

CONSTRUCTING THE HYBRID DITHERING MATRIX WITH EQUAL CLUSTERED DOT DENSITY

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Abstract: Hybrid halftone has great strengths over amplitude and frequency modulated halftone in offset printing. Modern CTP systems employ hybrid dithering algorithm, because dithering algorithm is a very efficient halftone algorithm. The shapes, distribution and density of clustered dots in halftone images depend on hybrid dithering matrix. This research proposes a new method to construct a hybrid dithering matrix with equal clustered dot density, based on a new geometry subdivision derived from Delaunay Triangulation. The matrix helps increase clustered dot density and maintain constraints of the offset printing procedure. This paper also discusses the uniformity of expanded central point set and quality of quadrilaterals of the geometry subdivision by comparing PSNR and quality factor value with that of the previous work.

1 INTRODUCTION

Halftone images are classified into three categories in terms of dot shape and distribution: AM, FM and Hybrid halftone images. (1) Amplitude Modulated (AM) halftone uses different size of clustered dots to represent different grey values. Each clustered dot belongs to a dot unit tiled in the halftone plane. It is better if the percentage of black pixels in the dot unit equals to the average grey value of the pixels in the same location of the continuous tone image. AM halftone is often used in laser printer, laser copier and traditional offset printing. (2) Frequency Modulated (FM) halftone randomly distributes dots (pixels) of output device and uses dot density to represent grey value. Dot density is defined as the number of dots in a unit area. The dot density of FM halftone is usually much higher than that of AM halftone. High dot density will make a smooth edge for line works and fine patterns. FM halftone fits for high resolution devices, such as ink jet printer. (3) Hybrid halftone dots vary both in size and density of clustered dots to represent different grey values. It combines the advantages of both AM and FM halftone and is widely used in CTP (Computer to Plate) systems.

In most cases, all halftone algorithms are often described as the following three computing procedures: point procedure, neighbouring point procedure and iteration procedure. Algorithm of

point procedure is very simple and highly efficient in speed. One typical point procedure, dithering algorithm, requires a dithering matrix that will be tiled in halftone plane. When a dithering algorithm halftones a continuous tone image, it first compares the value of the pixels of continuous tone image with the element values of dithering matrix in the corresponding position. Then, it sets the halftone pixel to "0" or "1", if the pixel value of the continuous tone image is greater or less than the value of the element of the matrix. Halftoning with dithering matrix is often referred to as screening.

This research originates from the observation of the unbalanced clustered dot densities shown in Figure 1 and Figure 2 together. It is a vital issue for hybrid halftone to maximize dot density in halftone image and maintain the constraints of offset printing procedure. There are two constraints of offset printing procedure: (1) a minimum clustered dot size that could be reliably reproduced in print; (2) a standard dot gain curve of offset printing. The positive clustered dot size in the light region and the negative clustered dot size in the shadow are small, when the clustered dot density is high. Therefore, we balance the dot density in the highlight and shadow of hybrid halftone image at all grey levels in order to maximize the clustered dot density.

The main contributions of this research are: (1) propose a new geometry subdivision of the halftone plane, of which all the faces (or regions) are quadri-

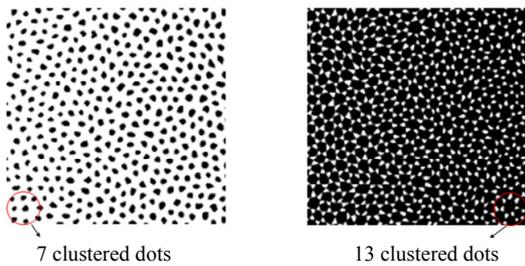


Figure 1: Detail of highlight. Figure 2: Detail of shadow.

laterals. The subdivision is constructed with points of a expanded central points set as a vertex set; (2) evaluate the quality of quadrilaterals in the proposed geometry subdivision in this research; (3) analyze the uniformity of the expanded central point set by comparing PSNR of the initial central point sets; (4) construct a dithering matrix with equal clustered dot density.

2 RELATED WORK

A lot of research concerning hybrid screening has been conducted since the 1990s. Screening is a technique to halftone continuous tone images with dithering algorithms. Printing industry usually uses screening technology in desk top printers, ink jet printers and RIPs (Raster Image Processor) for printing industry.

Barco first launched its hybrid screening product SAMBA for flexo printing. It combines AM and FM halftone in a single screening process. It halftones continuous tone images with FM halftone in tint under 10% and above 90%. It halftones continuous tone images with AM halftone in tint between 10% and 90%. So, it has the merits of FM halftone, for instance, fine detail of FM in the highlight instead of the demerits of FM halftone, for instance, excessive dot gain of FM in the middle tone.

Other companies, like Founder Electronic and Dainippon Screen, issued their hybrid screening products such as FAM and Spekta. However, few papers have been published on the design of these products.

Among the academic researches related to halftone technology, Ulichney first proposed the blue noise model for FM halftone (Ulichney, 1988). According to Ulichney's theory, the ideal halftone image distributes the same-sized dots as homogeneously as possible. By doing so, the spectral content of the image is composed entirely of high-frequency spectral components. And as blue is

the high-frequency component to visible white light, it is named blue noise model. However, blue noise model does not work for some printing procedures such as offset printing, because separate pixels can not be reproduced reliably. Hybrid screen is applied in offset printing instead of FM screen.

Later, Lau formalized the concept of green noise model for hybrid screen and proposed an algorithm, EDODF (Error Diffusion with Output-Dependent Feedback), to design such screens based on enforcing certain spatial-statistical characteristics of green noise (Lau et al, 1998; Lau et al, 2000). However, it is very difficult to control the shape of clustered dot with his method (see Figure 3).

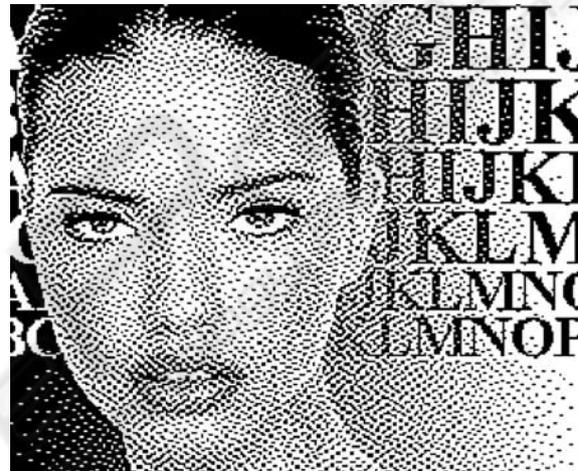


Figure 3: Sample image halftoned by EDODF algorithm.

Damera-Venkata and Lin used the void-and-cluster algorithm with a donut filter to create green noise screens (Damera-Venkata and Lin,2004).

Ostromoukhov proposed a hybrid screen design method based on stochastic seeding and Delaunay triangulation (Ostromoukhov and Hersch,1999). This method apparently has the strength to control the shape of the clustered dots. Tu tried other ways of filling tint in the Delaunay triangulation (Tu et al, 2000). The clustered dot densities in highlight and shadow have big difference for the result of research in (Ostromoukhov and Hersch,1999; Tu et al, 2000), as illustrated in Figure 1 and Figure 2.

Xu proposed two theorems to guide the optimization of the geometry subdivision from Delaunay triangulation, with which a dithering matrix with balanced clustered dot densities were constructed (Xu and Tan, 2009). We will elaborate on these theorems in Section 3.

3 GEOMETRY THEORY FOR OPTIMIZING HYBRID DITHERING MATRIX

In the existing research, constructing hybrid dithering matrix based on a geometry subdivision usually follows this procedure (Ostromoukhov and Hersch,1999; Tu et al, 2000): (1) randomly distribute some dots in a tile unit of a halftone plane as the vertex of geometry subdivision; (2) calculate a geometry subdivision to a tile unit of the halftone plane with the above central points as the vertex of the subdivision; (3) produce the dithering matrix by filling grey gradient tint in the regions of the subdivision above and make the vertex of the subdivision the darkest point and the centre of the region as the lightest point. Then, the grey mode image filled is converted directly to a dithering matrix by taking the pixel value of the image as the value of element of the matrix in the corresponding position. The requirements for the geometry subdivision are: (1) The narrow shape of region should be avoided to make the shape of clustered dot favourable in printing; (2) The vertex's number is expected to be as close as to the region's number of the subdivision. It is the best case as a result of balancing clustered dot density in all grey levels, if the vertex number equals the region's number.

In the previous research, (Ostromoukhov and Hersch,1999; Tu et al, 2000) first generate a pseudo random distribution of central point with space filling curve in a 1024x1024 square areas. Then, mark in a 1024x1024 reference image with a disk of radius r when a central point is added. After all pixels of the referenced image have been marked, the central point set is ready for the next step. When the disk covers pixels are beyond (0..1023, 0..1023), the pixels of coordinate mod by 1024 will also be marked. The mod operation makes the central point image mosaic seamlessly. Because Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation, they tend to avoid skinny triangles. Delaunay triangulation is a good choice of geometry subdivision to produce a hybrid dithering matrix with favourable clustered dot shape. However, the clustered dot density in highlight is greatly different from the clustered dot density in shadow area. Figure 1 and Figure 2 altogether show this difference. Figure 1 and Figure 2 are the enlarged detail of highlight and shadow of Figure 5 in (Ostromoukhov and Hersch,1999). There are 7 positive clustered dots inside the marked circle of Figure 1, whereas there are 13 negative clustered

dots inside the same sized circle of Figure 2. Because of the great differences of clustered dot densities and minimum dot size constraint for offset printing, a comparatively low clustered dot density has to be selected.

To explain the observation of Figure 1 and Figure 2 theoretically, the research in (Xu and Tan, 2009) deducted two theorems from Euler formula.

[Theorem 1] G is a Delaunay triangulation pattern that could be seamlessly tiled. The total number of vertex, edges and faces of G are V, E and F. $F = V/2 + 2$.

Proof: Because G is a geometry subdivision that can be tiled seamlessly, it can be converted to a multi-face object without hole and Euler formula can be applied to G.

$$V - E + F = 2 \tag{1}$$

Every triangle has three edges and every edge is shared by two triangles, so $3F = 2E$.

$$E = (3/2)F \tag{2}$$

Replace E in (1) with left side of (2):

$$\begin{aligned} V - (3F/2) + F &= 2 \\ V &= F/2 + 2 \end{aligned} \tag{3}$$

The geometry subdivision adopted to construct dithering matrix usually has more than 1,000 faces. Divide the two sides of equation (3) by F, we have $V:F \approx 1:2$. [Theorems 1] explains why the ratio of clustered densities in light and shadow areas is about 0.5.

[Theorem 2] G is a geometry subdivision that can be tiled seamlessly. The total vertex, edges and faces of G are V, E and F. If all the faces of the division are quadrilaterals, then $F = V - 2$.

Proof: Same as proof above:

$$V - E + F = 2 \tag{4}$$

If all faces of the G are quadrilaterals, every quadrilateral has 4 edges and every edge is shared by two quadrilaterals. So, $4F = 2E$

$$E = 2F \tag{5}$$

Replace E in equation (4) with 2F

$$V - 2F + F = 2$$

$$F = V - 2$$

[Theorems 2] suggests: if the geometry subdivision comprises more quadrilaterals, the ratio of the clustered dot densities in shadow and light areas is closer to 1. According to [Theorem 2],

an optimized algorithm is designed to merge triangles of Delaunay triangulation as much as possible while trying to keep quadrilaterals merged in good shape. In the research of (Xu and Tan, 2009), the ratio of the clustered dot densities was improved to 0.9 from 0.5 without any change to the central point set.

This research adopts an approach different from the optimizing algorithm in (Xu and Tan, 2009). It first expands pseudo random central point set. Then, it makes an all-quadrilateral subdivision with the expanded central point set. A dithering matrix of equal clustered dot densities is constructed, based on the all-quadrilateral geometry subdivision. Because this approach changes the initial central point set, a further study on the uniformity of the expanded central point set is conducted by comparing its PSNR with PSNR of the images of the central point set generated with different pseudo seeds.

4 ALL-QUADRILATERAL GEOMETRY SUBDIVISION

In the first step of this research, we generate an initial central point distribution over halftone plane by using the algorithm 1-3 of (Xu and Tan, 2009). The radius r defines minimum distance between central points. In the experiment, r is set to 14. In the second step, we calculate the Voronoi graph with the central point set. Delaunay triangulation is obtained by connecting the neighbouring central points in the two regions which share the same edge of the Voronoi graph.

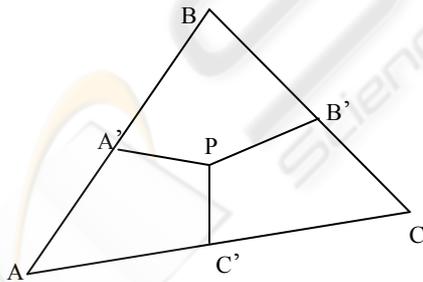


Figure 4: Split a triangle into quadrilaterals.

Now, take a triangle in Delaunay triangulation as an example to explain the principle to construct all-quadrilateral geometry subdivision. First, find the weight of the triangle ABC in point P. Next, connect P with three central points of the three edges of the triangle ABC. The triangle is split into three quadrilaterals (Figure 4). Then, all the triangles of

Delaunay triangulation will be converted to quadrilaterals in this way. The new subdivision is different from the optimized subdivision of (Xu and Tan, 2009) in the following two aspects:

(1) The new geometry subdivision is constructed with a central point set expanded from the initial central point set by adding the weight points and central points of edge of triangles.

(2) All regions of the new geometry subdivision are quadrilaterals, whereas the regions of the geometry subdivision optimized in (Xu and Tan, 2009) are a combination of quadrilaterals and triangles.

The vertex of the geometry subdivision is taken as the centre of the negative clustered dot. The image composed of the central points can also be regarded as a halftone image of minimum non-zero grey value. The image composed of the central points can mosaic seamlessly according to the previous description. We will further discuss the quality issue of quadrilaterals.

The region's narrow shape in the geometry subdivision should be avoided, because the clustered dot is formed in the region. By following the definition of Delaunay triangulation, a quadruple is defined as the quality factor the quadrilaterals (Xu and Tan, 2009). When merging triangles besides an edge into quadrilateral (Figure 5), quality factor of $A_1A_2A_3A_4$ (or edge A_1A_3) is calculated as follows:

Firstly, convert the angles of quadrilateral $A_1A_2A_3A_4$ to A_1', A_2', A_3' and A_4' according to formula (6). Then, sort the angles in ascend order and put it in a quadruple like $(A_{i_1}', A_{i_2}', A_{i_3}', A_{i_4}')$, where $i_1, i_2, i_3, i_4 \in \{1, 2, 3, 4\}$, $A_{i_1}' \leq A_{i_2}' \leq A_{i_3}' \leq A_{i_4}'$. The quadruple is called quality factor of quadrilateral $A_1A_2A_3A_4$.

$$A_i' = \begin{cases} A_i, & A_i \leq 90 \\ 180 - A_i, & 180 > A_i > 90 \\ 0, & A_i \geq 180 \end{cases} \quad (6)$$

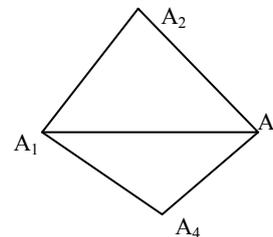


Figure 5: Defining quality factor of a quadrilateral.

For the convenience of comparing the quality of quadrilaterals in geometry subdivision of this research and (Xu and Tan, 2009), the quadruple is

converted to a real value QF in conformity with formula (7). The value QF ranges from 0 to 7.9.

$$QF = \log_{10}(A_{i4}' * 90^3 + A_{i3}' * 90^2 + A_{i2}' * 90 + A_{i1}') \quad (7)$$

The greater value of QF indicates better quality of the quadrilateral. We calculate QF of quadrilaterals in geometry subdivision of this research and (Xu and Tan, 2009) and use Table 1 to compare the quality of quadrilaterals in these two subdivisions. In Table 1, Min_QF, Max_QF and Average_QF are minimum, maximum and average value of QF respectively. Table 1 shows that quality factor of the quadrilaterals in this research is better than that in (Xu and Tan, 2009).

Table 1: Comparing QF of quadrilaterals.

	Min_QF	Max_QF	Average_QF
All-quadrilaterals subdivision	7.6857	7.8218	7.7920
Optimized subdivision in (Xu and Tan, 2009)	7.4150	7.8218	7.7150

5 UNIFORMITY OF THE EXPANDED CENTER POINT SET

The purpose of this research is to produce a hybrid dithering matrix with equal clustered dot densities. Because some extra central points besides initial central points are added in the process of obtaining all-quadrilateral geometry subdivision, the expanded central point set should be evaluated by comparing its uniformity with that of the initial central point set.

If the image of central points is regarded as a 1024x1024 halftone image, its average grey value g is defined as:

$$g = \frac{NC}{NP} \quad (8)$$

NC is the number of central points and NP is the total number of pixels of the image. The image of central point can also be viewed as the result of halftoning an image of constant grey value g . PSNR (Peak Signal Noise Ratio) is often used to measure the quality of image. In this research, the uniformity of central point set is measured by PSNR defined in the following procedure:

- (1) Calculate the average grey value g ;
- (2) Filter the central point image with HVS model and we get $x(i,j)$, the human perceived image of central point;
- (3) Calculate PSNR with formula (9), where g_{max} is maximum grey value. Let it be 1.0 here.

$$PSNR = \log\left(\frac{\sum_{j=1}^M \sum_{i=1}^N g_{max}^2}{\sum_{j=1}^M \sum_{i=1}^N (x(i,j) - g)^2}\right) \quad (9)$$

Greater PSNR means better uniformity for the central point image. The expanded central point image and initial central point image have different dot densities. To reduce factors affecting the comparing result, we scale down the expanded central point image to the same dot density of the initial central point image and crop it to 1024x1024. The reason why we filter the image with HVS model is that all printed halftone images are observed by human eyes. It is due to the HVS model that the halftone image looks similar to the original continuous tone image. Table 2 lists PSNR of the four central point images. D_1 is the expanded central point image both cropped and scaled. D_2 is the initial central point image. D_3 and D_4 are the central point image that is generated with radius 14 and different random seeds.

Table 2 shows PSNR of the tested pseudo random central point images. PSNR of D_2 , D_3 and D_4 varies in a range 43.3744~44.5205. PSNR of D_1 is out of this range. The quality (uniformity) of expanded central point image D_1 is inferior to the initial central point image D_2 . But PSNR of D_1 is very close to the above range. It is reasonable to think PSNR of D_1 is still within the acceptable range.

Table 2: PSNR of image of central point set.

	D_1	D_2	D_3	D_4
PSNR	43.1944	44.5205	43.3744	43.4002
g	0.0056	0.0051	0.0055	0.0055

6 CONSTRUCTING HYBRID DITHERING MATRIX

In Section 4, the triangle of Delaunay triangulation is split into three quadrilaterals to make an all-quadrilateral geometry subdivision. Thus, we fill these quadrilaterals with gradient tint to make a dithering matrix.

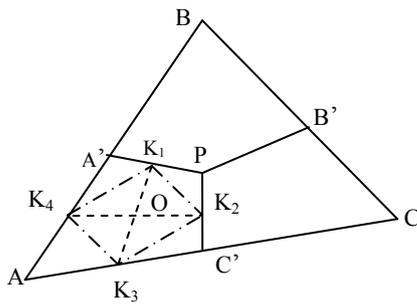


Figure 6: Filling gradient in quadrilaterals.

Take the quadrilateral PC'AA' as an example to show the procedure of filling quadrilateral with gradient tint (see Figure 6 also):

(1) Find the central point of all the four edges of the quadrilateral PC'AA' : K_1, K_2, K_3, K_4 ;

(2) Connect K_1 and K_3 with a line in between K_1K_3 . Connect K_2 and K_4 with a line in between K_2K_4 . O is the cross point of the line K_1K_3 and K_2K_4 . Take O as the centre of positive clustered dot and vertex P, C', A, A' as the centres of 4 negative clustered dots;

(3) Fill quadrilateral OK_1PK_2 with gradient tint

a) Filling triangle K_1PK_2 the gradient tint with 20 stages grey levels:

First, fill triangle K_1PK_2 with grey level $g=0.5$;

Then, shift K_1K_2 in the track of K_1P and K_2P to K_1' and K_2' while keeping the line $K_1'K_2'$ paralleled to K_1K_2 ; each step is $1/20$ of the distance from P to line K_1K_2 ; grey value g filled is increased by 0.025 in each step;

b) Fill the gradient tint into triangle K_1OK_2 with 19 grey level stages;

First, fill grey level $g=0.475$ into triangle K_1PK_2 ;

Then, shift K_1 and K_2 in the track of K_1O and K_2O to K_1' and K_2' while maintaining $K_1'K_2'$ paralleled to K_1K_2 ; each step is $1/20$ of the distance from O to line K_1K_2 , and grey value g filled is decreased by 0.025 in each step;

(4) The same filling procedure will be applied to quadrilaterals $OK_2C'K_3, OK_3AK_4$ and $OK_4A'K_1$.

The graphics of the dithering matrix image described above is programmed in PostScript language. With PostScript, it is easy to scale up and down with 'scale' operator to produce the dithering matrix of different clustered dot densities, which can also be adapted to the output devices with different resolutions.

The procedure to produce the dithering matrix is described as follows:

First, we print the postscript file of dithering matrix with Harlequin RIP and output it to a 600 DPI grey mode TIFF file. In Photoshop, the grey image is smoothed by Gaussian filter to bring the clustered dot a smooth edge. The parameters employed in Gaussian filtering are: Radius=4, Amount=100%. Next, we paste the above filtered image into a new image of two different layers in Photoshop. Then, we horizontally move the top layer rightwards. When the left edge of the top layer precisely matches the bottom layer, the distance by which the top layer has been moved is the width of pattern that can mosaic seamlessly. The size of the image pattern that can mosaic seamlessly is a 2134×2134 image. Figure 7 is the mosaic unit image. For clearness of the pattern, only a part of image is shown in Figure 7. In the following step, we directly convert the mosaic unit image to a 2134×2134 dithering matrix, in which the element value of the matrix is the pixel value of the unit mosaic image. Finally, we make a new dot shape named 'AccurateBalanced' with the dithering matrix and embed it into Harlequin RIP. Figure 8 and Figure 9 are two samples produced with this dot shape. From Figure 8 it is obvious that clustered dot densities (98%) in shadow and light area (2%) are the same. Figure 9 is an image halftone by the Harlequin RIP with 'AccurateBalanced' dot shape.

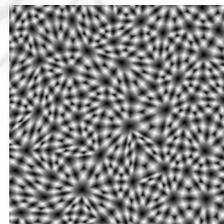


Figure 7: Dithering matrix.



Figure 8: Halftoned grey image.



Figure 9: Halftoned image sample.

7 CONCLUSIONS

This research proposes a new method to expand a Delaunay triangulation to an all-quadrilateral geometry subdivision. Based on the proposed subdivision, we construct a hybrid dithering matrix with equal clustered dot densities. The strengths of this result are described as follows: (1) The clustered dot density equals in all grey levels; (2) The quality factor of the quadrilaterals is improved when compared with that of the quadrilateral in the optimized geometry subdivision in (Xu and Tan, 2009); (3) The uniformity of the expanded central point set is proved to be acceptable by comparing its uniformity with the variance of the uniformity of central point distribution generated by different pseudo random seeds.

The new hybrid dither matrix has an obvious weakness: the shadow pattern consists of one big negative clustered dot surrounded by several small negative clustered dots (Figure 7). The big negative clustered dot is centred on the vertex of the geometry subdivision with five or more edges. The small negative clustered dots are centred on the vertex of 4 edges (central point of the edges of Delaunay triangle) or on the vertex of 3 edges (weight point of Delaunay triangle). The average number of edges connected to a vertex is 4 for an all-quadrilateral geometry subdivision. Further study deserves to be conducted to make a geometry subdivision with more 4 edges vertex to reduce the local unbalance of the clustered dot size.

REFERENCES

- Damera-Venkata N., Lin Q., 2003. AM-FM screen design using donut filters. *Proc. SPIE*,5293(469): 469–480.
- Lau D. L., Arce G. R., Gallagher N. C., 1998. Green-noise digital halftoning. *Proc. IEEE*,86(12):2424–2444.
- Lau D. L., Arce G. R., Gallagher N. C., 2000. Digital color halftoning with generalized error-diffusion and multichannel green-noise masks. *IEEE Trans. Image Process.*, 9(5):923–935.
- Ostromoukhov V., Hersch R. D., 1999. Stochastic clustered-dot dithering. *Journal of Electronic Imaging*, 8(4): 439–445.
- Tu Chang-He, Pan Rong-Jiang, Meng Xiang-Xu, 2000. An algorithm for building stochastic clustered-dot screens. *Chinese journal of computers*,23(9):931-937 (in Chinese).
- Ulichney R. A., 1988. Dithering with blue noise. *Proc. IEEE*, 76(1): 56-79.
- Xu Guoliang, Tan Qingping, 2009. Research on balancing clustered dot density for hybrid halftone dithering algorithm. *Chinese journal of computers*,32 (8): 1550 1559 (in Chinese).