

EMBEDDING HIERACHICAL DEFORMATION WITHIN A REALTIME SCENE GRAPH

A Simple Approach for Embedding GPU-based Realtime Deformations using Trilinear Transformations Embedded in a Scene Graph

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Abstract: Scene graphs are widely used as a description of spatial relations between objects in a scene. Current scene graphs use linear transformations for this purpose. This limits the relation of two objects in the hierarchy to *simple* transformations like sheer, translation, rotation and scaling. In contrast to this, we want to represent and control deformations that result from propagating the *dynamics* of objects to *deformable attached objects*. Our solution is to replace the linear 4x4 matrix-based transformation of a scene graph by a more generic trilinear transformation. The linear transformation allows the composition of the transformation hierarchy into one transformation. Our approach additionally allows the handling of deformations on the same level. Building on this concept we present a system capable of real-time rendering. The computations of the applied deformations of the scene graph are performed in *real-time* on the GPU. We allow the approximation of *arbitrary nonlinear transformations and deformations* by utilising grids of trilinear transformations in our system. As an application we show geometric attachments on deformable objects and their deformation on a scene graph level.

1 INTRODUCTION

In complex scenes it is useful to organise the scene elements in a hierarchy to achieve a structured scene management. These scene graphs bundle the objects in groups and assign spatial relations between them. This allows a simple approach for the grouping and construction of larger building blocks, which are represented by linear transformations. While grouping relates objects *spatially* to each other, it is sometimes necessary to attach an object to the surface of another object. For this problem the grouping mechanism of the graph is sufficient, *as long the surface of the object does not change*. An example of such a scenario is the attachment of accessories on a piece of garment or an animated 3D avatar or object in a computer game. In this cases it is necessary to update the accessories' transformation in respect to the surface of the animated object.

If the accessory has a *static* nature (for example a button) this problem can be still solved using a linear transformation system. However, it is often necessary to attach elements on several points on a *deforming* object. Several examples can be shown when modelling garments or in computer games: Accessories on an animated character or garment, objects following the curvature of a landscape, etc. For solving the deformation problem itself there exist a lot of techniques that can handle this problem (Chen et al., 2005). However, there is still the problem of having a hierarchy of deformations, created by grouping of objects within the scene graph. This hierarchy is problematic, since several deformations need to be applied. It is necessary to flatten this hierarchy into one deformation per Object instance in order to draw it efficiently. This motivates us to use a *deformation* system, which can be *concatenated* over the hierarchy in the same sense as it is performed with the linear

transformations within a standard scene graph. Additionally, we propose to choose a deformation system capable of simulating linear transformations as well. This allows the scene graphs' linear transformation system to be replaced by the more general one. Due to its simplicity, we have chosen trilinear transformations for this replacement.

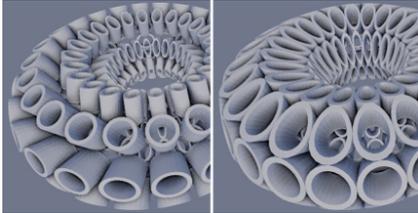


Figure 1: Wrapping a 2D grid of tubes around a torus using the scene graph's transformation capabilities only: Using linear transformations this results in single tubes sticking out from the torus. With our approach the tubes stay in contact with each other as they did in the 2D grid.

The used transformation describes a warping of the 3D space. So any kind of geometrical structure can be used in conjunction with this transformation type.

There are two benefits when using this approach. First, the whole process of a deformation is simplified, since the scene graph is now able to handle the deformation itself without the need of external structures. Second, it allows simple GPU based implementations, while the handling of the transformation system itself stays similar to matrix based systems. Using GPU based transformation, geometry can be directly deformed during rendering, removing the necessity to produce and store intermediate deformation results, see Figure 1.

Our main contribution is an approach for embedding a *deformation* into a scene graph system by replacing the linear 4x4 matrix based transformation of a scene graph by a more *generic trilinear* transformation. Just as linear transformations can be combined through composition, trilinear transformations can be composed to allow hierarchical transformations. Additionally, we allow assembling several transformations into a grid for the approximation of *arbitrary non-linear transformations*. While the composition of transformations is performed inside the CPU during scene graph traversal, all geometric transformations are computed on the GPU. The composition allows the GPU to transform the vertices of the geometry in *constant time*. This is independent of the depth and complexity of the transformation hierarchy attached to it. It is independent of the number of applied deformations over the hierarchy. This is shown in Figure 2.

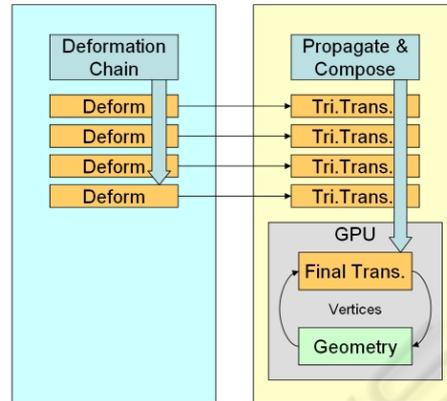


Figure 2: Our approach: Instead of transforming each geometry vertex in each deformation node, we first approximate the arbitrary non-linear transformations with our trilinear transformation system (left box to right box). During the scene graph traversal we can now use propagation and composition of the trilinear transformations. This leads to one combined transformation used within the vertex transformation stage inside the GPU (right box).

In the next section we review and describe basic concepts for deformation and scene graphs. Then we refer to the concept behind trilinear transformations and we describe our usage of this kind of transformation. In the implementation section we present the composition mechanism we use in the transformation process of geometry and normals. As an application we present the use of this transformation concept to handle geometric attachments on the surface of deformable geometries. The results section discusses the abilities and results of the presented approach and is followed by conclusions and future work.

2 BACKGROUND

2.1 Scene Graphs

Graph structures deal efficiently with hierarchical relations of objects within scenes. These days, scene graphs are widely used within graphic applications. Several systems and application programming interfaces (APIs) like X3D, Open Inventor (Wang et al., 1997), OpenGL Performer (Rohlf and Helman, 1994), Java3D (Sowizral et al., 1998), OpenSG (Reiners et al., 2002), Open Scene Graph or the NVIDIA NVSG provide scene graph based scene management functionality. Being powerful toolkits, scene management and the rendering subsystem are often mixed and difficult to exchange. To circumvent this, (Rubinstein et al., 2009) present a scene graph system

which is especially designed to fit to different rendering methods. This is done by allowing the rendering process to use a retained mode, to start the rendering rendering process itself after scene traversal.

2.2 Transformation and Deformation

Candidates for replacing the linear transformation system of a scene graph are presented in (Gomes et al., 1998). The authors give a survey over different transformation techniques with a focus on warping and morphing techniques. The presented 2D techniques can be easily extended to 3D. An overview of existing deformation and animation techniques for 3D objects is given in (Chen et al., 2005). Physical simulation often uses deformation techniques to apply the simulation result to a target mesh from a lower resolution control mechanism (Nealen et al., 2005)

An early deformation technique with a focus on proper handling of the surface normals can be found in (Barr, 1984). The author presents a group of deformation methods, which additionally allow the computations of proper deformations of the normal. The nesting of several subsequent deformations is presented in (Raviv and Elber, 1999) with a focus on freeform sculpting and modelling. Free Form Deformations (Sederberg and Parry, 1986) allow an intuitive way to manipulate objects with deformation using a control grid. Both methods use local and global deformations to create level of detail mechanisms for modifying an object.

Nowadays, deformation topics have moved from definition and structuring to a more animation and modelling related view. A number of different approaches have been proposed to control the deformation of high polygon models. In (Sumner et al., 2007) a handle-based approach for manipulating high polygon models is presented. In (Eigensatz and Pauly, 2009) the authors present a different deformation method based on the manipulation of parts of the surface's properties. In (Langer and Seidel, 2008) the authors propose a deformation method, which extends the concept of barycentric coordinates in order to achieve smooth transition between the deformation elements. In (Botsch et al., 2007) the authors present a method which is based on elastic coupling of cells to achieve a smooth transition between user constraints. In order to generate skin deformation on 3D characters, user specified chunks are deformed by using a finite element method to create realistic looking deformations of the mesh (Guo and Wong, 2005).

A highly efficient GPU-based approach for creating wrinkles on textile materials using deformation was proposed by (Loviscach, 2006). (Popa et al.,

2009) use deformations as a tool to model wrinkles of garments, which have been captured from video frames resulting in highly detailed 3D capture result.

2.3 Deformation Embedded Within the Scene Graph

The presented methods show scene handling and the use cases for deformation and deformation processes. As the presented scene graph systems do not take into account deformation as a low level transformation process, the aforementioned publications dealing with deformation focus mainly on animation and modelling aspects. Even though the idea of using a hierarchy of deformations is not new (Sederberg and Parry, 1986) (Raviv and Elber, 1999), the previous work leave out the possibility of handling hierarchical deformations in conjunction with object placing and grouping in 3D scenes for real time applications on a scene graph level. Our scenario requires an evaluation of all transformations of the whole scene per frame. The nonlinearity of the transformation adds the functionality to not only group objects, but to apply these groups to curvatures, greatly increasing the scene graph's functionality. In the next section we show our approach to embed deformation into the scene graph of a real time rendering system.

3 EMBEDDING TRILINEAR TRANSFORMATIONS

In this section we describe the trilinear transformation and how we use it as replacement of the linear transformation within a scene graph.

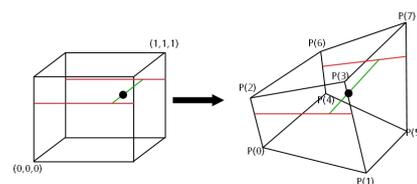


Figure 3: Trilinear transformation: a point within a unit cube is transformed by using its three coordinates as coefficients for a trilinear interpolation between the eight corners of the cuboid.

3.1 Trilinear Transformations

As described in (Gomes et al., 1998) the trilinear transformation is an extended version of the bilinear transformation. It is a function, mapping points of R^3 to R^3 defined by 8 points forming a cuboid, see Figure 3.

In contrast to linear transformations each corner point of the cuboid defines its own coordinate system. It is created by the three adjacent corner points. A hierarchy of these transformations describes a hierarchy of local subspaces within a world space. This is completely different to a linear system, which would describe a hierarchy of local coordinate systems within a world coordinate system.

3.2 Vertex Transformations

In general there are two methods to perform the trilinear transformation of a point in space. The first one uses the coordinates of the cuboid directly to transform vertices, the second one uses a polynomial representation allowing fast transformation of many vertices. Additionally, the necessary coefficients allow a simple test, whether the transformation represents a parallelepiped or a real cuboid.

The first methods uses the corner points $\vec{p}_0 \dots \vec{p}_7$ of the cuboid (see Figure 3) and the transformation T, defined as

$$T(x,y,z) = \begin{pmatrix} \vec{p}_0 \\ \vec{p}_1 \\ \vec{p}_2 \\ \vec{p}_3 \\ \vec{p}_4 \\ \vec{p}_5 \\ \vec{p}_6 \\ \vec{p}_7 \end{pmatrix}^T * \begin{pmatrix} (1-x)(1-y)(1-z) \\ x(1-y)(1-z) \\ (1-x)y(1-z) \\ xy(1-z) \\ (1-x)(1-y)z \\ x(1-y)z \\ (1-x)yz \\ xyz \end{pmatrix} \quad (1)$$

Here x,y and z are the coordinates of a point to transform in the range $[0..1]$. This equation can be used to directly transform a point. A nice feature of this approach is the direct usage of the image (cuboid) of the transformation.

However, with multiplication and sorting by x,y,z we get a second transformation process using a *polynomial representation*, where a coefficient matrix C is created from the corner points of the cuboid:

$$C = \begin{pmatrix} \vec{p}_7 - \vec{p}_6 - \vec{p}_5 + \vec{p}_4 - \vec{p}_3 + \vec{p}_2 + \vec{p}_1 - \vec{p}_0 \\ \vec{p}_6 - \vec{p}_4 - \vec{p}_2 + \vec{p}_0 \\ \vec{p}_5 - \vec{p}_4 - \vec{p}_1 + \vec{p}_0 \\ \vec{p}_3 - \vec{p}_2 - \vec{p}_1 + \vec{p}_0 \\ \vec{p}_4 - \vec{p}_0 \\ \vec{p}_2 - \vec{p}_0 \\ \vec{p}_1 - \vec{p}_0 \\ \vec{p}_0 \end{pmatrix}^T \quad (2)$$

Additionally, a parameter vector \vec{v} is built from the coordinates of the point in space which we want to transform:

$$\vec{v} = (xyz, yz, xz, xy, z, y, x, 1) \quad (3)$$

The transformation is now performed by multiplying the vector \vec{v} with the matrix C

$$\vec{v}' = C * \vec{v} \quad (4)$$

Since matrix C is valid for all points to be transformed, the only computations be to be done are the construction of \vec{v} and its multiplication with C . We describe linear transformations as a special case of the trilinear transformation by keeping the coordinate systems constant over the volume defined by the cuboid. This is the case, if the cuboid represents a parallelepiped. This allows a detection of linear cases after propagation and composition, see Figure 4.

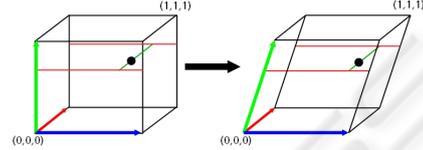


Figure 4: Warping of a unit cube using linear transformation results in a parallelepiped. This is a special case of a cuboid.

Taking a closer look at the matrix C and the generation of coefficients two things are clear. The right part of C describes a linear 4×3 matrix. It contains a coordinate system plus an offset. The left side of C consists of 4 vectors describing the difference between the parallelepiped defined by the right 4×3 matrix and the intended cuboid. If the transformation describes a linear transformation this left side of C contains only zeros. This knowledge allows one to test whether a trilinear transformation is truly a deformation or just a linear transformation. This is a nice feature, since this test can be performed after composing the overall transformation for a geometry directly before rendering. This test allows one to limit the higher computational effort necessary for deformation to the objects really needing deformation without the need of an extra protocol.

Both methods describe a warping of the whole space. This allows handling any geometric structure which can be represented in that space.

3.3 Approximating Arbitrary Deformations and Composition

Until now we discussed simple trilinear transformations using only a single cuboid. We will call this type simple transformation. In order to cope with real deformations we need a more flexible tool. Fortunately, trilinear transformations have a local character and can be attached side by side to form control grids. These grid structures can be used to approximate complex arbitrary deformations, like it is proposed by (Rezk-Salama et al., 2001). Additional refinements of the grid are presented to allow local details of the deformation. Since our aim is to directly

use 3D textures of the GPU to store these grids, we decided to use simple uniform grids. Additionally, this allows us to use trilinear interpolation of the texture stage to perform the necessary interpolation within a single cuboid of the grid.

Since we intend to combine assembled transformations with simple transformations, we have to take care of the following sampling issue: transforming the cuboid of a transformation is a sampling process that will only approximate the original transformation. If this is another simple trilinear transformation, this is no problem. A problem arises when an assembled transformation is sampled by one with lower frequency. This will lead to under-sampling. We prevent this problem by propagating the transformation with the highest resolution.

4 IMPLEMENTATION

We additionally use bounding boxes to map parts of the scene to unit cubes. These additional bounding boxes are defined for each non-linear transformation. All content of the bounding box is warped into the cuboid. So in addition to the transformation hierarchy of the scene graph there is a bounding box hierarchy. The images of the child transformations have to be contained inside the parent's bounding box.

4.1 The Transformation Process

In order to create propagation and composition we use one transformation (\mathcal{G}) to warp the other (\mathcal{L}). In principle the composition (\mathcal{C}) of two trilinear transformations $\mathcal{C} = \mathcal{G} * \mathcal{L}$ is computed by transforming the positions of \mathcal{L} 's cuboid grid by the transformation \mathcal{G} .

A problem arises from this composition. Our goal is to compose two trilinear transformations into a new one. Since composing a child nodes transformation with the parent nodes transformation does not result mathematically in a new trilinear transformation. This happens due to the ability of the trilinear transformation to be able to map lines to curves, which is always the case, if the image cuboid does not resemble a parallelepiped. To circumvent this problem we use the child's transformation cuboid to approximate the deformation represented by the parent's cuboid. Besides from introducing an approximation error this allows us to get one final composed transformation at a geometry leaf, which is trilinear, and has to be applied to the geometry in the vertex processing stage.

Looking at the transformation hierarchy inside a scene graph we now perform the propagation and

composition process the following way: We propagate the transformation from the world space towards the local space of the geometry. At each step we have to transform a more local transformation (\mathcal{L}) with the composition of the more global transformations (\mathcal{G}). The new composed transformation (\mathcal{C}) is now given by transforming the grid data of the local transformation \mathcal{L} by the old composed transformation \mathcal{G} . As a result the new composition transformation is a transformed copy of \mathcal{L} .

As described in section 3.3 this composition of these transformations resembles a sampling process. To prevent loss of information we have to choose a sampling grid for \mathcal{L} being equal or finer in structure of the cells than the grid performing the transformation. Otherwise we have to handle this problem by increasing the resolution of \mathcal{L} 's grid, e.g. by resampling. We chose the grid size in the following way: Since we use a bounding box hierarchy, \mathcal{L} and \mathcal{G} having the same resolution will automatically result in a proper sampling of \mathcal{G} since \mathcal{L} is smaller in size. Otherwise we check which transformation uses the highest resolution and use its grid resolution for computing \mathcal{C} . On the GPU we use interpolated 3D textures to represent the transformation grids. Since a 3D texture already has a domain defined in a unit cube, it is only necessary to perform the mapping from the bounding box to a unit cube in advance. Positions are represented by float values instead of colours in the texture.

We represent linear transformations only by using transformations with a grid size of one (one cuboid). As an optimisation this allows us to check whether the transformation represents a parallelepiped. If the final transformation contains a larger grid we always consider it to handle a non-linear transformation.

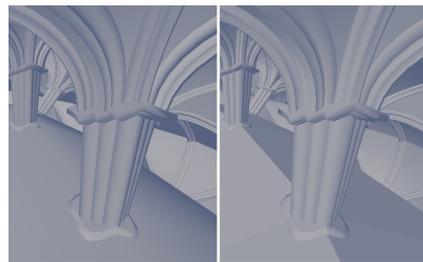


Figure 5: For rendering the lighting of the scene it is necessary to handle the deformation of the geometry's surface normals. We use two methods. Method 1 (left) simply transforms the normal with the given transformation. The resulting normal is not necessarily orthogonal to the surface, creating a smooth normal transition at the cuboid's boundary. Method 2 (right) shows the results transforming the surface's tangent space. Since the normal is now orthogonal to the surface, visible seams at the cuboid's border are noticeable.

4.2 Normal Transformations

Since groups of trilinear transformations are of C_0 continuity at their borders, special care have to be taken when computing the normals for lighting the contained objects (Rezk-Salama et al., 2001). There are several methods for computing the normals used for shading the surfaces (see (Akenine-Möller et al., 2008)). For our implementation we have chosen two computation methods. Using the first method, we transform the tangent space representation of the normal. This results in normals orthogonal to the deformed surface. With the second method, we transform the normal directly. This results in a smooth lighting transition between cuboids. In both methods, transformations have to be performed with respect to the position of the vertex the normal or tangent space belongs to. This has to be done due to the position dependency of the coordinate system in a non parallelepiped cuboid. Both methods are valid and can be used with this approach (see Figure 5).

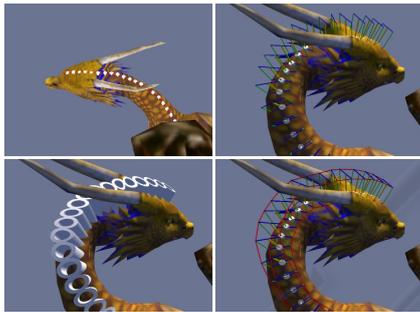


Figure 6: Automated handling of attached geometry in our method: At first we place a set of anchor points on the figurine (top left). Barycentric coordinates allow updating these points if the figurine is deformed or animated. These points represent local coordinate system (top right). These coordinate systems are used to create a deformation grid on the fly (bottom right). The control grid enables the scene graph to deform the attached geometry (bottom left).

5 APPLICATION EXAMPLE

In order to attach geometry to another surface we need additional control structures, keeping the deformation nodes up to date if the surface position changes, see Figure 6. An attachment consists of a list of anchors placed on the surface of the target's geometry, a transformation controlled by the anchors, and the geometry object which has to be attached. An anchor is defined by two points, which are registered and mapped onto a triangle of the basic mesh using barycentric coordinates. The first point resembles the anchor position. The second point is used to specify the orien-

tation of the tangent vector. From these two points and the triangle's surface normal a coordinate system is computed. The transformation has to compute a trilinear transformation for this list of anchors, depending on which behaviour the attachment has to have. An update of the anchors local coordinates system is performed by barycentric interpolation of the points of the triangle. This allows the computation of anchor position and alignment by only using the information of the geometry, see Figure 6. This concept is independent of the animation concept, which is used on the basic mesh, the anchor is applied to. The coordinate system defined by an anchor resides within the local space of the geometry. Thus the transformations dealing with the attachments have to work within the same space. In our example we use arrays of anchors to produce a 1D or 2D deformation grids used by the cuboids.

6 EXPERIMENTAL RESULTS

We have tested the system with several scenes (see Figure 7). We focused on two aspects of our implementation. Besides measuring the speed difference between trilinear transformations and linear transformations we had to differentiate between the GPU and the CPU part. In order to measure the vertex throughput in the GPU we used a high polygon model as deformation target. The high polygon count was created by attaching several highly tessellated spheres to an animated object. The chain of spheres is deformed according to the animation of the base mesh. For the CPU side we created a helix using a large number of small objects to measure the composition throughput.

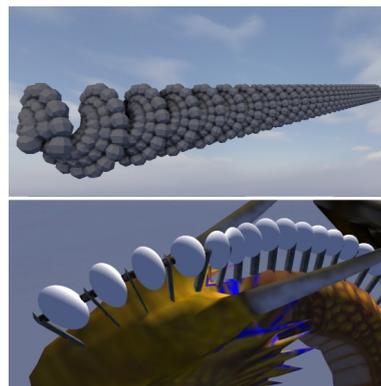


Figure 7: The scenes we used for experimentation: For the CPU tests, we created a helix consisting of many small low resolution spheres to create a high amount of composition. For testing the vertex throughput of the GPU, we attached some high resolution spheres to an animated model. These spheres are deformed with respect to the animation, using our method.

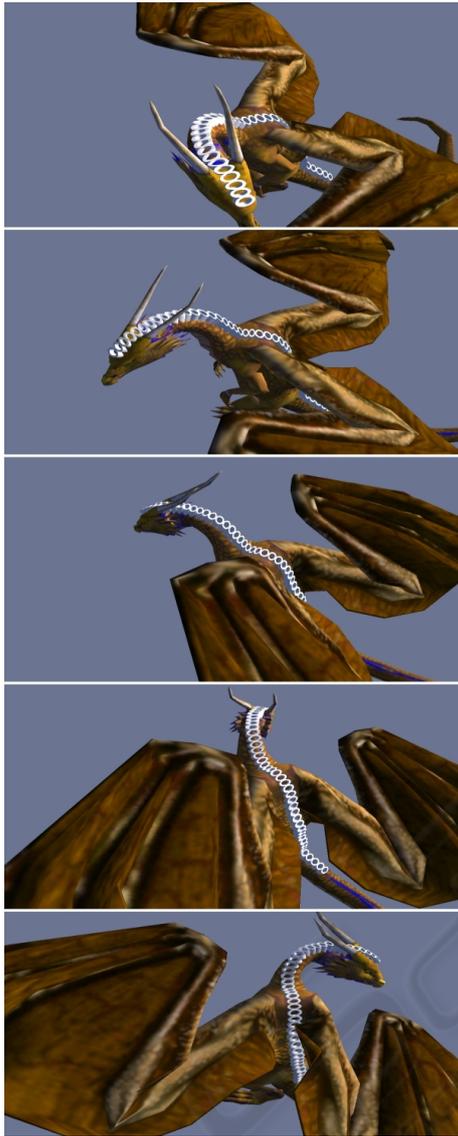


Figure 8: Results: 5 Frames from the vertex animated figurine with a geometric attachment on its back, presenting the deformation of the attached geometry. The whole process is directly managed by the scene graph.

In both tests we compared linear vs. trilinear throughput in frames per second.

The tests were performed on a system containing a GForce 8800 and an Intel Core 2 Quad 6600. The tables show a comparison between the use of trilinear transformation and the linear transformed scene in frames per second. Additionally, the number of objects and triangles are exposed. The software was implemented in OpenGL. All geometry was stored in Vertex Buffer Objects (VBO) on the GPU side. The composition of the trilinear transformations within the scene graph's hierarchy is performed on the CPU.

Deformation of the geometry leaf nodes is done per GPU vertex shader. The trilinear transformation part of the used shader is roughly two and a half times more complex compared to the four scalar products of a linear transformation.

scene	objects	Tri.	FPS/L	FPS/T
spheres	185	1.5M	19.5	14.9
helix	4617	164k	16.5	13.3
helix	2566	656k	10.8	8.4

An experiment was performed with a 1.5 million triangles object to compare the GPU side of the transformation stage. Even though the vertex shader for the trilinear transformation is much more complex than the linear one, we achieve 75 percent of the frame rate from the linear vertex shader. The composition test (CPU) had different results. Using a large number of low poly models resulted in a performance decrease to 80 percent (second row). Increasing the polygon rate by decreasing the amount of objects (third row) shifts the performance decrease to 77 percent (third row). Expectedly, this behaviour results from moving slowly the amount of operations from composition (CPU) towards transformation (GPU).

7 CONCLUSIONS AND FUTURE WORK

We have presented an approach to *embed deformations in a scene graph system* by replacing the 4x4 matrix based transformation system by a more generic one. This *extends* the usage of transformation nodes to warping. We achieve this by combining the idea of trilinear transformation with the hierarchical organisation structure of a scene graph. We support arbitrary deformations by using an approximation scheme. As a large benefit we perform *all geometric* transformations on the GPU within *constant time*. The composition of the scene graph's transformations is still performed inside the CPU. Theoretically, this could lead to a performance bottlenecks, but only if a huge multitude of complex transformations have to be composed and send to the GPU. This has not been observed by any of our experiments. As an application we presented the attachment of arbitrary geometry to the surface of other deformable or dynamic geometry. For handling surface normals we have presented two methods. The first one performs a direct deformation of the normals. The second method guarantees orthogonality to the deformed surface by using the tangent space for normal representation. According to

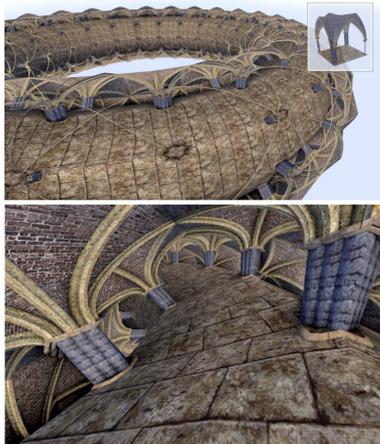


Figure 9: A geometric tile (top right) of a hall mapped on a torus (top). Inside the 3D model (bottom). The embedded transformation system allows directly the deformation of the geometric tiles to achieve a seamless (C_0) joining at their borders.

(Gomes et al., 1998) the inverse of a trilinear transformation can be computed analogously to the bilinear transformation's inverse. As we did not need the inversion for our rendering concept we did not cover this issue. However, the inverse is often a critical feature, for example for ray / objects intersections. Additionally it allows mapping one deformation onto another. Through an inversion, the integration of structure based animation concepts (like skeleton based character animation) could be mapped into the scene graph structure. The inversion problem and C_0 continuity are current drawbacks of the used trilinear deformation. So a more complex, but invertible and more continuous transformation scheme could be of higher benefit in these cases.

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