A MULTI-ESTIMATION SCHEME FOR CONTROLLING THE BEVERTON-HOLT EQUATION IN ECOLOGY

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Abstract: This paper proposes an adaptive control algorithm to govern the solution of the Beverton-Holt equation under parametrical uncertainties and the potentially presence of additive disturbances. The control strategy is based on a multi-estimation scheme with a supervisor choosing on-line the active estimation model used to parameterize the controller. The tracking of a reference sequence with local modifications of the carrying capacity sequence around its nominal values is achieved with such a control strategy.

1 INTRODUCTION

Models based on the Beverton-Holt Equation (BHE) are very common in Ecology for the study of the evolution of species in their habitats, (Barrowman et al., 2003). Such models rely on more general discrete recursive equations proposed in (Stevic, 2010, Elsayed and Iricanin, 2009, Iricanin and Stevic, 2009a, 2009b). The BHE is a nonlinear equation given by (Beverton and Holt, 1957):

$$x_{k+1} = \frac{\mu_k K_k x_k}{K_k + (\mu_k - 1) x_k}, \; k \in \mathbf{N}_0 := \mathbf{N} \cup \{0\} \tag{1}$$

where $\mathbf{N}$ is the set of natural numbers, $x_0 > 0$ the initial species population size, x_k and x_{k+1} the population sizes at time instants kT (*spawning stock)* and (k+1)T (*recruitment*), respectively, with T being the sampling period, and $\mu_k \in \mathbf{R}_0^+ := \mathbf{R}^+ \cup \{0\}$ and $K_k \in \mathbf{R}^+$ the population *intrinsic growth rate* and the *environment carrying capacity* at the time instant kT, respectively, with $\mathbf{R}^+$ being the set of positive real numbers. The intrinsic growth rate sequence $\{\mu_k\}_0^\infty$ is determined by life cycle and demographic properties like species growth rate, survivorship rate and so on. The carrying capacity sequence $\{K_k\}_0^\infty$ is a characteristic of the habitat depending on resources availability, temperature, humidity and so on. Typically, $\mu_k > 1$ and so $\{\mu_k\}_0^\infty$ as $\{K_k\}_0^\infty$ are cyclic sequences as a consequence of periodic fluctuations are common in biological problems. The carrying capacity sequence is susceptible of being locally modified by means of small changes of temperature, humidity and so on around nominal values. This fact can be used to control the species population size in a closed or semi-closed habitat (De la Sen and Alonso-Quesada, 2008, De la Sen and Alonso-Quesada, 2009). Such control strategies take advance of the linearity of the Beverton-Holt inverse equation (BHIE) (Stevic, 2006) so that conventional techniques developed for linear control systems may be used in order to govern the BHIE solution and then also the BHE one. In such works, the controllers are designed for matching a prescribed reference model by the BHE model possibly affected by the presence of additive disturbances. The reference models are another BHE with suitable intrinsic growth rate and carrying capacity sequences. The paper (De la Sen and Alonso-Quesada, 2008) considers the perfect knowledge of the sequences defining the standard BHE while the research in (De la Sen and Alonso-Quesada, 2009) extends the discussion to the adaptive case since the intrinsic growth rate and carrying capacity sequences are partially or fully unknown. In both cases the environment carrying capacity may be locally modified around its reference values to achieve the prescribed behaviour.

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The matching objective by local modifications of the carrying capacity sequence is only available and practical if the BHE to be controlled as well as the BHE used as the reference model are locally deviated from each other. However, such a condition may not be guaranteed, at least in an adaptive control context where some system parameters are unknown. In this sense, the *main contribution* of the present paper lies in the *design of an adaptive control scheme with a set of potential reference models, instead of a unique one, to be matched in order to circumvent such a drawback.* For such a purpose, *the reference models included in the set are suitably chosen such that at least one of them is sufficiently closed to the unknown BHE at any sampling time.* This quality may be guaranteed with the inclusion of *a large number of reference models in the control scheme and a well distribution of them within the BHE parameters space.* An estimation algorithm is associated to each reference model. Such estimators work in parallel and a supervisor activates on-line the estimation algorithm providing the closest estimated model to the unknown BHE at each sampling instant. The closeness is measured by means of the estimation error associated to each algorithm. The supervisor function implies the switching among the estimated models provided by the estimators included in the adaptive control scheme. Then, *a minimum residence time is maintained in operation the active estimated model in order to achieve a good tracking behaviour and the stability of the control system* (De la Sen and Alonso-Quesada, 2006, Narendra and Balakrishnan, 1997). In this way, the adaptive control scheme works with a time-varying reference model, what is compatible in a species population system subject to periodic fluctuations.

2 PROBLEM STATEMENT

The change of variable $s_k = x_k^{-1}$ in (1) leads directly to the BHIE (Stevic, 2006):

$$s_{k+1} = a_k s_k + b_k u_k\,,\ s_0 = x_0^{-1} > 0 \tag{2}$$

where $a_k = \mu_k^{-1}$, $b_k = 1 - a_k$ and $u_k = K_k^{-1}\ \ \forall k \in \mathbf{N}_0$. Note that the inverse carrying capacity can act as a control action. If an additive disturbance sequence $\left\{\eta_k^0\right\}_0^\infty$ exists, one gets a more general version of (2):

$$s_{k+1} = a_k\left(s_k - u_k\right) + u_k + \eta_k^0 \tag{3}$$

The disturbance may include the effects in the solution of parametrical uncertainties, for instance, in the intrinsic growth rate, or effects, like migrations or local migrations which are not taken into account in the standard BHE. The following assumptions are considered related to the BHE:

Assumptions 1.

(i) $1 + \varepsilon_\mu \le \mu_k < \infty$ and $\varepsilon_K \le K_k < \infty\ \ \forall k \in \mathbf{N}_0$ and some $\varepsilon_\mu,\ \varepsilon_K \in \mathbf{R}^+$.

(ii) $\left|\eta_k^0\right| \le \overline{\eta}_k < \infty\ \ \forall k \in \mathbf{N}_0$ with $\left\{\overline{\eta}_k\right\}_0^\infty$ known. ***

Remark 1.

(i) The BHIE is stable and controllable since both $a_k \le \dfrac{1}{1+\varepsilon_\mu} < 1$ and $b_k \ge \dfrac{\varepsilon_\mu}{1+\varepsilon_\mu} > 0\ \ \forall k \in \mathbf{N}_0$ are derived from Assumption 1(i).

(ii) All solutions of the BHE and BHIE are uniformly bounded and positive provided that $x_0 > 0$ if both Assumptions 1 hold (De la Sen and Alonso-Quesada, 2009). ***

Since the control action is the inverse of the environment carrying capacity it is not admitted a large deviation from its nominal values for tracking purposes in a practical situation. That means that the reference model to be matched by the current BHE has to be sufficiently closed to it. Such a reference model might be another BHE as follows:

$$x_{k+1}^* = \frac{\mu_k^* K_k^* x_k^*}{K_k^* + \left(\mu_k^* - 1\right)x_k^*}\ \ \forall k \in \mathbf{N}_0 \tag{4}$$

which defines the suitable solution through the appropriate reference values of the intrinsic growth rate and the environment carrying capacity sequences, $\left\{\mu_k^*\right\}_0^\infty$ and $\left\{K_k^*\right\}_0^\infty$ with $\mu_k^* > 1\ \ \forall k \in \mathbf{N}_0$. Its corresponding reference BHIE is:

$$s_{k+1}^* = a_k^* s_k^* + b_k^* r_k \tag{5}$$

with reference input $r_k = \left(K_k^*\right)^{-1}$, and parameter sequences $a_k^* = \left(\mu_k^*\right)^{-1}$ and $b_k^* = 1 - \left(\mu_k^*\right)^{-1}\ \ \forall k \in \mathbf{N}_0$.

Assume that the carrying capacity $\left\{K_k^*\right\}_0^\infty$ and the intrinsic growth rate $\left\{\mu_k^*\right\}_0^\infty$ sequences of a reference BHE are given together with the sequences $\left\{\delta_k\right\}_0^\infty$ and $\left\{\lambda_k\right\}_0^\infty$, with $\delta_k \in [0,\ 1)$ and $\lambda_k \in \left[0,\ 1/\mu_k^*\right)$ $\forall k \in \mathbf{N}_0$. The following definition and proposition are concerned with the available BHE models for tracking, with a sufficiently small tracking error, a given BHE reference solution by local modifications of the environment carrying capacity (De la Sen and Alonso-Quesada, 2009).

Definition 1. A class $\mathbb{C}_{BHE}\left(K_k^*, \mu_k^*, \delta_k, \lambda_k\right)$ of BHEs exists parameterized by some sequences $\left\{K_k\right\}_0^\infty$ and $\left\{\mu_k\right\}_0^\infty$ such that $K_k \in \left[\frac{1-\delta_k}{1+\lambda_k}K_k^*,\ \frac{1+\delta_k}{1-\lambda_k}K_k^*\right]$ and $\mu_k \in \left[\frac{(1+\lambda_k)\mu_k^*}{1+\lambda_k\mu_k^*},\ \frac{(1-\lambda_k)\mu_k^*}{1-\lambda_k\mu_k^*}\right]$ $\forall k \in \mathbf{N}_0$. ***

Proposition 1. If **(i)** the upper-bound sequence $\left\{\overline{\eta}_k\right\}_0^\infty$ for the absolute value of the additive disturbance $\left\{\eta_k^0\right\}_0^\infty$ of the BHIE associated to a BHE belonging to the class $\mathbb{C}_{BHE}\left(K_k^*, \mu_k^*, \delta_k, \lambda_k\right)$ is such that $\overline{\eta}_k \le \frac{\delta_k\left(\mu_k^*-1\right)}{(1+\delta_k)\mu_k^* K_k^*}$ $\forall k \in \mathbf{N}_0$ and **(ii)** the initial condition s_0 fulfils $s_0 \ge s_0^*\left(1-\gamma_0\right) > 0$ for some monotonically increasing sequence $\left\{\gamma_k\right\}_0^\infty$ with $\gamma_k \in \mathbf{R}^+$, $\gamma_k < 1$ $\forall k \in \mathbf{N}_0$ and such that $\underset{k\in N_0}{\mathrm{Max}}\left\{\frac{1}{1+\delta_k}\left[\delta_k + \left(\frac{\varepsilon_\mu - 2}{\varepsilon_\mu + 2}\right)^2\right]\right\} \le \gamma_0 < 1$, then:

(i) The control law:

$$u_k = \begin{cases} t_k r_k + f_k s_k - \omega_k & \text{if} \quad s_k \le \dfrac{2}{(1+\delta_k)K_k^*} \\ r_k - \omega_k & \text{otherwise} \end{cases} \tag{6}$$

with the parameter sequences given by:

$$t_k = \frac{1-a_k^*}{1-a_k}\ ;\ f_k = 1 - t_k\ ;\ \omega_k = \frac{\overline{\eta}_k}{1-a_k} \tag{7}$$

guarantees $K_k \in \left[\frac{1-\delta_k}{1+\lambda_k}K_k^*,\ \frac{1+\delta_k}{1-\lambda_k}K_k^*\right]$ $\forall k \in \mathbf{N}_0$,

(ii) $s_k \ge s_k^*\left(1-\gamma_k\right)$ $\forall k \in \mathbf{N}_0$, where $\left\{s_k\right\}_0^\infty$ is the solution of the BHIE and $\left\{s_k^*\right\}_0^\infty$ the solution of the inverse of the reference BHE of the class $\mathbb{C}_{BHE}\left(K_k^*, \mu_k^*, \delta_k, \lambda_k\right)$ and

(iii) the BHE solution $\left\{x_k\right\}_0^\infty$ is upper-bounded by the sequence $\left\{\frac{x_k^*}{1-\gamma_k}\right\}_0^\infty$, where $\left\{x_k^*\right\}_0^\infty$ is the solution of the reference BHE of such a class. ***

Remark 2. Proposition 1 implies that given any BHE belonging to an arbitrary class $\mathbb{C}_{BHE}\left(K_k^*, \mu_k^*, \delta_k, \lambda_k\right)$, local modifications of the carrying capacity around the reference sequence $\left\{K_k^*\right\}_0^\infty$ can be used to achieve the control objective. Namely, a sufficiently small tracking error between the solutions of the given BHE and that of the reference one of such a class can be obtained. ***

3 ADAPTIVE CONTROL

An estimation scheme is incorporated to solve the control problem in the case that the intrinsic growth rate sequence $\left\{\mu_k\right\}_0^\infty$ of the BHE (or the sequence $\left\{a_k\right\}_0^\infty$ of the BHIE) is unknown. In the context of adaptive control, the BHIE (3) can be written as:

$$s_{k+1} = a\left(s_k - u_k\right) + u_k + \eta_k \tag{8}$$

for some unknown constant $a = \mu^{-1}$ with $\left\{\eta_k\right\}_0^\infty$ given by:

$$\eta_k := \left(a_k - a\right)\left(s_k - u_k\right) + \eta_k^0 \tag{9}$$

In this way, the nominal parameter a of the BHIE is constant and $\left\{\eta_k\right\}_0^\infty$ incorporates the deviations of the intrinsic growth rate with respect to the unknown constant μ and, possibly, other unstructured disturbance contributions in $\left\{\eta_k^0\right\}_0^\infty$. If $\eta_k \equiv 0$, the resulting particular case of (8) is called the nominal BHIE.

The estimation algorithm provides an estimated BHIE given by:

$$\hat{s}_{k+1} = \hat{a}_k\left(s_k - u_k\right) + u_k \tag{10}$$

where $\hat{a}_k$ denotes the estimate of a at the k-th sample. Moreover, an estimation error given by:

$$e_{k+1} := s_{k+1} - \hat{s}_{k+1} = -\tilde{a}_k\left(s_k - u_k\right) + \eta_k \tag{11}$$

is associated to the estimation algorithm where $\tilde{a}_k := \hat{a}_k - a$ is the parametrical error. Finally, the tracking error between the solution of the BHIE to be controlled an that of the reference model (5) is:

$$\varepsilon_{k+1} := s_{k+1} - s^*_{k+1} = e_{k+1} + \hat{s}_{k+1} - s^*_{k+1} \tag{12}$$

The tracking error depends on the estimation error and the deviation of the estimated model from the reference one. Then, *the use of a multi-estimation scheme and a supervisor choosing the estimation algorithm providing the smallest estimation error*, instead of the use of a unique estimation algorithm, *will improve the tracking objective*. Furthermore, the deviation between the estimated model and the reference one can be sufficiently small if (i) each estimation algorithm is associated to a different BHE reference model defining a class $\mathbb{C}_{BHE}\left(K^*_k, \mu^*_k, \delta_k, \lambda_k\right)$ and (ii) each one includes a parameter projection for guaranteeing the closeness of both corresponding estimated and reference models. In this way, *if the multi-estimation scheme is composed by a large number of reference model/estimation algorithm pairs and the reference models are well distributed within the definition domain of $\{a_k\}_0^\infty$, then at least one of the estimated models will be sufficiently close to the unknown BHE to be controlled for all time*. As a consequence, such an unknown BHE solution will be able to track that of the reference model associated to the estimation algorithm activated by the supervisor by means of locally modification of the environment carrying capacity around its nominal values. Both reasons motivate the use of a multi-estimation scheme with several estimation algorithms working in parallel in the adaptive control scheme. Furthermore, such a scheme makes that the reference model to be tracked is online changed by the supervisor, what is of interest for ecologic system subject to periodic fluctuations.

3.1 Multi-estimation Scheme

A set $S_e := \{1, 2, \ldots, n_e\}$ of estimation algorithms working in parallel is considered. Each one is associated to a different class $\mathbb{C}_{BHE}\left(K^*_k, \mu^*_k, \delta_k, \lambda_k\right)$. All of them use a least-squares algorithm with a parameters projection and a dead-zone. The projection is used to obtain an estimation model belonging to the corresponding class and the dead-zone for dealing with the presence of potentially disturbances affecting to the nominal BHIE. Each algorithm is defined by:

$$\bar{a}^{(i)}_{k+1} = \hat{a}^{(i)}_k + \frac{\sigma^{(i)}_k \left(s_k - u_k\right) e^{(i)}_{k+1}}{1 + \beta^{(i)}_k \left(s_k - u_k\right)^2} \tag{13.a}$$

$$\hat{a}^{(i)}_{k+1} = \mathrm{Proj}\left\{\bar{a}^{(i)}_{k+1}\right\} = \begin{cases} \dfrac{1-\lambda^{(i)}_{k+1}\mu^{*(i)}_k}{\left(1-\lambda^{(i)}_{k+1}\right)\mu^{*(i)}_k} & \text{if} \quad \bar{a}^{(i)}_{k+1} < \dfrac{1-\lambda^{(i)}_{k+1}\mu^{*(i)}_k}{\left(1-\lambda^{(i)}_{k+1}\right)\mu^{*(i)}_k} \\ \dfrac{1+\lambda^{(i)}_{k+1}\mu^{*(i)}_k}{\left(1+\lambda^{(i)}_{k+1}\right)\mu^{*(i)}_k} & \text{if} \quad \bar{a}^{(i)}_{k+1} > \dfrac{1+\lambda^{(i)}_{k+1}\mu^{*(i)}_k}{\left(1+\lambda^{(i)}_{k+1}\right)\mu^{*(i)}_k} \\ \bar{a}^{(i)}_{k+1} & \text{otherwise} \end{cases} \tag{13.b}$$

with $\quad \hat{a}^{(i)}_0 \in \left[\dfrac{1-\lambda^{(i)}_0\mu^{*(i)}_0}{\left(1-\lambda^{(i)}_0\right)\mu^{*(i)}_0}, \dfrac{1+\lambda^{(i)}_0\mu^{*(i)}_0}{\left(1+\lambda^{(i)}_0\right)\mu^{*(i)}_0}\right] \subseteq (0, 1)$

where $e^{(i)}_k = s_k - \hat{s}^{(i)}_k$, $\forall k \in \mathbf{N}_0$ and $\forall i \in S_e$, is the estimation error of the i-th algorithm at the sampling instant kT. The real sequence $\left\{\beta^{(i)}_k\right\}_0^\infty$ is such that $\beta^{(i)}_k > 0 \;\; \forall k \in \mathbf{N}_0$ and $\left\{\sigma^{(i)}_k\right\}_0^\infty$ a relative dead-zone defined as:

$$\sigma^{(i)}_k = \begin{cases} 0 & \text{if} \quad \left|e^{(i)}_{k+1}\right| \le \varsigma^{(i)}\bar{\eta}_k \\ \dfrac{2\beta^{(i)}_k\left(\varsigma^{(i)} - 1 - \varsigma^{(i)}_1\right)}{\varsigma^{(i)}} & \text{if} \quad \left|e^{(i)}_{k+1}\right| > \varsigma^{(i)}\bar{\eta}_k \end{cases} \tag{13.c}$$

for some prefixed real constants $\varsigma^{(i)} > 1$ and $\varsigma^{(i)}_1 \in \left(0, \; \varsigma^{(i)} - 1\right)$ where $\{\bar{\eta}_k\}_0^\infty$ is a known upper-bound for $\{|\eta_k|\}_0^\infty$ [see Assumption 1(ii)].

Such an algorithm meets the following properties (De la Sen and Alonso-Quesada, 2009).

Lemma 1.

(i) The sequence $\left\{\hat{a}^{(i)}_k\right\}_0^\infty$ is bounded and converges asymptotically to a finite $\hat{a}^{(i)}_\infty$,

(ii) The sequences $\left\{\left(\dfrac{\sigma^{(i)}_k}{1+\beta^{(i)}_k\left(s_k - u_k\right)^2}\right)^{1/2} \left|e^{(i)}_{k+1}\right|\right\}_0^\infty$ and $\left\{\left(\dfrac{\sigma^{(i)}_k}{1+\beta^{(i)}_k\left(s_k - u_k\right)^2}\right)^{1/2} \bar{\eta}_k\right\}_0^\infty$ are bounded and both tend asymptotically to zero. ***

3.2 Supervisory System

This element chooses on-line one of the estimation algorithms which compose the multi-estimation scheme, namely, that closest to the unknown BHE. For such a purpose, a cost function given by:

$$F_k^{(i)} = \sum_{j=0}^{k} \rho^{k-j} \left(e_j^{(i)}\right)^2 \tag{14}$$

with the forgetting factor $\rho \in (0,1) \cap \mathbf{R}^+$, is evaluated by the supervisor $\forall k \in \mathbf{N}_0$ and $\forall i \in S_e$ and the estimation algorithm minimizing such a function is activated. Furthermore, the supervisor maintains activated such an algorithm at least a minimum number N_{min} of sampling periods before switching to a different one. This residence time prevents against the instability of the control system caused by an eventual great amount of switches concentrated in a short time interval. Then, the switching law is given by:

$$c_k = \begin{cases} c_{k-1} & \text{if} \quad k - k' < N_{min} \\ \ell & \text{otherwise} \end{cases} \tag{15}$$

where $k'T$ is the sampling instant at which the last switching occurred before the current time instant kT and $\ell := \mathrm{Min}\left\{ i \in S_e \;\middle|\; F_k^{(i)} \le F_k^{(j)} \;\forall i, j \in S_e \right\}$.

3.3 Adaptive Control Law

An adaptive control law with the same structure as (6)-(7) by replacing the true parameter a_k by its estimate $\hat{a}_k^{(c_k)}$ and by deleting the correcting signal for disturbances ω_k is used to generate the suitable value for the carrying capacity sequence at each sampling time. The super-index (c_k) denotes the estimation algorithm which is maintained active by the supervisor at the current sampling instant kT. The control term relative to the disturbances is omitted since such disturbances are treated by the inclusion of the dead-zone in each estimation algorithm. Such a control law is:

$$u_k = \begin{cases} t_k \left(r_k - s_k\right) + s_k & \text{if} \quad s_k \le \dfrac{2}{\left(1 + \delta_k^{(c_k)}\right) K_k^{*(c_k)}} \\ \left(K_k^{*(c_k)}\right)^{-1} & \text{otherwise} \end{cases} \tag{16}$$

where $\left\{K_k^{*(c_k)}\right\}_0^\infty$ is the carrying capacity sequence of the active reference model at the current sampling instant and the sample-dependent controller parameter is given by:

$$t_k = \frac{1 - a^{*(c_k)}}{1 - \hat{a}_k^{(c_k)}} \tag{17}$$

i.e., the control is parameterized from the active estimated model at the current sampling instant.

3.4 Stability Analysis

The following additional assumption has to be considered for proving the closed-loop stability.

Assumption 2. There exist known finite nonnegative real constants ϑ_1 and ϑ_2 such that $\overline{\eta}_k \le \vartheta_1 + \vartheta_2 \underset{0 \le j \le k}{\mathrm{Max}} \left\{ s_j \right\}$. ***

Remark 3. This assumption implies a slow growing of the unknown disturbances with respect to the solution of the BHIE. This is a reasonable assumption used in adaptive control theory since a complete lack of knowledge of disturbances makes impossible the stabilization in the general case (De la Sen and Alonso-Quesada, 2006, Feng, 1999).

The control system stability is based on the following features: **(i)** the adaptive control law (16) with any of the estimation algorithms maintained active by the supervisor for all time stabilizes the control system while achieving a sufficiently small tracking error if the unknown BHE is closed to the reference BHE model corresponding to such an estimation algorithm for all time by means of locally modifications of the environment carrying capacity (De la Sen and Alonso-Quesada, 2009), and **(ii)** the switching law in the supervisory system guarantees a minimum residence time in the active estimation algorithm, which is crucial to avoid instability caused by an eventual high concentration of switches in a short time interval (Narendra and Balakrishnan, 1997). In summary, the switching law allows to change the reference model to be tracked by the current BHE to ensure the closeness between such a BHE and the active reference one.

4 NUMERICAL EXAMPLE

A BHE (1) defined by an unknown intrinsic growth rate sequence $\left\{\mu_k\right\}_0^\infty$, which is given by:

$$\mu_k = \begin{cases} 1.65 & \text{if} \quad 360\cdot j \le k < 360\cdot j+29 \\ 1.6 & \text{if} \quad 360\cdot j+30 \le k < 360\cdot j+59 \\ 1.65 & \text{if} \quad 360\cdot j+60 \le k < 360\cdot j+89 \\ 1.75 & \text{if} \quad 360\cdot j+90 \le k < 360\cdot j+119 \\ 1.85 & \text{if} \quad 360\cdot j+120 \le k < 360\cdot j+149 \\ 1.95 & \text{if} \quad 360\cdot j+150 \le k < 360\cdot j+179 \\ 2 & \text{if} \quad 360\cdot j+180 \le k < 360\cdot j+209 \\ 1.95 & \text{if} \quad 360\cdot j+210 \le k < 360\cdot j+239 \\ 1.85 & \text{if} \quad 360\cdot j+240 \le k < 360\cdot j+269 \\ 1.8 & \text{if} \quad 360\cdot j+270 \le k < 360\cdot j+299 \\ 1.75 & \text{if} \quad 360\cdot j+300 \le k < 360\cdot j+329 \\ 1.7 & \text{if} \quad 360\cdot j+330 \le k < 360\cdot j+359 \end{cases} \quad (18)$$

and a known nominal environment carrying capacity sequence $\left\{K_k^{nom}\right\}_0^\infty$, given by:

$$K_k^{nom} = \begin{cases} 185 & \text{if} \quad 360\cdot j \le k < 360\cdot j+29 \\ 180 & \text{if} \quad 360\cdot j+30 \le k < 360\cdot j+59 \\ 185 & \text{if} \quad 360\cdot j+60 \le k < 360\cdot j+89 \\ 200 & \text{if} \quad 360\cdot j+90 \le k < 360\cdot j+119 \\ 205 & \text{if} \quad 360\cdot j+120 \le k < 360\cdot j+149 \\ 210 & \text{if} \quad 360\cdot j+150 \le k < 360\cdot j+179 \\ 220 & \text{if} \quad 360\cdot j+180 \le k < 360\cdot j+209 \\ 215 & \text{if} \quad 360\cdot j+210 \le k < 360\cdot j+239 \\ 210 & \text{if} \quad 360\cdot j+240 \le k < 360\cdot j+269 \\ 200 & \text{if} \quad 360\cdot j+270 \le k < 360\cdot j+299 \\ 190 & \text{if} \quad 360\cdot j+300 \le k < 360\cdot j+329 \\ 185 & \text{if} \quad 360\cdot j+330 \le k < 360\cdot j+359 \end{cases} \quad (19)$$

$\forall k, j \in \mathbf{N}_0$, with $T = 1$ day as sampling period, is considered. i.e., both are piecewise constant periodic sequences with period equal to 1 year. Note that each line of (18)-(19) corresponds to values of the sequences during a month approximately. The nominal carrying capacity sequence is susceptible of locally modifications in order to achieve the control objective. Such a *control objective* is that the BHE solution $\left\{x_k\right\}_0^\infty$ tracks a suitably chosen close reference sequence $\left\{x_k^{*(c_k)}\right\}_0^\infty$ with a sufficiently small tracking error. The reference sequence is chosen on-line by a supervisor among four potential sequences $\left\{x_k^{*(i)}\right\}_0^\infty$, for $i \in S_e := \{1, 2, 3, 4\}$, each one issued by the BHE reference model defining a different class $\mathbb{C}_{BHE}^{(i)}\left(K_k^{*(i)}, \mu_k^{*(i)}, \delta_k^{(i)}, \lambda_k^{(i)}\right)$. Such reference models have been chosen so that at least one of them be sufficiently close to the unknown BHE at each sampling time. The four classes used in the example are defined by the same carrying capacity $\left\{K_k^{*(i)}\right\}_0^\infty = \left\{K_k^{nom}\right\}_0^\infty$, $\left\{\delta_k^{(i)}\right\}_0^\infty$ and $\left\{\lambda_k^{(i)}\right\}_0^\infty$ sequences with $\delta_k^{(i)} = 0.0421$ and $\lambda_k^{(i)} = 0.0526$ $\forall k \in \mathbf{N}_0$ and $\forall i \in S_e$ and different sequences $\left\{\mu_k^{*(i)}\right\}_0^\infty$ for the reference intrinsic growth rates, namely, $\mu_k^{*(1)} = 1.55$, $\mu_k^{*(2)} = 1.95$, $\mu_k^{*(3)} = 2.05$ and $\mu_k^{*(4)} = 1.75$ $\forall k \in \mathbf{N}_0$.

The unknown BHE to be controlled is associated to the BHIE given by (8)-(9) with an unknown parameter $a = 0.6$, which would correspond to a constant intrinsic growth rate $\mu = 1.6667$. Such a parameter has to be estimated to parameterize the adaptive control law (16) by using (14), (15) and (17). For such a purpose, four estimation algorithms working in parallel are included in the multi-estimation scheme, each one associated with each potential reference sequence $\left\{x_k^{*(i)}\right\}_0^\infty$ with $i \in S_e$. Each algorithm is defined by (13) with the same sequence $\left\{\beta_k^{(i)}\right\}_0^\infty$, namely $\beta_k^{(i)} = 10^{10}$ $\forall k \in \mathbf{N}_0$, and the same parameters $\varsigma^{(i)} = 1.011$ and $\varsigma_1^{(i)} = 0.01$ for all of them, i.e. $\forall i \in S_e$. Moreover, the constants $\vartheta_1 = 2\times 10^{-5}$ and $\vartheta_2 = 10^{-6}$ are used to build the upper-bound of the contribution of the unmodeled dynamics. The estimation algorithms are, respectively, initialized with $\hat{a}_0^{(1)} = 0.66$, $\hat{a}_0^{(2)} = 0.49$, $\hat{a}_0^{(3)} = 0.46$ and $\hat{a}_0^{(4)} = 0.55$. Note that each estimated model is initialized within its corresponding class and they cannot leave from them due to the projection included in each estimation algorithm.

The initial population of the species is $x_0 = 300$ and that of the reference sequence $x_0^{*(c_0)} = 300$ with $c_0 = 1$ being the initialization for the switching law of the supervisor. The results obtained with the adaptive control system with the multi-estimation scheme are displayed in the following figures.

Figures 1 and 2 show the time evolution of the population size, active reference model solution and tracking error sequences in a year approximately. An acceptable tracking of the active reference by the supervisor can be observed from such figures. Figure 3 shows that local modifications of the inverse of the environment carrying capacity are sufficient to achieve such a tracking performance. Note that the control sequence is within the domain delimited by the lower and upper bounds associated

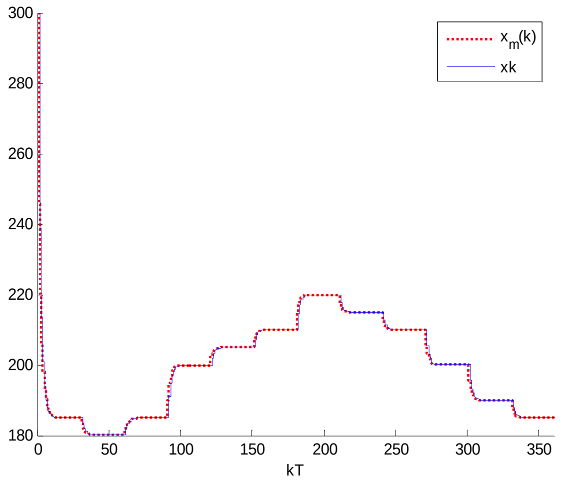

Figure 1: Evolution of the population size and the reference sequence activated by the supervisor.

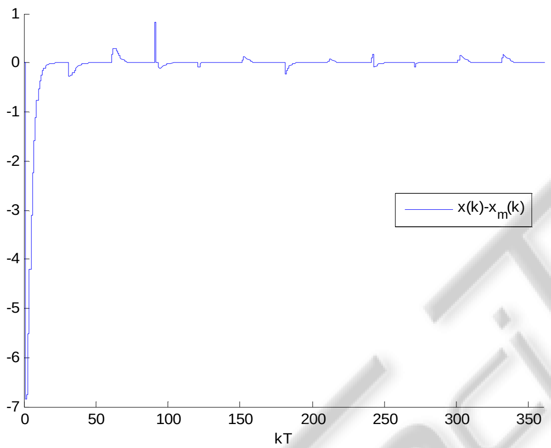

Figure 2: Evolution of the tracking error.

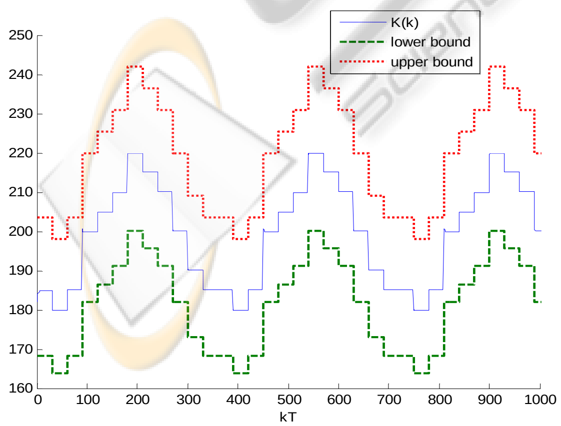

Figure 3: Evolution of the control sequence (inverse of the environment carrying capacity).

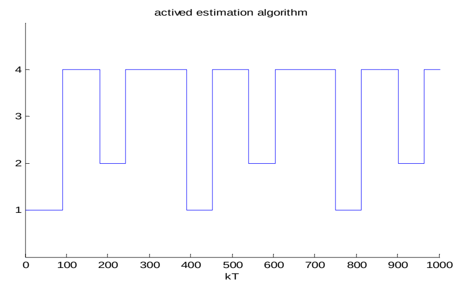

Figure 4: Estimation algorithm/reference model pair activated by the supervisor.

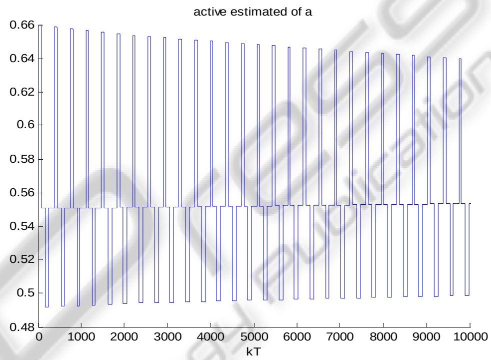

Figure 5: Evolution of the estimated of the active algorithm.

to local modifications around nominal values of the carrying capacity. Figure 4 displays the estimation algorithm which is online activated by the supervisor during the simulation. The active algorithm is changed by the supervisor several times during a year, which is reasonable due to the periodic fluctuations in the species intrinsic growth rate. Figure 5 displays the time evolution of the estimated of the unknown parameter corresponding to the active estimation algorithm.

Finally, the performance indexes given by $J_i(k)=\sum_{j=0}^{k}\left(x_j^{(i)}-x_j^{*(i)}\right)^2 \quad \forall k\in\mathbf{N}_0$ and $\forall i\in S_e$, if an adaptive control algorithm with a unique estimation algorithm (without supervisor) is used, or given by $J_m(k)=\sum_{j=0}^{k}\left(x_j^{(c_j)}-x_j^{*(c_j)}\right)^2$ if the multi-estimation scheme is used, are considered in order to compare the tracking performance of the developed multi-estimation scheme with the tracking results obtained with any of the single estimations algorithms working alone. Both indexes are measures of the tracking error accumulated during the simulation. Figure 6 below displays the performance indexes corresponding to the four simulations with the single estimation algorithms and the simulation with the multi-estimation scheme incorporating the supervisor. Note that the best behaviour is obtained with the multi-estimation scheme, what motivates the use of the adaptive control strategy developed in this paper.

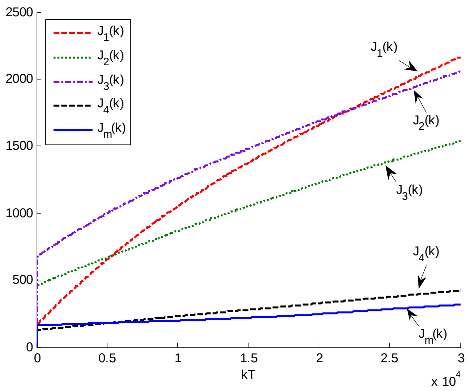

Figure 6: Tracking performances indexes.

5 CONCLUSIONS

BHE models are commonly used in Ecology to describe the time evolution of species populations in their habitats. Actually these models are subject to parametrical uncertainties what motivates the use of adaptive control techniques for such a purpose. The design of an adaptive control system with a multi-estimation scheme to achieve the solution of the BHE tracks a desired reference signal has been developed. The proposed use of a multi-estimation scheme instead of a single estimation one is due to two reasons, mainly. On one hand, Ecology systems are usually time-varying in the sense that their parameters suffer periodic fluctuations. On the other hand, the signal used as control is the inverse of the carrying capacity sequence, which depends on the habitat characteristics. Then, locally modifications of such a sequence around their nominal values are only available to control the BHE solution. This constraint makes that a suitable tracking performance is only guaranteed if the BHE and the reference model are locally deviated from each other. Then, a set of potential reference models, each one associated to an estimation algorithm, instead of a unique one improves the tracking behavior as it has been illustrated by some simulation results.

Future research will extend these adaptive control techniques to other Ecological systems as, for example, epidemic propagation models.

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