# MANIPULATION OF PARAMETRIC SURFACES THROUGH A SIMPLE DEFORMATION ALGORITHM

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Abstract: In this paper, we present a novel but simple physics based method to manipulate parametric surfaces. This method can deal with local deformations with an arbitrarily complicated boundary shape. We firstly map a deformation region of a 3D surface to a circle on a 2D parametric plane. Then we derive an approximate analytical solution of a set of fourth order partial differential equations subjected to sculpting forces and the boundary conditions of the circle. With the obtained solution, we show how to create a deformed surface and how sculpting forces and the shape control parameters affect the shape of a deformed surface. Finally, we provide some examples to demonstrate the applications of our proposed method in surface manipulation.

## 1 INTRODUCTION

Surface manipulation is at the heart of geometric modelling and has attracted a lot of research attention.

Depending on whether physics of object deformation is introduced or not, surface manipulation can be divided into purely geometric and physics based. Purely geometric surface manipulation achieves the intended shapes by manually changing the positions of surface points or control points. Physics based surface manipulation obtains different surface shapes by applying virtual forces to deform the surfaces.

Directly manipulating surface points of polygonal models or control points of NURBS models is a commonly used method for purely geometric surface manipulation. In addition, extrusion, blending, sweeping, skinning, filleting, chamfering, and Boolean operations etc. are also frequently applied in shape manipulation (Fleming 1999; Maestri 1999).

In order to improve the efficiency and capability of surface manipulation, free from deformation methods were developed. By simulating the deformations caused by twisting, bending, tapering, or similar transformations of geometric objects, Barr (1984) proposed new operations for shape manipulation. Following Barr's work, Sederberg and Parry (1986) developed a more general approach called free-form deformation (FFD). This method embeds an object in a lattice and achieves the deformations of the object by deforming the lattice. By using the initial lattice points to define an arbitrary trivariate Bézier volume, and allowing the combination of many lattices to form arbitrarily shaped spaces, Coquillart (1990, 1991) introduced Extended Free-Form Deformations (EFFD). Free-form deformation was also investigated by Lamousin and Waggenspack (1994), MacCracken (1996), Hirota et al. (2000), and Feng et al. (2002, 2006).

Purely geometric surface manipulation methods are simpler and more efficient than the physics based methods. However, purely geometric methods do not follow any underlying physical laws. Therefore, if an object is to be modelled by such methods, the quality depends on the skills and perception of modellers. For a same object, different modellers may create somewhat different shapes.

This issue may be resolved by introducing the underlying physics governing the deformation of deformable materials. The surface manipulation based on this consideration is called physics based. It considers material properties and physical laws relevant to surface deformation. This approach has a potential to create more realistic looking objects.

Employing the elasticity theory, Terzopoulos and his colleagues (1987) and (1988) introduced dynamic differential equations for flexible materials



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such as rubber, cloth and paper. This work was extended from elasticity to viscoelasticity, plasticity and fracture (Terzopoulos and Fleischer 1988). By minimizing the energy functional under user controlled geometric constraints and loads, Celniker and Gossard (1991) presented a curve and surface finite element method for shape manipulation. Based on a primal formulation and a hybrid formulation derived from the theory of pure elasticity, Güdükbay and Özgüç (1994) investigated a physically based modeling algorithm to animate deformable objects. In order to deal with mass distributions, internal deformation energies, and other physical quantities of shape manipulation of NURBS, a dynamic NURBS was developed by Terzopoulos and Qin (1994). This method was further investigated to tackle the surfaces with symmetries and topological variability which leads to a dynamic NURBS swung surface (Qin and Terzopoulos 1995). By extending triangular B-splines to triangular NURBS and using Lagrangian mechanics, Qin and Terzopoulos (1997) developed the mathematical model of dynamic triangular NURBS and manipulated the surfaces defined over arbitrary, nonrectangular domains through the finite element solution of the mathematical model. Applying sculpting forces on a surface and formulating and minimizing the energy functional of the surface, Vassilev (1997) proposed a method to manipulate deformable B-spline surfaces. Using the model of a bar network, Léon and Veron (1997) and Guillet and Léon (1998) dealt with the deformation of free-form surfaces. Considering non-homogeneous material properties and conducting the finite element calculations of deformable objects in local frames, McDonnell and Qin (2007) presented a new, physics based shape manipulation method.

Surfaces can also be described by the solution to a partial differential equation subjected to suitably defined boundary conditions. Partial differential equations (PDEs) based modelling was first introduced by Bloor and Wilson (1989, 1990). In order to cope with more complicated surface modelling problems, Bloor and Wilson proposed a spectral approximation method (1996) and a perturbation method (2000). Using the partial differential equation (PDE) method, Ugail et al. (1999) examined how practical surfaces can be constructed interactively in real time. Kubeisa et al. (2004) addressed the problem of interactive design of higher order PDEs. In the work carried out by Ugail (2004), the generation of the spline of a PDE surface and parameterization of the surface by using the spline were investigated. By studying the so-called harmonic and biharmonic Bézier surfaces, Monterde and Ugail (2004) presented a new method of surface generation. By defining the trim curves to be a set of boundary conditions, Ugail (2006) proposed a method to trim PDE surfaces. How Bézier surfaces can be generated from boundary information through a general 4th-order PDE was tackled by Monterde and Ugail (2006). Generalizing the governing partial differential equation to arbitrary order, complex shapes were designed as single patch by Ugail (2007). Incorporating dynamic effects into a fourth order PDE, You and Zhang studied creation of 3D deformable moving surfaces (2003). Using a sixth order PDE and a semi-analytical and semi-numerical solution, Zhang and You (2004) presented a method for surface modelling.

This paper will focus on surface manipulation using an approximate analytical solution to fourth order partial differential equations. It maps an arbitrary deformation region in 3D coordinate space to a circle in 2D parametric plane, achieves the approximate analytical solution of the deformation within the circle, and uses it to manipulate surfaces.

## 2 MATHEMATICAL MODEL

The deformations of a surface can be simulated through those of a thin elastic plate. When subjected a lateral load $q$, the mathematical model describing surface deformations is

$$\frac{\partial^4 \xi}{\partial u^4} + 2\frac{\partial^4 \xi}{\partial u^2 \partial v^2} + \frac{\partial^4 \xi}{\partial v^4} = \frac{q_\xi}{D} \quad (\xi = x, y, z) \tag{1}$$

subjected to the following boundary conditions

$$\xi = 0, \qquad \frac{\partial \xi}{\partial u} = 0, \qquad \frac{\partial \xi}{\partial v} = 0 \quad (\xi = x, y, z) \tag{2}$$

where

$$D = \frac{Eh^3}{12(1-\mu^2)} \tag{3}$$

and $E$ and $\mu$ are Young's modulus and Poisson's ratio, and $h$ is the thickness of a surface.

## 3 SOLUTION

Since the analytical solution of Eq. (1) under the boundary conditions (2) on a circular boundary is obtainable, we take the boundary defined by parametric variables $u$ and $v$ to be

$$u^2 + v^2 - 1 = 0 \tag{4}$$

For the deformation which has both positional and tangential continuities at boundary (2), we take the following functions as the solution of Eq. (1)

$$\xi = m_\xi (u^2 + v^2 - 1)^2 \quad (\xi = x, y, z) \tag{5}$$

where $m_\xi$ is an unknown constant.

Substituting Eq. (5) into (2), boundary conditions are satisfied exactly.

Substituting Eq. (5) into (1), we determine the unknown constant $m_\xi$ and obtain the analytical solution of Eq. (1).

# 4 APPLICATIONS

In order to use the above method to determine the deformations of a 3D surface, we relate a deformation region with an arbitrary boundary shape to a circle.

As shown in Figure 1, we use the length of the boundary of the deformation region and the circle to determine the corresponding points $P$ and $P'$ between them. Then, we find a point $O$ on the 3D surface which corresponds to the geometric centre of the deformation region. The surface curve $OP$ is related to the straight line $O'P'$. With such a treatment, we obtain the one-to-one relationship between all the points on the 3D surface and those within the circle.

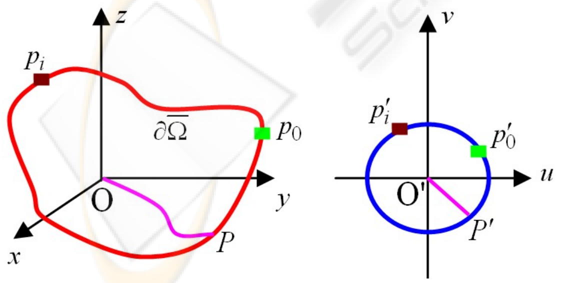



Figure 1: Parameterization of boundary $\partial\overline{\Omega}$ and deformation region.

Finally, we apply a sculpting force $q_\xi$, and use the above method to calculate the deformations of the 3D surface, and superimpose these deformations to the original surface to create the deformed surface. In the subsections below, we will demonstrate this through a number of examples.

## 4.1 Surface Deformations within a Triangle

In this subsection, we investigate how to deform a triangle.

As indicated in Figure 2, by calculating the length of the triangle and the circle, we find the points A′, B′ and C′ on the circle which correspond to the three vertices A, B and C of the triangle, respectively.

Then we calculate the geometric centre O of the triangle from its three vertices. This geometric centre O is related to the centre O′ of the circle.

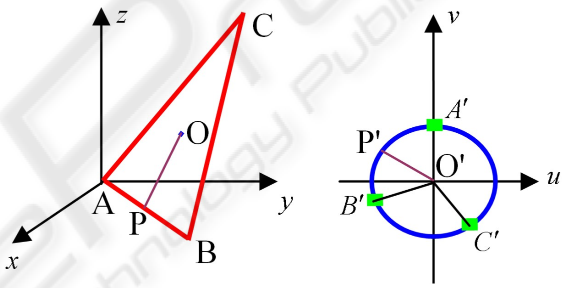
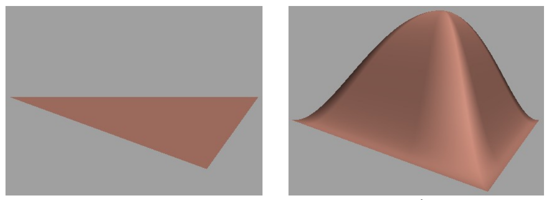



Figure 2: Parameterization of a triangle.

For an arbitrary point $P$ on the boundary of the triangle, we find its corresponding point $P'$ on the circle. The points on the line $OP$ are related to the points on the line $O'P'$. The same method is used to determine the one-to-one relationship of the points between the triangle and the circle.



a b

Figure 3: Surface deformation within a triangle region.

For a triangle deformation region indicated in Figure 3a, we set Young's modulus $E = 10000$, Poisson's ratio $\mu = 0.3$, surface thickness $h = 0.1$ and the sculpting force $q_z = 100$. The deformation of the triangle was obtained and depicted in Figure 3b.

## 4.2 Effect of Material and Geometric Properties

In this subsection, we examine how material and geometric properties affect the shape of a surface.

The deformation region in a 3D coordinate space was shown in Figure 4a. It was mapped into a circle. Basic parameters are taken to be: the material properties $E = 10000$, $\nu = 0.3$, geometric property $h = 0.1$, and sculpting force $q_z = 100$.

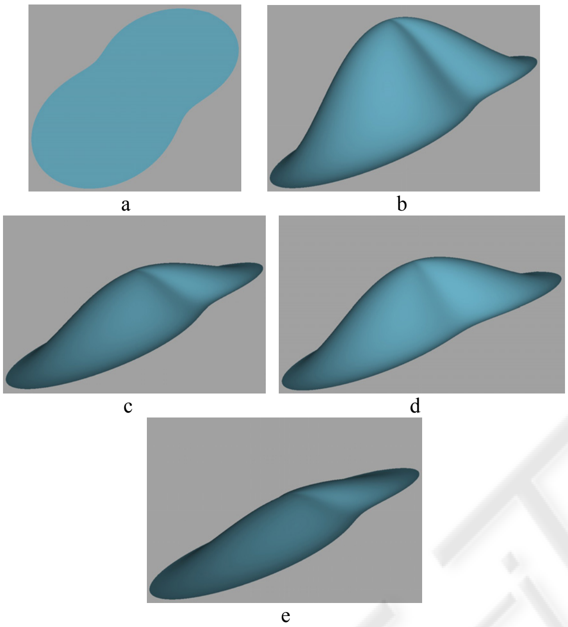


Figure 4: Effect of material and geometric properties.

The deformed surface indicated in Figure 4b was obtained. Only raising Young's modulus to 20000 and keeping all other basic parameters unchanged, the deformation was reduced and the deformed shape in Figure 4c was generated. When Poisson's ratio of the basic parameters was increased to 0.6, the deformation given in Figure 4b was dropped to that in Figure 4d. Increasing the surface thickness to 0.15 also decreases the deformation and produces the shape in Figure 4e.

## 4.3 Effect of Sculpting Forces

Here we study how sculpting forces affect surface deformations. The deformation region in a 3D coordinate space is an ellipse. The original surface shape within the ellipse is depicted in Figure 5a. The surface will be deformed in $z$ direction. Young's modulus E is taken to be $E = 30000$, Poisson's ratio is set to $\nu = 0.3$. Applying a sculpting force $q_z = 50$ on the surface, the surface was pulled upwards and the deformed shape was indicated in Figure 5b. Raising the sculpting force to 200, the deformation was greatly increased as indicated in Figure 5c. Changing both the direction and size of the sculpting force, i. e., setting the sculpting force to -120, the surface was push downwards and the deformed shape in Figure 5d was created. These images indicate that sculpting forces are very useful in surface manipulation.

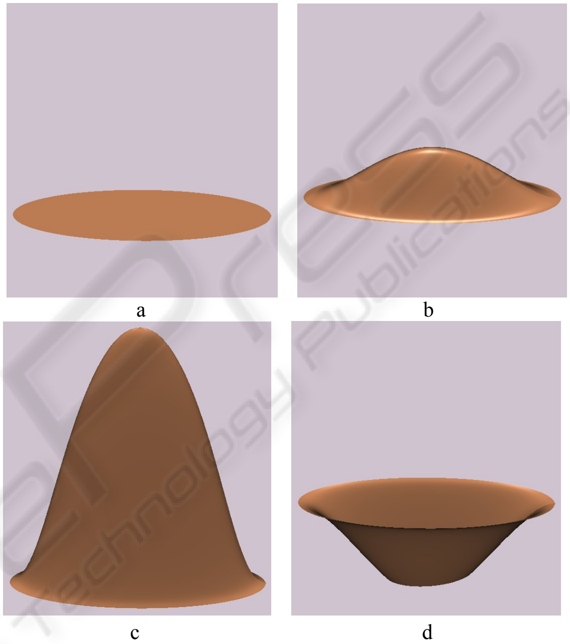


Figure 5: Effect of sculpting force.

## 4.4 Local Deformations

In this subsection, we discuss how to use our proposed method to achieve complex local deformations of 3D models.

For a 3D surface model, we first interactively specify the region which will be deformed. Then we extract the boundary curve of the deformation region. Finally, the above method is used to determine the corresponding relationship between the deformation region and a circle.

Here we give an example to deform a male chest. The undeformed chest was shown in Figure 6a. The boundary of the deformation region on the chest was shown in Figure 6f. By applying different sculpting forces to the deformation region, different deformed shapes were obtained and depicted in Figures 6b, 6c and 6d. A local view of the deformation shape in Figure 6b was given in Figure 6e.

## 5 CONCLUSIONS

A physics based surface manipulation method has been proposed through the above work. For doing this, we examined the relationship between a deformation region in 3D coordinate space and a circle in 2D parametric plane and formulated the corresponding boundary conditions. By constructing proper trial functions, we obtained an approximate analytical solution which exactly satisfies both positional and tangential continuities at the circle and the partial differential equations. With the application examples given in this paper, we discussed how to use the solution to carry out surface manipulation.

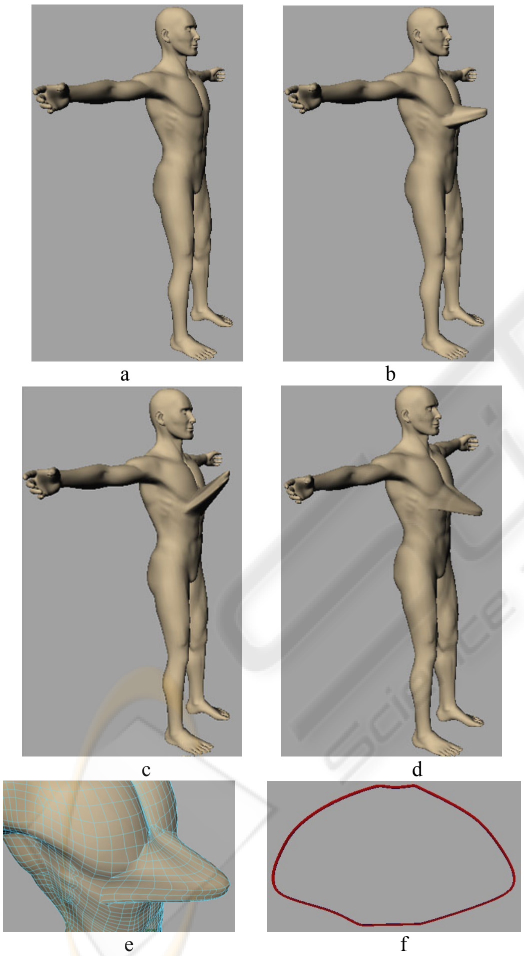



Figure 6: Deformation of a male chest.

The method proposed in this paper can be easily developed into an interactive software tool whereby surface manipulation can be performed easily and in real-time. We intend to develop such a tool in the future.

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