \sqsubseteq B^j$ is applicable (see* Figure 2 *and* Step 6c*).*

It is clear from the above rules that we obtain ever shorter statements when we start from a propositional combination of global statements, provided we are dealing with finite statements and a finite number of localities.

Lemma 1. *(a)* Invariance of $\mathscr{A}^k$, cf. (Haarslev et al., 2000)*). Assume that Γ together with each ontology $\mathscr{O}(k)$ is system-wide consistent, then our sending operation preserves system-wide consistency. I.e. for all k, , if $\mathscr{A}'^k$ is derived from $\mathscr{A}^k$ by application of our rules for layered tableaux, then $\mathscr{A}'^k$ is also consistent whenever $\mathscr{A}^k$ is.*

(b) Model Existence *If our $\langle DL, +PL\rangle$ yields a clash free and complete tableau, then our $\langle DL, +PL\rangle$ has a model.*

Proof. We shall rely on what happens in the local tableaux and we shall use the results of (Haarslev et al., 2000), especially Definition 27, in constructing a canonical interpretation $\mathscr{I}^k$ for each locality and then follow Theorem 28 of the same work.

Let $\mathfrak{T}$ be a layered tableau for our $\langle$DL, +PL$\rangle$ which is complete and clash free. Then for each of the final *ABoxes* $\mathscr{A}^i$ (for any local ontology i), the local tableau is clash free and complete. Hence by Theorem 3, we can construct a canonical interpretation structure $\mathscr{I}^i$ as in (Haarslev et al., 2000) for each of these $\mathscr{A}^i$ and know that $\mathscr{I}^i \models \mathscr{A}^i$. Now let $\mathfrak{M} = \{\mathscr{I}^i | i \in \mathbb{K}\}$: this is our global model. We then prove that $\mathfrak{M}$ satisfies every global statement $\mathbf{X} \in \Gamma$ by induction on the structure of $\mathbf{X}$.

We only consider here $\mathfrak{M} \models_\gamma \mathbf{X}$ where $\mathbf{X} = (A^i \sqsubseteq B^j)$ as this is the only non-trivial case. Since we have a layered tableau that is clash free and complete, then if $c : A$ was in the tableau for i, $c : B$ was added successfully to j, and conversely, if $c : \neg B$ was in the tableau for j, then $a : \neg A$ was added to the tableau for i, and moreover, these were only added provided consistency was maintained. So if $\mathscr{I}^i_{\mathscr{C}} \models a : A$ in the canonical interpretation for the local tableau of i, then $\mathscr{I}^j_{\mathscr{C}} \models c : B$. Conversely, by De Morgan's laws if $\mathfrak{M} \models_j a : \neg B$ then $a \in O^i$ and $\mathfrak{M} \models_i a : A$. □

Completeness. Since a tableau is *complete* if it has no clashes and no more tableau completion rules can be applied to it., we define a layered tableau to be *complete* if no transformation rule (tableau completion rule) can be applied to it and it is clash free.

Theorem 5. *Let $\mathscr{A}^k$ be an augmented ABox in the tableau of locality k. If $\mathscr{A}^k$ is consistent then there exists at least one completion $\mathscr{A}'^k$ of $\mathscr{A}^k$ computed by applying our completion rules.*

Use Lemma 1 *(a)* for each local tableau, then these local tableaux determine the truth values of the atomic propositions (which are basic global statements) in tier-1. If the set of global formulae were inconsistent, then it would contain $(a : C)^k$ and $(a : \neg C)^k$ for some concept term C in k. But then $\mathscr{A}^k$ would be inconsistent, which is a contradiction. □

Theorem 6. *(a) Our ⟨DL, +PL⟩ is system-wide consistent iff our ⟨DL, +PL⟩ has a layered tableau which is clash free and complete.* □

*(b) (*Decidability*) Checking for the consistency of our ⟨DL, +PL⟩ system is a decidable problem provided Γ and the number of localities is finite.* □

Theorem 7 (Transitive Subsumption Propagation)**.** *Every complete model of a set Γ of global formulae containing the Bridge Rules, $A^i \sqsubseteq B^j$ and $B^j \sqsubseteq C^k$, is a model of $A^i \sqsubseteq C^k$.* □

The proof is a straightforward exercise in semantics.

3 CONCLUSIONS

This paper serves to support the contention that the tiered logic method TLM transfers the nice properties of local logics into the tiered scheme. ⟨DL, +PL⟩, using the TLM, exhibits soundness, completeness and decidability in a similar way to DL in tier-0. TLM provides a contextualized DL system without the overhead of heavy duty theoretical machinery in contrast with DDL and $\mathscr{E}$-Connections. ⟨DL, +PL⟩ has been simulated using the distributed reasoning tool RACER (Haarslev and Möller, 2001) (now called RacerPro) with consistent results.

On a more recherché note, the problem of the negation of Bridge Declarations (see Remark 1), seems an interesting one; one which has been treated in a different way in (Zimmermann and Duc, 2008).

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