# ASYMPTOTIC ANALYSIS OF PHASE CONTROL SYSTEM FOR CLOCKS IN MULTIPROCESSOR ARRAYS

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Abstract: New method for the rigorous mathematical analysis of electronic synchronization systems is suggested. This method allows to calculate the characteristics of phase detectors and carry out a rigorous mathematical analysis of transient process and stability of the system.

## 1 INTRODUCTION

In recent years, it has actively produced and used array processors systems, which face the problem of generation of synchronous signals and the mutual synchronization of processors.

In realizing parallel algorithms, the processors must perform a certain sequence of operations simultaneously. These operations are to be started at the moments of arrival of clock pulses at processors. Since the paths along which the pulses run from the clock to every processor are of different length, a mistiming in the work of processors arises. This phenomenon is called a clock skew.

The elimination of the clock skew is one of the most important problems in parallel computing and information processing (as well as in the design of array processors).

Several approaches to solving the problem of eliminating the clock skew have been devised for the last thirty years.

In developing the design of multiprocessor systems, a way was suggested Kung, 1988 for joining the processors in the form of an H-tree, in which (Fig. 1) the lengths of the paths from the clock to every processor are the same. However, in this case the clock skew is not eliminated completely because of heterogeneity of the wires (Kung, 1988). Moreover, for a great number of processors, the configuration of communication wires is very complicated. This leads to difficult technological problems.

Among the disadvantages we note the deceleration of performance of parallel algorithms. In addition to the problem of eliminating the clock skew, another important problem arose. The increase in the number of processors in multiprocessor systems required an increase in the power of the clock. But the powerful clock came to produce significant electromagnetic noise. Not so long ago a new method for eliminating the clock skew and reducing the generator's power was suggested. It consists of introducing a special distributed system of clocks controlled by phase-locked loops (Fig. 2). This approach enables one to reduce significantly the power of clocks.

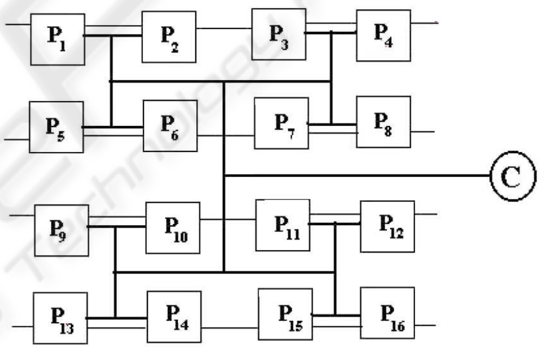


Figure 1: H-tree.

Phase-locked loops (PLLs) are widely used in telecommunication and computer architectures. They were invented in the 1930s-1940s (De Bellescize, 1932; Wendt & Fredentall, 1943) and then the theory and practice of PLLs were intensively studied (Viterbi, 1966; Lindsey, 1972; Gardner, 1979).



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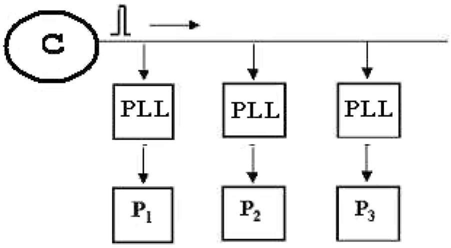


Figure 2: Distributed system of clocks controlled by PLLs.

Various methods for analysis of phase-locked loops are well developed by engineers and considered in many publications (see, e.g., (Best, 2003; Kroupa, 2003; Egan, 2007)) but the problems of construction of adequate nonlinear models and nonlinear analysis of such models are still far from being resolved and require using special methods of qualitative theory of differential, difference, integral, and integro-differential equations (Leonov et al., 1992; Leonov et al., 1996; Abramovitch, 2002; Margaris, 2004; Kudrewicz & Wasowicz, 2007; Kuznetsov, 2008; Leonov, 2006).

In this paper new method for the rigorous mathematical analysis of electronic synchronization systems is suggested. This method consists in considering a phase synchronization system on three levels:

1) a level of electronic realizations;

2) a level of phase and frequency relations between inputs and outputs in block diagrams;

3) a level of differential and integro-differential equations, and performing the asymptotic analysis of high-frequency periodic oscillations.

This method allows one to calculate the characteristics of phase detectors and make a rigorous mathematical analysis of transient and stability of the system (Leonov, 2006; Kuznetsov et al., 2008; Kuznetsov et al., 2009; Leonov et al., 2009).

## 2 ASYMPTOTIC ANALYSIS AND PHASE DETECTORS CHARACTERISTICS CALCULATION

Consider a differentiable $2\pi$-periodic function $g(x)$, having two and only two extremums on $[0,2\pi]: g^- < g^+$, and the following properties.

For any number $\alpha \in (g^-, g^+)$ there exist two and only two roots of the equation $g(x) = -\alpha$:

$$0 < \beta_1(\alpha) < \beta_2(\alpha) < 2\pi.$$

Consider the function

$$F(\alpha) = 1 - \frac{\beta_2(\alpha) - \beta_1(\alpha)}{\pi}$$

if $g(x) < -\alpha$ on $(\beta_1(\alpha), \beta_2(\alpha))$ and the function

$$F(\alpha) = -\left(1 - \frac{\beta_2(\alpha) - \beta_1(\alpha)}{\pi}\right)$$

if $g(x) > -\alpha$ on $(\beta_1(\alpha), \beta_2(\alpha))$ and $a < b$, $\omega$.

Suppose, $\omega$ is sufficiently large relative to the numbers $a, b, \alpha, \pi$.

**Lemma 1.** The following relation

$$\int_a^b \text{sign}\,[\alpha + g(\omega t)]dt = F(\alpha)(b-a) + O(\frac{1}{\omega}) \quad (1)$$

is satisfied.

Lemma 1 results from the formula for definitions of $F(\alpha)$.

Consider now the propagation of pulse high-frequency oscillations through linear filter (Fig. 3)

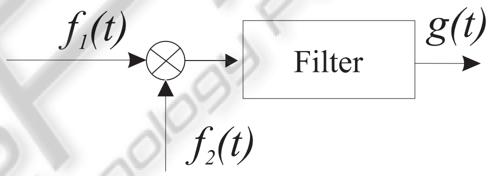


Figure 3: Multiplicator and filter.

Here

$$f_j(t) = A_j \text{sign}\,\sin(\omega_j(t)t + \psi_j), \quad (2)$$

$$g(t) = \alpha(t) + \int_0^t \gamma(t-\tau) f_1(\tau) f_2(\tau) d\tau, \quad (3)$$

$\otimes$ is a multiplicator, $A_j > 0$, $\psi_j$ are certain constants, $j = 1, 2$, $\gamma(t)$ is a impulse response of linear filter and $\alpha(t)$ is an exponentially damped function, linearly depending on initial state of filter at moment $t = 0$.

A high-frequency property of generators can be reformulated as the following condition.

Consider a large fixed time interval $[0, T]$, which can be partitioned into small intervals of the form

$$[\tau, \tau+\delta],\ (\tau \in [0, T]),$$

where the following relations

$$\begin{aligned} &|\gamma(t) - \gamma(\tau)| \le C\delta, \quad |\omega_j(t) - \omega_j(\tau)| \le C\delta, \\ &\forall t \in [\tau, \tau+\delta],\ \forall \tau \in [0, T], \end{aligned} \quad (4)$$

$$|\omega_1(\tau) - \omega_2(\tau)| \le C_1,\ \forall \tau \in [0, T], \quad (5)$$

$$\omega_j(\tau) \ge R,\ \forall \tau \in [0, T], \quad (6)$$

are satisfied.

We shall assume that $\delta$ is small enough relative to the fixed numbers $T, C, C_1$ and $R$ is sufficiently large relative to the number $\delta$ : $R^{-1} = O(\delta^2)$.

The latter means that on small intervals $[\tau, \tau+\delta]$ the functions $\gamma(t)$ and $\omega_j(t)$ are "almost constant" and the functions $f_j(t)$ on them are rapidly oscillating. Obviously, such a condition occurs for high-frequency oscillations.

Consider now $2\pi$-periodic function $\varphi(\theta)$ of the form

$$\varphi(\theta) = A_1 A_2 \left(1 - \frac{2|\theta|}{\pi}\right), \quad \theta \in [-\pi, \pi] \qquad (7)$$

and a block-scheme in Fig. 4

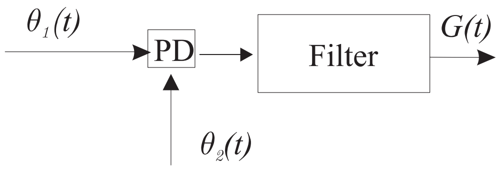


Figure 4: Phase detector and filter.

Here $\theta_j(t) = \omega_j(t)t + \psi_j$ are phases of the oscillations $f_j(t)$, PD is a nonlinear block with the characteristic $\varphi(\theta)$ (being called a phase detector or discriminator) with the output

$$G(t) = \alpha(t) + \int_0^t \gamma(t-\tau)\varphi(\theta_1(\tau) - \theta_2(\tau))d\tau. \qquad (8)$$

**Theorem 1.** *If conditions (4)–(6) are satisfied, then for the same initial states of filter we have*

$$|G(t) - g(t)| \le D\delta, \ \forall t \in [0, T]. \qquad (9)$$

*Here D is a certain not depending on $\delta$ number.*

**Proof.** It is readily seen that

$$g(t) - \alpha(t) = \int_0^t \gamma(t-s) A_1 A_2 \mathrm{sign}\,[\cos((\omega_1(s) -$$
$$-\omega_2(s))s + \psi_1 - \psi_2) - \cos((\omega_1(s) + \omega_2(s))s +$$
$$+\psi_1 + \psi_2)]ds =$$
$$= A_1 A_2 \sum_{k=0}^{m} \gamma(t-k\delta) \Bigg[ \int_{k\delta}^{(k+1)\delta} \mathrm{sign}\,[\cos((\omega_1(k\delta) -$$
$$-\omega_2(k\delta))k\delta + \psi_1 - \psi_2) - cos((\omega_1(k\delta) + \omega_2(k\delta))s +$$
$$+\psi_1 + \psi_2)]ds + O(\delta^2) \Bigg], \ t \in [0, T].$$

Here the number $m$ is such that

$$t \in [m\delta, (m+1)\delta].$$

By Lemma 1 this implies the estimate

$$g(t) = \alpha(t) + A_1 A_2 \Bigg( \sum_{k=0}^{m} \gamma(t-k\delta)\varphi(\theta_1(k\delta) -$$
$$-\theta_2(k\delta))\delta \Bigg) + O(\delta) = G(t) + O(\delta).$$

This relation proves the assertion of Theorem 1.

Consider now a block-scheme of typical phase-locked loop [1–6] (Fig. 5)

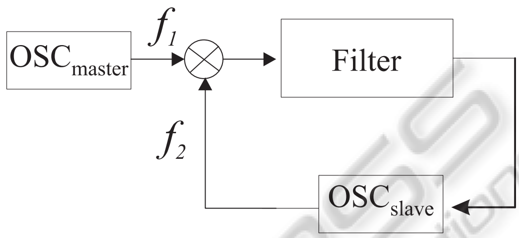


Figure 5: Phase-locked loop with multiplicator.

Here $\mathrm{OSC_{master}}$ is a master oscillator, $\mathrm{OSC_{slave}}$ is a slave (tunable) oscillator and block $\otimes$ is a multiplier of oscillations of $f_1(t)$ and $f_2(t)$.

From Theorem 1 it follows that for pulse generators, at the outputs of which there are produced signals (2), this block-scheme can be asymptotically changed (for high-frequency generators) to a block-scheme on the level of frequency and phase relations (Fig. 6)

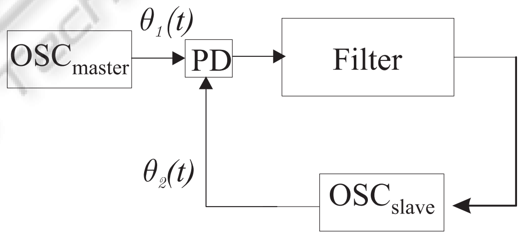


Figure 6: Phase-locked loop with phase detector.

Here PD is a phase detector with characteristic (7).

Thus, here on basis of asymptotical analysis of high-frequency pulse oscillations (Lemma 1 and Theorem 1) a characteristic of phase detector (7) is computed.

We give now a scheme for computing characteristics of phase detector for PLL with squarer. Consider a block-scheme in Fig. 1 with

$$f_1(t) = A_1^2(1 + \mathrm{sign}\sin(\omega_1(t)t + \psi_1))^2$$
$$f_2(t) = A_2 \mathrm{sign}\sin(\omega_2(t)t + \psi_2)$$

Consider then a block-scheme in Fig. 2, where PD is a block with characteristic $F(\theta) = 2A_1\varphi(\theta)$.

**Theorem 2.** *If conditions (4)–(6) are satisfied, then for the same initial states of filter the relation*

$$|G(t) - g(t)| \le D\delta, \quad t \in [0, T]$$

*holds true. Here D is a certain independent of* $\delta$ *number.*
Proof of Theorem 2 is similar to that of Theorem 1.

Finally it may be remarked that for modern processors a transient process time in PLL is less than or equal to 10 sec. and a frequency of clock oscillators attains 10Ghz . Given $\delta = 10^{-4}$ (i.e. partitioning each second into thousand time intervals), we obtain an expedient condition for the proposed here asymptotical computation of phase detectors characteristics:

$$\omega^{-1} = 10^{-10} = 10^{-2}(\delta^2) = O(\delta^2).$$

## 3 CONCLUSIONS

Thus consideration of phase synchronization system at three levels (electronic realizations; phase and frequency relations differential and integro-differential equations) make it possible to calculate the characteristics of the phase detector and perform rigorous mathematical analysis of the stability of the system.

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