Image Formalization via Descriptive Image Algebras

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Abstract. In the framework of the Descriptive approach to image analysis and understanding, the concept of Image Formalization Space (IFS) is introduced. The elements of IFS are the different image representation forms through which a raw image passes while it is transformed into a recognizable form, i.e., into an image model. The structure and the main objects of the IFS are characterized and discussed. A system of concepts and a formal apparatus of descriptive image models are introduced to state the main axioms constituting the conceptual foundation for image formalization in image analysis and recognition. The topology of IFS is given by descriptive image algebra defining operations on pixels and their configurations, on pixel values, and on image states constructed using these operations.

1 Introduction

A breakthrough challenge of theoretical computer science is to find automated ways of processing, analyzing, evaluating, and understanding information represented in the form of images. For developers of automated systems designed to handle images, as well as for end users in automated or interactive modes, this automation is expected to help to develop, adapt and check methods and algorithms for image recognition, understanding, and evaluation: (1) choose optimal or suitable methods and algorithms for image recognition, understanding and evaluation; (2) check the quality of initial data and whether they can be used in solving the image recognition problem; (3) apply standard algorithmic schemes for image recognition, understanding, evaluation, and search.

To ensure automation, we need to develop and evolve a new approach to analyzing and evaluating information represented in the form of images. For this purpose, the Algebraic Approach of Yu.I. Zhuravlev [9] has been modified for the case when the initial information is represented in the form of images. The result is the descriptive approach to image analysis and understanding (DAIA) proposed and justified by I.B. Gurevich [2] and developed by his students [3, 4].

By the middle of 1990s, it had become obvious that the following points are critical to the development of image analysis and recognition: (1) understand the nature of initial information - images, (2) find methods of image representation and description that allow constructing image models designed for recognition problems, (3) establish a mathematical language for a unified description of image models and

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their transformations that allow constructing image models and solving recognition problems, (4) construct models to solve recognition problems in the form of standard algorithmic schemes that allow, in the general case, moving from the initial image to its model and from the model to the sought solution.

The DAIA gives a unified conceptual structure that helps develop and implement these models and the mathematical language [2, 3, 4]. The main purpose of the DAIA is to structure and standardize different methods, operations and representations used in image recognition and analysis. The DAIA provides the conceptual and mathematical basis for image mining, with its axiomatic and formal configurations giving ways and tools for representing and describing the images to be analyzed and evaluated.

In this work, we give a brief review of the main results concerning the system of concepts characterizing the initial information - images - in recognition problems and descriptive image models designed for recognition problems. The work consists of three main sections (along with the Introduction and Conclusions). Section 1 deals with the concepts needed to formally describe and represent images within the DAIA as well as forms and mathematical objects that reproduce an image in the course of constructing descriptive image models (DIM). We introduce the concept of Image Formalization Space and define the basic states – aspects of image representations – generated in the course of reducing images to a recognizable form (RF - recognizable form). Section 2 formalizes the ways of characterizing images, transformations, and objects needed to describe images in a form suitable for recognition algorithms. Finally, we consider the scheme for DIM synthesis. The system of concepts and the formal apparatus of DIM introduced form the necessary background to state the main axioms of the mathematical theory of image analysis and recognition. Section 3 gives a brief review of a new class of image algebras, viz. descriptive image algebras (DIA), as the sought algebraic language to describe, compare and standardize image analysis, processing and recognition algorithms. The topology of IFS is given by DIA. With these algebras, we can combine and standardize procedures of processing image models and their transformations.

2 Image Description Formalization

To develop methods of automating image recognition, we need to find efficient ways to formalize images so as to reflect image semantics, information hidden in its internal structure and the structure of the external bonds within the real world part (scene) reproduced by the image. No systematic mathematical methods for image formalization and analysis have been developed thus far. The overwhelming majority of image handling methods are heuristic, and their advantages depend on the way they use "non-depictive" tools to convey the "depictive" features of images.

In applied mathematics and computer science, constructing and applying mathematical and simulation models of objects and procedures used to transform them is a conventional method of standardization. It was largely the necessity to solve complicated recognition problems and develop structural recognition methods and specialized image languages that generated the interest in formal descriptions -

models of initial data -and formalization of descriptions of procedures for their transformation in pattern recognition (and especially in image recognition in the 1960s) [1, 5, 6, 8].

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The system of concepts we introduced provides the basis for formal definition of methods for synthesizing image models and descriptive image models designed for image analysis and recognition problems. Defining the system of concepts, we take into account the following properties of images: (1) An image consists of a collection of points and a set of values associated with these points. (2) Manipulation of images in image analysis involves operations on images and on different types of values and quantities associated with these images. (3) An image is endowed with two types of information, i.e., it is defined as spatial relationships between its points and some types of numerical or other descriptive information associated with these points. (4) A point set is a topological space. It consists of a collection of objects called points and a topology which provides for the nearness of two points, the connectivity of a subset of a point set, the neighborhood of a point, boundary points, curves, and arcs.

By its nature, the image is an object of complex information structure that reproduces information on the initial scene, using the values of brightness of discrete elements of the image, viz. pixels, patterns of image fragments, sets of pixels and spatial and logical relations between patterns, sets of pixels and individual pixels. What make images different from other tools for data representation is that they are highly informative, visual, structured and natural in terms of human perception. An image is a mix of initial (non-processed, "real") data, their realizations, and some deformations. The realizations (as well as appropriate descriptions) reflect the informational and physical nature of objects, events, and processes reproduced by the image, while deformations are due to the technical characteristics of the tools used to record, form, and transform the image in the course of constructing a hierarchy of descriptions. Thus, when developing methods for formal description of images, in addition to the brightness values of image pixels, we need to take into account the extra information associated with it explicitly and implicitly.

To formalize an image description and its conceptual structure, it is natural to assume that the initial image is given not only by its digital implementations but also by context and semantic information that shows the ways of obtaining and forming the image and/or some of its specific aspects.

The process of **image model synthesis** consists of a set of transformations applied sequentially to a raw image. As a result, we have sequentially changing image “states” corresponding to different degrees of formalization.

Thus, we can introduce the concept of **Image Formalization Space (IFS)**.

Definition 1. The **IFS** is a set of image “states” (a raw image, image realization, image representation, realization of image representation, image model [3]). The IFS is a metric space, i.e., its elements are image states (phases of formal descriptions of images). In this sense, the IFS is a phase space. The topology of this space is given by some algebraic system, i.e. via some image algebra defining operations on pixels and their configurations, on pixel values, and on image states constructed using these operations.

To construct formalized image descriptions, transformations from the set of transformations admissible for this type of images have to be applied to the entire information available on the image. Thus, we need to study, first, the types of information contained in the image (**the space of initial data**) and, second, transformations that can be applied to initial images to reduce them to a form supported by recognition algorithms (**the space of transformations**).

Descriptions of the ways of sequential and/or parallel application of transformations from the space of transformations to the initial information from the space of initial data constitute a set of schemes for constructing formal descriptions of images (**the space of image representations**).

To be able to apply recognition algorithms to the obtained formal image descriptions, we need to implement the constructed schemes (implement image representations), i.e., construct image models that result from reducing the initial image (taking into account the entire information on the image) to a form supported by recognition algorithms, i.e., to an RF. The space of image representations is an intermediate space between the space of initial data and **the space of image models**.

Thus, the construction of image models involves synthesizing objects from the following sets: (1) initial data - images; (2) image transformations that reduce images to an RF; (3) image representations, viz. schemes for constructing formal image descriptions; (4) realizations of image representations, viz. image representations with chosen values of transformation parameters and structural elements included in a representation; (5) image models.

3 Descriptive Image Models

To characterize images, the DAIA uses the following concepts: initial information (an image together with its legend), its transformations, image representations, realizations of image representations, and models. To define the types of states undergone by the initial image in the course of constructing its descriptive model and establishing the relations between these types, we introduce additional objects. These are structuring elements, generating rules, semantic and context information on images, digital representations of images, classes of image representations, classes of image models, and the correct image model.

To define the main stages of image reduction to an RF, we consider some notions.

The DAIA assumes [4] that an image is described by a set of initial information $\{\tilde{I}_0\}$.

Lemma 1. The set of initial information $\{\tilde{I}_0\}$ consists of two sets $\{\tilde{I}'\}$ and $\{\tilde{B}\}$: (1) the set of realizations $I' \in \{\tilde{I}'\}$ of the image *I* that represent the given object or scene such that $I' = \{(x, f(x))\}_{x \in D_f}$ is the set of points x belonging to the domain of definition D_f of the image realization I' and the range of $f(x)$ at each point of D_f; 2) semantic and context information on the image $\{\tilde{B}\}$.

Definition 2 [4]**. The Image Representation** is a formal scheme designed to obtain a standardized formal description of surfaces, point configurations, and shapes that form the image and the relations between them.

Definition 3. The Image Model is a formal description constructed by applying a realization of image representation to initial image representations.

Definition 4. The Realization of an Image Representation is an image realization with chosen parameter values of the transformations involved in the representation.

The synthesis of an image model is shown schematically in Fig. 1. The scheme for DIM synthesis is based on the following principles: (1) the initial information consists of the initial image realizations $\{\tilde{I}'\}$ (image state 1) and semantic and context information $\{\tilde{B}\}$ on the initial image; (2) an image representation $\{\tilde{\Re}(\tilde{\rho})\}$ (image state 2) is constructed by applying the transformations $\{\tilde{O}\}$ chosen on the basis of $\{\tilde{B}\}$ to initial image or a fragment of the initial image; (3) an image representation is realized by choosing the transformation parameters $\tilde{\rho}_0$ and structural elements $\{\tilde{S}_0\}$ which help to process not only an image itself but also its fragments (the image representation realization is $\{\tilde{\Re}(\tilde{\rho}_0)\}$, which is image state 3); (4) an initial image model $\{\tilde{M}\}$ (image state 4) is constructed by applying $\{\tilde{\Re}(\tilde{\rho}_0)\}$ to the image realizations $\{\tilde{I}'\}$ or to image realizations after the structural elements have been applied to them $\{\tilde{S}_0\} * \{\tilde{I}'\}$ (to the fragments of the initial image).

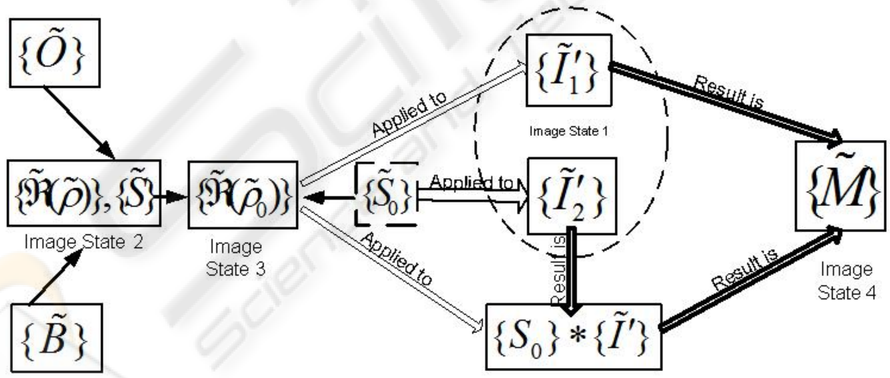

Fig. 1. Synthesis of a descriptive image model.

The DAIA deals with three classes of admissible image transformations, viz. procedural, parametric, and generating transformations [3]. These classes generate three classes of representations and three classes of image models. Three specialized schemes for parametric, procedural, and generating image model synthesis were constructed. The fundamental difference between these schemes and the general scheme for image model synthesis lies in the corresponding transformations of image realizations (parametric, procedural, and generating transformations) and in the use of generating rules for synthesizing the generating DIM.

Proposition 1. Any T-model of the image *I* generates some realization of the image *I′*, i.e., the I-model of the image *I*.

Proposition 2. Any P-model of the image *I* generates some new semantic and context information about the image *I*.

Proposition 3. Any G-model of the image *I* generates some new semantic and context information about the image *I*.

The following axioms constitute the conceptual foundation for image formalization.

Axiom 1. Any image *I* can be unambiguously put into correspondence with the totality of sets ($\{\tilde{I}_0\}$, $\{\tilde{O}\}$, $\{\tilde{M}\}$), where $\{\tilde{I}_0\}$ is the set of initial information, $\{\tilde{O}\}$ is the set of transformations applicable to $\{\tilde{I}_0\}$, and $\{\tilde{M}\}$ is the set of results produced by applying transformations to the initial information.

Axiom 2. The set of transformations $\{\tilde{O}\}$ of image states is given by the set of structuring elements $\{\tilde{S}\}$, the set of generating rules $\{\tilde{R}\}$ and three subsets of transformations: (1) procedural transformations $\{\tilde{O}_T\}$, (2) parametric transformations $\{\tilde{O}_P\}$, and (3) generating transformations $\{\tilde{O}_G\}$.

Axiom 3. The Image Formalization Space (IFS) includes the Space of Initial Data (image realizations), the Space of Realizations of Image Representations, and the Space of Image Models. The IFS is a metric space with the following types of elements: the states of an image (image aspects – image realizations and the phases of its formalized descriptions), image representations, realizations of image representations, and image models.

Axiom 4. The topology of the Image Formalization Space can be given via some algebraic system – an image algebra, defining operations over image pixels, its values, and image states constructed by applying the operations.

Axiom 5. The process of reducing an image to an easy-to-recognize form is totally realized in the Image Formalization Space by Descriptive Image Algebras defined over IFS elements – image transformations and image states.

4 Descriptive Image Algebras

As shown by the attempts made to create it, the formal apparatus to represent image processing and analysis procedures in a uniform and compact form should be based on a formal system of image representation and transformation that meets the following conditions:

– points, sets, models, transformations, and morphisms can be used as objects;
– each object is a hierarchic structure constructed of primitive objects by some transformations;

- each transformation is a hierarchic structure constructed of basic transformations by some transformations.

This formal system (which is essentially a formal language and formalisms based on it) should allow:

- constructing formal configurations (for instance, algebraic structures) that make it possible to apply methods from different branches of mathematics and computer science in image processing, analysis and recognition;
- constructing accurate and compact image descriptions handy in terms of both the way to interpret the actions performed and the development of new methods;
- describing transformations over images in the form of compact sets of simple transformations both in the machine-independent form and in the form adapted to particular architectures;
- creating specialized sub-languages to describe images and transformations over them in certain classes of image recognition and evaluation problems;
- increasing the efficiency of software implementation;
- choosing the most efficient programming languages in terms of formal structures that describe known algorithms of image processing, analysis and recognition.

Having analyzed the requirements to its functionality and mathematical characteristics, we can see that the sought formal system should represent a certain, special class of algebras that makes it possible to write any image transformation algorithm as a combination of elementary basic operations. Thus, this class of algebras should allow handling both main image models - analysis and recognition objects - and main models of transformations that allow synthesizing and implementing basic procedures of formal image description, processing, analysis and recognition efficiently.

The DAIA defines a new class of image algebras, viz. DIA, as the sought algebraic language to describe, compare and standardize image analysis, processing and recognition algorithms. DIA makes the process of constructing and applying algorithmic schemes of image mining flexible and standardized. To give problems, objects and transformations associated with image mining, we use hierarchic structures that result from applying DIA operations to the set of primitive problems, primitive elements of the image and basic transformations. Within such approach, we can vary methods of solving the sub-problem - use operations of image analysis as DIA elements, keeping the overall image mining technique unchanged.

The sources of DIA development are investigations of 1970-1980's in area of pattern recognition and image analysis "algebraization". An appearance and investigations of new algebra follows directly from results of Yu.I.Zhuravlev algorithm's algebra [10] and studies on varied image algebras by S.Sternberg[9] and G.Ritter[7].

Conducted investigation of publications centre around algebraic methods in image analysis and recognition defines primary specific of new mathematic language: 1) DIA were created for integration and standardization of procedures of image model synthesis and their transformations; 2) DIA operands are image models (also initial images themselves) and image transformations; 3) DIA operations are image analysis and processing transformations; standard algebraic operations; algebraic closures, linear combination and superposition of these operations; 4) algorithmic schemes for formal image description, processing, analysis and recognition consist of elements described by DIA and any used in scheme transformation is given by structures constructed by applying DIA operations to a set of DIA operands; 5) to provide DIA correspondence with requirements for mathematical object "algebra" restrictions on basic DIA operations are introduced.

We recall the definitions of DIA (definition 5, 6).

Definition 5 [3]**.** An algebra is called a **DIA** if its operands are either image models (both the image itself and the set of its related values and characteristics can be chosen as a model) or operations over images or both.

Main DIA investigations were directed on study of DIA with one ring (DIA1K) (definition 6), which is classical algebra of Van der Varden with non-classical operands. To define DIA with several rings we propose to use a notion "graduated algebra" and in case DIA with two rings a notion "superalgebra". Further DIA specific is defined by algebra properties.

Definition 6 [3]**.** The ring U, which is the finite-dimensional vector space over some field P, is a **DIA with one ring** if its operands are either image models or operations over images.

What makes DIA special is that

- by imposing restrictions on basic DIA operations, new mathematical constructions (DIA) ensure that we use the concept of algebra in its strict classical sense;
- basic DIA operations are introduced both over images and over arbitrary formal representations of images as well as over image transformations.

The latter explains why this new type of algebras has the word "descriptive", viz. dealing with image descriptions, in its name. Using the concept of «algebra» in its strict classical sense in the DIA definition, we can single out basic DIA operations for different types of operands, thus having the set of complete systems to describe image analysis problems.

To construct algorithmic schemes for solution of image recognition problem we should extract basic DIA1K classes, used for description of each scheme step. To provide this fact we should introduce specialized DIA1K over initial and intermediate information in image processing, analysis and recognition problems (DIA1K over image models and over image realizations) and specialized DIA1K over transformations of images and image models for generation of new image models.

On the basic of image model synthesis scheme (Fig. 1) there were extracted specialized DIA1K classes for image model description and construction (Table 1). Each DIA1K class generates rules for DIM construction by this DIA1K (theorems and proposals were formulated and proved). Formulated theorems and proposals define algebra property "to generate DIM".

The Table 1 gives the basic DIA1K classes.

Table 1. Basic classes of DIA1K.

Class	Algebra Operands	Algebra Operations	Algebra generates
1.	Realizations of initial images or realization of image fragments	Procedural Image Transformations	Procedural image models, image realizations
2.	Procedural image transformations	Operations on procedural transformations	Procedural image models, image realizations
3.	Parametric image transformations	Operations on parametric transformations	Parametric image models
4.	Generating image transformations	Operations on generating transformations	Generating image models
5.	Image Representations	Operations on image representations	DIM; multi model and multi aspect image representations
6.	Procedural Image Models	Operations on procedural image models	Procedural image models, image realizations
7.	Parametric Image Models	Operations on parametric image models	Parametric Image Models
8.	Generating Image Models	Operations on generating image models	Generating Image Models

For more detailed information on DIA, see [3].

5 Conclusions

We introduce the concept of Image Formalization Space (IFS) as a set of image "states" (a raw image, image realization, image representation, realization of image representation, image model). The topology of this space is given by some algebraic system, i.e. via some image algebra defining operations on pixels and their configurations, on pixel values, and on image states constructed using these operations.

Being developed as the fundamental basis for the mathematical theory of image analysis and recognition, the DAIA allows introducing the axiomatics of the sought theory. Five axioms were represented.

The DAIA defines a new class of image algebras, viz. DIA, as the sought algebraic language to describe, compare and standardize image analysis, processing and recognition algorithms. The topology of IFS is given by DIA. DIA makes the process of constructing and applying algorithmic schemes of image mining flexible and standardized. Main DIA investigations were directed on study of DIA with one ring (DIA1K).

Basic DIA1K should satisfy following requirements: 1) to provide effective representation of images and their models; 2) to provide representation of DIM [4] as elements of information technology for image analysis, recognition and understanding: a) to provide generation of basic DIM classes (P-, G-, T-, I- models);

б) to provide transformations of basic DIM classes; 3) to be elements of a basis of the set of all DIA1K used for description image processing, analysis and recognition problems; 4) to satisfy main propositions and conditions defined algebraic constructions.

Eight classes of basic DIA were extracted.

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