

ON-LINE PLANAR AREA SEGMENTATION FROM SEQUENCE OF MONOCULAR MONOCHROME IMAGES FOR VISUAL NAVIGATION OF AUTONOMOUS ROBOT

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Keywords: Variational image analysis, Fractional differentiation, Optical flow.

Abstract: We introduce an on-line segmentation of a planar area from a sequence of images for visual navigation of a robot. We assume that the robot moves autonomously in a man-made environment without any stored map in the memory or any markers in the environment. Since the robot moves in a man-made environment, we can assume that the robot workspace is a collection of spatial plane segments. The robot is needed to separate a ground plane from an image and/or images captured by imaging system mounted on the robot. The ground plane defines a collision-free space for navigation. We develop a strategy for computing the navigation direction using a hierarchical expression of plane segments in the workspace. The robot is required to extract a spatial hierarchy of plane segments from images. We propose an algorithm for plane segmentation using an optical flow field captured by an uncalibrated moving camera.

1 INTRODUCTION

Spatial reasoning is a fundamental process for visual navigation and localisation of autonomous robots (Kuipers and Byun, 1991; Wagner et al., 2004). Segmentation of image is a fundamental problem in image understanding. Segmentation is categorised in supervised and unsupervised method. In this paper, we deal with unsupervised on-line segmentation for visual navigation of a robot. Assuming that the robot moves on a planar ground plane, the robot is needed to separate a ground plane from an image and/or images captured by imaging system mounted on the robot.

Segmentation is a methodology to extract meaningful parts from a image and a video sequence. As a human-interface tool for editing images on a computer screen, marker-based semi-supervised and supervised segmentation techniques are studied. A powerful method used as a back-end of the method is graph cut. For visual navigation, the robot required to use unsupervised on-line segmentation algorithm. Therefore, the robot cannot use marker-based segmentation method. Furthermore, the most important segment on an image is a free-space on which

the robot can navigate without colliding to obstacles. The ground plane defines a collision-free space for navigation. Therefore, we introduce a visual navigation algorithm for a robot which moves in a man-made indoor environment. In a man-made environment planer surface on polyhedral objects are dominant geometrical features. Therefore, configurations of planer segments are essential quarry for obstacles. Assuming that a robot is deriving on a flat plane with polyhedral obstacles, we develop a method to hierarchically separate planar segments in a scene using an image sequence captured by a imaging system mounted on the robot. Since a robot moves, the imaging system mounted on the robot automatically captures a sequence of images. This series of images derives optical flow sequence. The depth of planer segment affects to the observed optical flow. Therefore, the robot can separate planar areas based on the depths from the camera on the robot. We assume that for the collision free navigation a robot decide the direction to tern using the configurations of planes in front of the robot.

Model-based methods for image segmentation have been proposed. Homography-based methods

(Chum et al., 2005; Yang et al., 2005) use plane-to-plane homography for detecting a plane. Motion segmentation with layers is proposed in refs. (Wang and Adelson, 1994; Weiss, 1997). Brox et al. proposed an algorithm for image segmentation by the level set method (Brox et al., 2006). We use the dominant-plane model for segmenting multiple planar areas. Since the dominant plane is a planar area in the robot workspace and it corresponds to the largest part of an image, our algorithm does not require any restrictions on the camera motion or geometric configurations between the camera and objects. The hierarchical detection of dominant planes allows the robot to achieve spatial reasoning without any three-dimensional reconstruction of the scene, since the dominant plane is a binary feature on the image plane. Furthermore, since the dominant plane is a binary feature, the algorithm is robust against the outliers that are derived in the process of optical-flow computation.

2 DOMINANT PLANE AND OPTICAL FLOW

2.1 Dominant Plane

We define the dominant plane in an image.

Definition 1. *The dominant plane as the planar area in the robot workspace corresponding to the largest part of an image or at least the half of an image.*

Similarly with the previous paper (N.Ohnishi and Imiya, 2005), we accept the following five assumptions.

Assumptions

1. The ground plane is the planar area.
2. The camera mounted on the mobile robot is looking downward.
3. The robot observes the world using the camera mounted on itself for navigation.
4. The camera on the robot captures a sequence of images since the robot is moving.
5. Obstacles occupy at most 1/2 region in an image captured by the robot.

Therefore, if there are no obstacles around the robot, and since the robot does not touch the obstacles, the ground plane corresponds to the dominant plane in the image observed through the camera mounted on the mobile robot. Assuming that the dominant plane in the image corresponds to the ground plane on which

the robot moves, the detection of the dominant plane enables the robot to detect the feasible region for navigation in its workspace.

2.2 Optical Flow on the Dominant Plane

Assuming that the camera displacement is small, on each layer the corresponding points $\mathbf{x} = (x, y)^\top$ and $\mathbf{x}' = (x', y')^\top$ in the dominant planes between a pair of successive two images are connected with an affine transform such that $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$, where \mathbf{A} and \mathbf{b} are a 2×2 affine-coefficient matrix and a 2-dimensional vector.

We can estimate the affine coefficients using the RANSAC-based algorithm (Fischler and Bolles, 1981). Using estimated affine coefficients, we can estimate optical flow on the dominant plane $\hat{\mathbf{x}} = (\hat{x}, \hat{y})^\top$, $\hat{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b} - \mathbf{x}$, for all points \mathbf{x} in the image. We call $\hat{\mathbf{x}}$ the *planar flow on the l th layer*, and $\hat{\mathbf{x}} = (x, y, t)$ the *planar flow field* at time t , which is a set of planar flow $\hat{\mathbf{x}}$ computed for all pixels in an image.

If an obstacle exists in front of the robot, the planar flow on the image plane differs from the optical flow on the image plane. Since the planar flow vector $\hat{\mathbf{x}}$ is equal to the optical flow vector $\dot{\mathbf{x}}$ on the dominant plane, we use the difference between these two flows to detect the dominant plane. We set ϵ to be the tolerance of the difference between the optical flow vector and the planar flow vector. Therefore, for the optical flow equation $\nabla I^\top \dot{\mathbf{x}} + \partial_t I = 0$ of an image I observed at time t if the inequality

$$|\dot{\mathbf{x}} - \hat{\mathbf{x}}| < \epsilon, \\ \text{s.t. } \hat{\mathbf{x}} = (\mathbf{A}\mathbf{x} + \mathbf{b}) - \mathbf{x}, \nabla I^\top \dot{\mathbf{x}} + \partial_t I = 0 \quad (1)$$

is satisfied, we accept point \mathbf{x} as a point on the dominant plane (N.Ohnishi and Imiya, 2005).

Our algorithm is summarised as follows:

1. Compute optical flow field $\mathbf{u}(x, y, t)$ from two successive images.
2. Compute affine coefficients of the transform $\mathbf{A}\mathbf{x} + \mathbf{b}$ by random selection of three points.
3. Estimate planar flow field $\hat{\mathbf{u}}(x, y, t)$ from affine coefficients.
4. Match the computed optical flow field $\mathbf{u}(x, y, t)$ and estimated planar flow field $\hat{\mathbf{u}}(x_1, y_1, t)$ using eq. (1).
5. Assign the points $|\dot{\mathbf{x}} - \hat{\mathbf{x}}| < \epsilon$ as the dominant plane. If the dominant plane occupies less than half the image, then return to step 2.
6. Output the dominant plane $d(x, y, t)$ as a binary image.

2.3 Hierarchical Plane Segmentation

Using the dominant-plane-detection algorithm iteratively, we develop an algorithm for multiple-plane segmentation in an image.

Our basic algorithm detects the dominant plane in an image. After removing the region corresponding to the dominant plane from the image, we can extract the second dominant planar region from the image. Since the first dominant plane is assumed to be the ground plane, the second dominant plane corresponds to an obstacle. Then it is possible to extract the third dominant plane by removing the second dominant planar area. This process is expressed as

$$D_k = \begin{cases} \mathbf{A}(R \setminus D_{k-1}), & k \geq 2, \\ \mathbf{A}(R), & k = 1, \end{cases} \quad (2)$$

where \mathbf{A} , R , and D_k stand for the dominant-plane-extraction algorithm, the region of interest observed by the camera, and the k th dominant planar area, respectively. The algorithm is stopped after a predetermined iteration time or when the size of the k th dominant plane is smaller than a predetermined size.

The iterative plane-segmentation algorithm is summarised as follows:

Algorithm 1: Hierarchical Segmentation.

```

repeat
  if the dominant plane cannot be detected
  then stop;
  Remove the dominant plane area from the
  image;
until predetermined number of times  $K$  ;

```

The procedure of the algorithm is shown in Fig. 1. In the experiments, we set the predetermined number of the iteration K in algorithm 4 as $K = 3$.

Setting R to be the root of the tree, this yields derives a binary tree such that

$$R \langle D_1, R \setminus D_1 \langle D_2, R_2 \setminus D_2 \langle \dots \rangle \rangle \rangle. \quad (3)$$

Assuming that D_1 is the ground plane on which the robot moves, D_k for $k \geq 2$ correspond the planar areas on the obstacles. Therefore, this tree expresses the hierarchical structure of planar areas on the obstacles. We call this tree the binary tree of planes. Using this tree constructed with the dominant-plane-detection algorithm, we obtain the topological configuration of planes in an image. Even if an object exists in an image and it lies on D_k , $k \geq 2$, the robot can navigate while ignoring this object, using the binary tree of planes, as shown in Fig. 2. In accordance with the spatial configuration of planar areas, the robot can decide the navigation direction.

3 SPATIAL REASONING USING HIERARCHY OF PLANE SEGMENTS

In this section, we apply an algorithm based on the extension of eqs. (2) and (3) to the spatial reasoning used for the robot navigation.

For an image R^2 , setting D and \bar{D} to be the dominant plane area in R^2 and its conjugate $\bar{D} = R^2 \setminus D$, respectively, we have the tree structure

$$R^2 \langle D, \bar{D} \rangle. \quad (4)$$

Applying the dominant-plane-detection algorithm to \bar{D} , we have the tree structure

$$R^2 \langle D, \bar{D} \langle D_1, \bar{D}_1 \rangle \rangle. \quad (5)$$

Here, we affix the labels L , R , and M to these trees, where L , R , and M express the locations of the dominant plane in the hierarchy on the image plane. From the property of the dominant plane clarified in section 2.1, we have the following tree structures from the hierarchical extraction of the dominant planes. These structures are derived from the geometrical configuration of obstacles in workspace from a sequence of images captured with a camera mounted on the robot.

We have the possibilities

$$T_L = R^2 \langle D_L, D_R \langle D_{RL}, D_{RR} \rangle \rangle, \quad (6)$$

$$T_M = R^2 \langle D_M, \bar{D}_M \rangle, \quad (7)$$

$$T_R = R^2 \langle D_L \langle D_{LL}, D_{LR} \rangle, D_R \rangle \quad (8)$$

$$T_D = \emptyset. \quad (9)$$

Equation (9) means that no dominant plane in front of the robot exists, which is shown as property in section 2.1. Therefore, these three trees correspond to spatial configurations of planes in front of the robot, as shown in Fig. 4. The correspondence between the trees and spatial configurations indicates the direction of collision-free paths for the mobile robot. In accordance with these trees, which express the configurations of the free space and obstacles in front of the robot, the mobile robot decides which direction to move. For the robot to move on the dominant plane without colliding with obstacles, the rule of the robot motion is as follows.

These trees describe the following four geometrical configurations.

1. Obstacles and the free space for the robot to move exist in the left and right, respectively. Therefore, the robot should move to the right.
2. Obstacles and the free space for the robot to move exist in the right and left, respectively. Therefore, the robot should move to the left.

3. The free space for the robot to move exists in front of the robot. Therefore, the robot can move forward.
4. Obstacles exist in front of the robot. Therefore, the robot should turn 180 degrees to move to the backward.

Algorithm 2: Obstacle Avoidance Rule.

```

if  $T(t) = T_L(t)$  then
  | the robot turns to the left;
  |  $t := t + 1;$ 
else if  $T(t) = T_R(t)$  then
  | the robot turns to the right;
  |  $t := t + 1;$ 
else
  |  $T(t) = T_M(t);$ 
  | the robot moves forward;
  |  $t := t + 1;$ 
end
else
  |  $T(t) = T_D(t);$ 
  | the robot turns 180 degrees;
  |  $t := t + 1;$ 
end

```

While the robot is moving, it obtains the sequence of trees $T(t)$ such that

$$\begin{aligned}
 T(t) &= T_M, \\
 T(t+1) &= T_M, \\
 T(t+2) &= T_R, \\
 T(t+3) &= T_R, \\
 &\vdots
 \end{aligned}
 \tag{10}$$

When the robot detects the transition of trees, that is,

$$T(t+1) \neq T(t), \tag{11}$$

the robot is required to control its direction for navigation. This property is used to derive the control rule as follows.

Algorithm 3: Direction Control Rule.

```

if  $T(t) = T(t-1)$  then
  | the robot moves in the direction of the label
  | of  $T_*(t-1)$ ;
else
  | the robot moves in the direction of the label
  | of  $T_*(t)$ ;
  |  $t := t + 1;$ 

```

Using the labels of the trees, this rule is expressed as the automaton given in Fig. 3. The label of the cell corresponds the direction in which the robot moves. The automaton accepts the binary tree corresponding to the spatial configuration of the dominant plane.

We show an experimental example of mobile robot navigation using a hierarchy of plane segments. The specifications of the mobile robot used for this experiment are summarised in Table 1. In the experiments, the mobile robot moves in a room. If the mobile robot detects a wall, it turns to the left or right in accordance with the binary tree computed from the spatial configuration of planar areas. The control rule is described in the previous section.

The results in Fig. 14 and property 2 described in section 2.1 show that the robot would turn and return to the start point if there are many obstacles between the start point and destination. Therefore, we prepared the environments with the sparse configuration of obstacles. For the experiments in the real environment, on the basis of property 2 derived in section 2.1, we prepared four scenarios for the experiments.

Obstacle Configuration of Experiments

E.1 Wall Following. The robot moves following the walls of the man-made room. In this experiment, the robot was required to compute the positions to start turning and to stop turning using a sequence of trees extracted from an image sequence using optical-flow technique.

E.2 Corridor Passing. In this experiment, the robot was required to achieve centring in the corridor using a sequence of trees extracted from an image sequence using optical-flow technique. Since the robot, at present, does not numerically compute the configuration of the obstacles, the corridor width is to be the twice of the width of the robot. The robot is required to estimate the corridor centre using topological information derived as a sequence of trees

E.3 Random Box. Some boxes of the same size as the robot are randomly and sparsely distributed in the workspace. There is a corridor path in front of the robot.

E.4 Door Closing. A student closed the door to the corridor on the straight path in front of the robot. The closing of the door causes the transition of the obstacle configuration in the workspace and the map of the workspace. Therefore, this scenario provides a dynamic environment in the robot workspace.

For the first scenario, we show all the snapshots, views from the robot, and the trees extracted from these images. The experimental result for mobile robot navigation using the hierarchy of plane segments is shown in Fig. 5. Snapshots of the mobile robot, images captured by the mobile robot and binary trees extracted from these images are shown. Using

Table 1: Specifications of our mobile robot.

Name	Magellan Pro, AAI Systems, Inc.
Size	Circular - 16-inch diameter
Weight	50pounds
Drive	2-wheel
CPU	800MHz, AMD-K6 processor
Main memory	256MB
OS	Red Hat Linux
Compiler	GNU C++ Compiler
Camera	SONY EVI-D30

the sequence of these trees, we obtain the control sequence $\langle M L L L M \rangle$ from this image sequence. The left row of Fig. 5 shows the robot navigating by this sequence of directions. This example shows that the hierarchy of plane segments is acceptable as a cue for mobile robot navigation.

In the practical experiments, the velocity of the robot was approximately 5cm/s. The robot autonomously moved in the room in our lab without colliding with obstacles and walls using the algorithms described in the previous section.

Figure 6 shows snapshots of the corridor passing of the robot without optical and geometrical calibration. The width of the corridor is approximately twice of the width of the robot. The robot started from the outside of the corridor and passed through the corridor centre without colliding with the corridor walls. Therefore, the camera configuration on our robot allows the collision-free navigation if the width of the corridor is at least twice the width of the robot.

Figure 7 shows a sparse-obstacle environment. In this environment, although the obstacles are as large as the robot, there is a corridor path in front of the robot. Therefore, the robot detects a free space to pass and decides a straight path to move in front of the robot.

Figure 8 shows a dynamic environment. The door to the corridor is closed by a student while robot is moving to the door. Furthermore, the student returned to the outside of the view field of the robot. After the door is closed, the robot recognises the door as a part of the wall and decides the path to the left. Since our robot does not use any maps of the workspace, the robot detects the wall in front of the robot and decides the navigation direction to avoid the colliding with the wall.

As described in section 3, our assumption on the configuration of the obstacles is that the robot should observe the first dominant plane as the free space on the ground floor. The configurations for real experiments satisfy this assumption. Therefore, as an exten-

sion of the featureless visual navigation introduced in our previous paper (N. Ohnishi and Imiya, 2005), the hierarchical expression of dominant planes yields the control information for the navigation direction.

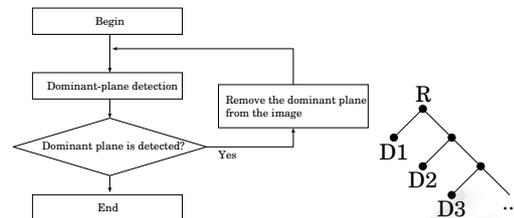


Figure 1: Extraction binary hierarchy of plane segments. (a) Iterative plane-segmentation algorithm using dominant-plane detection. (b) Binary tree extracted from hierarchical structure of planar areas. R is the root of the tree and D_k are planes on the image.

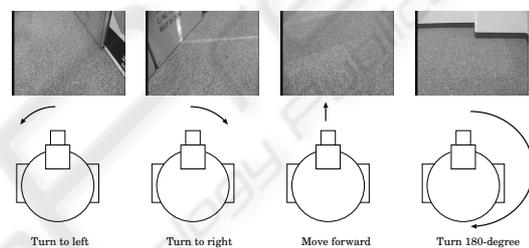


Figure 2: The configuration of planes determines the robot motion. The top row shows examples of the patterns of plane configurations captured by the camera mounted on the robot. The bottom row shows the robot motion corresponding to each plane configuration observed by the camera mounted on the robot.

4 CONCLUSIONS

In the previous papers (Fermin and Imiya, 1997; Imiya and Fermin, 1999), we developed a RANSAC-based motion analysis algorithm for two- and three-dimensional motions, respectively. Furthermore, we developed a RANSAC-based free space analysis method for visual navigation of the autonomous robot, using a dominant-plane-detection strategy (N. Ohnishi and Imiya, 2005). The dominant-plane-based navigation method (Ohnishi and Imiya, 2006) detects the free space from the appearance of the workspace using images captured by the camera/cameras mounted on the robot. The method used the mathematical relation between optical flow computed from a series of images captured by a camera mounted on the robot and homography transform between captured ground plane in a series of images.

In this paper, we extended these RANSAC-based results to spatial reasoning to derive the navigation

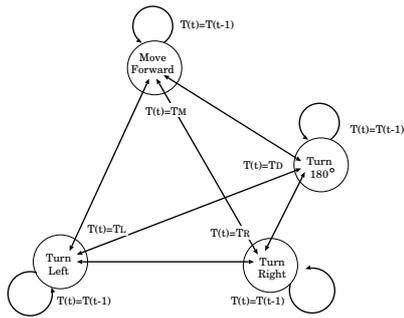


Figure 3: Automaton for the direction control rule. The labels of the cells show the directions in which the robot moves. The automaton accepts the sequence of binary trees corresponding to the spatial configuration of the dominant plane. The state of the automaton changes in accordance with Algorithms 5 and 6.

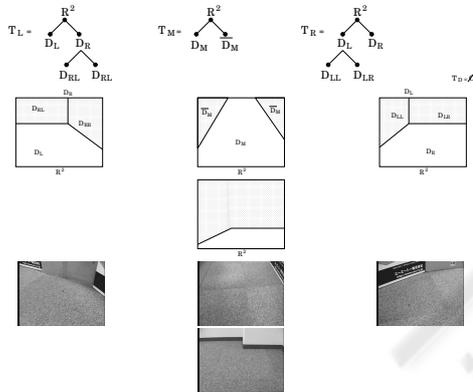


Figure 4: Binary trees and corresponding images of hierarchical structure of dominant planes. R^2 is an image plane and D_* are hierarchical dominant planes. $\overline{D}_M = R^2 \setminus D_M$ and \emptyset .

direction in the visual navigation process. We extend the dominant-plane detection to planer-area detection in an image using the same property between optical flow and the homography transform of the planar area in the workspace. For the extraction of planes, we apply the dominant-plane detection algorithm hierarchically to an image. This series of hierarchically extracted planes expresses configurations of planar areas in the workspace. We can extract the second dominant plane from the obstacle area, and the third dominant plane from the obstacle area to the second dominant plane. This set theory property of the dominant planes enables us to define the higher order dominant planes which describe the appearance configurations of planer segments in the space. These plane configurations allow the extraction of the corridors for the robot in the polygonal world.

The camera geometry of the imaging system mounted on the robot is uncalibrated, that is, for the

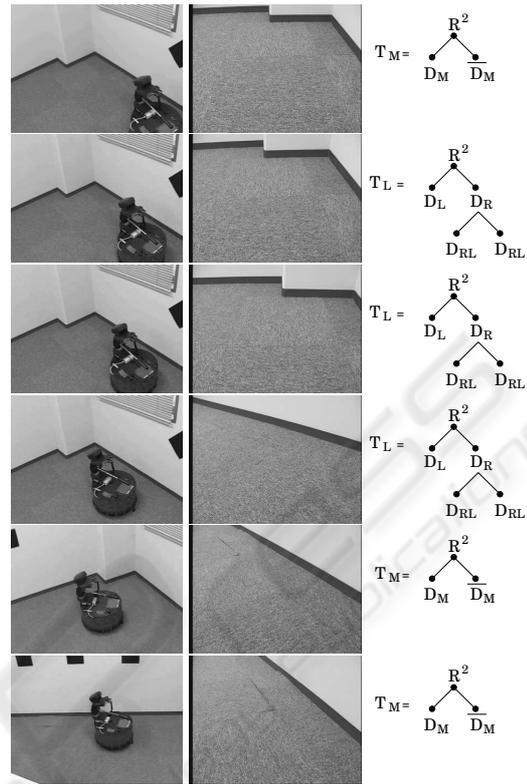


Figure 5: Experimental result for mobile robot navigation using hierarchy of plane segments. The left, middle, and right columns show snapshots of the mobile robot, images captured by the robot, and corresponding binary trees, respectively. This example shows that the hierarchy of plane segments is acceptable as a cue for describing the spatial configuration of planar areas in front of the robot for mobile robot navigation.

spatial reasoning of the navigation direction, the robot does not use any parameters in the imaging system and the robot. For the detection of the widths of the corridors and the sizes of the obstacles, we are required to calibrate the imaging system of the robot geometrically, since for geometrical reconstruction of three-dimensional geometric configuration, geometrical information such that the height of the camera centre from the ground plane, the downward angle of the optical axis of the camera, the view-angle of camera, the focal-length of the camera, the distance between optical centre of the camera and gravity centre of the robot are used(Young-Geun and Hakil, 2004). Therefore, although our robot cannot calculate the sizes of obstacles and the widths of corridors from images, the robot can decide the configurations of obstacles and corridors from the images. We assume that the widths of the corridors in the workspace are sufficient for the robot to pass through, since we are interested in spatial reasoning for the direction control for robot navigation. Our experiments showed that the navigation



Figure 6: Experimental result for mobile robot navigation using hierarchy of plane segments. The robot passes through the corridor, whose width is about the twice of the width of the robot, without collision to the wall of the corridor.



Figure 7: A sparse real environment. Although obstacles are large, the robot decides a straight path in a sparse environment.

direction is computed using the topological configuration of the ground floor and obstacles in the front view of the robot captured by the imaging system mounted on the robot.

Appearance-based object recognition is a stable and robust method of volumetric shape recognition from a series of images (Murase and Nayar, 1995). The method is introduced to robotics (Jones et al., 1997; Ulrich and Nourbakhsh, 2000). Applying the appearance-based method to the localisation of the



Figure 8: A dynamic environment. After the door to the corridor is closed, the robot recognises that the door is a part of wall and find a path to the left.

mobile robot, Jones, Andresen, and Crowley (Jones et al., 1997; Ulrich and Nourbakhsh, 2000) developed a method to achieve the localisation of the robot from images without three-dimensional reconstruction of the spatial positions of the objects from an image sequence. The appearance-based method allows the acquisition of spatial features from images without reconstructing the spatial locations of objects in the space. The algorithm which we developed in this paper is an algorithm for robot navigation without reconstructing the three-dimensional locations of obstacles and landmarks in the workspace. In this sense, our algorithm can be categorised as an appearance-based navigation strategy.

The appearance-based navigation is suitable for the small-payload robot, since the method enables the robot to navigate without any maps in the memory and special purpose procedures for the landmark extraction. In applications, the combination of the prepath planning and landmark-based localisation allows the stable, robust and safe navigation of the robot. As shown in a real experiment, our method allows the robot to navigate autonomously even if the configuration of the obstacles in the workspace is changed. In the experiment, the transition of geometric configuration of obstacles in workspace is caused by closing the door to the corridor. Furthermore, a student, who is an obstacle to the robot, walked to the door to close it and returned from the view field of the camera mounted on the robot. This control property is suitable for the collaboration of the robot with human beings, since the motion of the human causes the tentative transition of configuration of the obstacles, which

is not described in the original map on the workspace.

Psychologically, it is known that the optical-flow field is a basic cue for the understanding of motion (Vaina et al., 2004). Our results also suggest that the optical-flow field is a cue for determining the obstacle configuration in a workspace.

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