VISUALIZATION OF UNCERTAIN CONTOUR TREES

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Abstract: Contour trees can represent the topology of large volume data sets in a relatively compact, discrete data structure. However, the resulting trees often contain many thousands of nodes; thus, many graph drawing techniques fail to produce satisfactory results. Therefore, several visualization methods were proposed recently for the visualization of contour trees. Unfortunately, none of these techniques is able to handle uncertain contour trees although any uncertainty of the volume data inevitably results in partially uncertain contour trees. In this work, we visualize uncertain contour trees by combining the contour trees of two morphologically filtered versions of a volume data set, which represent the range of uncertainty. These two contour trees are combined and visualized within a single image such that a range of potential contour trees is represented by the resulting visualization. Thus, potentially erroneous topological structures are visually distinguished from more certain structures. Moreover, topological structures can be revealed that are otherwise obscured by data errors. We present and discuss results obtained with a prototypical implementation using well-known volume data sets.

1 INTRODUCTION

Most of the topological structure of scalar volume data sets can be efficiently represented by its contour tree (Freeman and Morse, 1967), which is also a useful data structure for several algorithms in volume visualization.

Unfortunately, contour trees of real volume data sets tend to be extremely large even for small volume data sets (thousands or millions of nodes and edges) since noise in the data results in many local extrema and, therefore, in many nodes of the contour tree. Thus, it is challenging to visualize contour trees computed from real data. Moreover, most researchers agree that one coordinate of the visualized nodes should correspond to an isovalue in the volume data set. With this requirement, however, it is impossible to avoid crossing edges in the graph layout if straight edges are employed.

In recent years, various techniques have been proposed to visualize large contour trees of real volume data sets using dot-and-line diagrams in two and three dimensions with and without colors and/or icons. Moreover, functions, surfaces, and terrains have been proposed as well as combinations of stacked graphs and Sankey diagrams.

However, the problem of uncertain volume data has not been addressed by these approaches although some uncertainty is usually inevitable, for example because of measurements or simulations with finite precision. Moreover, contour trees are susceptible to arbitrarily small changes of the data of individual voxels, which can determine whether isosurface components are separated or form a single component, and therefore result in discrete changes of the contour tree. The lack of visualization techniques for uncertain contour trees is particularly remarkable as uncertainty visualization for many other visualization methods have been suggested in recent years and their importance is widely accepted (Griethe and Schumann, 2006; Johnson and Sanderson, 2003).

While some parts of contour trees are uncertain, other parts may be stable against certain kinds of errors and noise. One objective of this work is therefore to propose a visualization of contour trees that conveys the difference between the uncertain parts of a contour tree and its stable structures. Additionally, our technique allows us to reveal further topological structures, which might have been concealed by noise or other data errors.

The data flow of the proposed technique is illustrated in Figure 1. Our approach is based on the observation that the uncertainty of volume data corresponds to the uncertain existence of components of isosur-
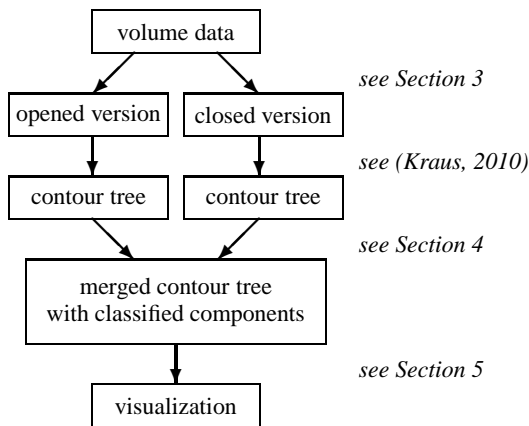
Figure 1: Data flow of the proposed technique.

Figure 2: Visualization of the contour tree of the fuel data set with markers for the 10 isosurfaces shown in Figure 3.

faces and the uncertain separation between these components. In order to detect uncertain components and uncertain separations, we compute two versions of the volume data set by means of grayscale morphology as discussed in Section 3. To this end, we propose a new variant of the opening operator that is capable of preserving small components of isosurfaces.

While one of the two versions of the data set includes the stable components of isosurfaces, the second version includes also uncertain and suspected components. By computing contour trees for both versions and matching their nodes, we can therefore determine which components and separations between components should be considered uncertain as explained in Section 4. In order to visualize the resulting classification as described in Section 5, we extend a recently proposed visualization of contour trees for error-free data (Kraus, 2010).

The main contribution of this work is therefore the use of grayscale morphology to determine the uncertainty of the elements of a contour tree. In particular, a new variant of the opening operator is suggested for this purpose. Moreover, we show how to combine multiple contour trees of different versions of a data set in one visualization and how to visually convey the uncertainty of their elements.

Results of a prototypical implementation of our algorithm are presented in Section 6, while our conclusions and plans for future work are discussed in Section 7.

2 RELATED WORK

One of the first discussions of the contour tree was published by Freeman and Morse (Freeman and Morse, 1967). Efficient algorithms for the computa-
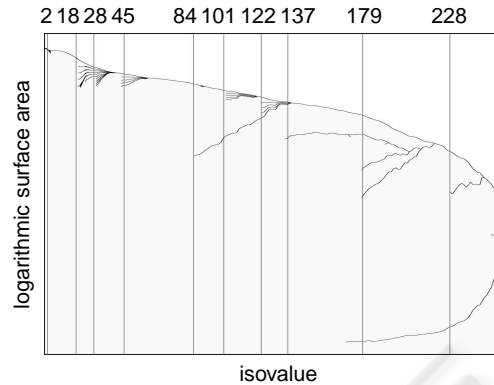
tion of contour trees were published, for example, by Carr et al. (Carr et al., 2004) and Pascucci et al. (Pascucci and Cole-McLaughlin, 2002; Pascucci et al., 2004).

Visualizations of contour trees are often based on dot-and-line diagrams in two dimensions (Bajaj et al., 1997; Pascucci and Cole-McLaughlin, 2002; Carr et al., 2004) or three dimensions (Takahashi et al., 2004a; Takahashi et al., 2004b; Pascucci et al., 2004). Furthermore, the use of icons has been proposed by Shinagawa et al. (Shinagawa et al., 1991). As it is difficult to match the resulting graph drawing to the scalar field, the use of colors has been proposed to identify connected components (Carr et al., 2004; Weber et al., 2007b; Weber et al., 2007a). Furthermore, Weber et al. suggested the metaphor of a topological landscape (Weber et al., 2007b). Note that all these visualization techniques are limited to the visualization of one error-free contour tree at a time.

In our work, contour trees are computed by the method published by Kraus (Kraus, 2010), who also presented a visualization technique for contour trees. As depicted in Figure 2, contour trees are visualized within a logarithmic plot of the area of isosurfaces as a function of the corresponding isovalues. Alternatively, a histogram plot could also be employed. This visualization can be considered a combination of stacked graphs (see the works by Havre et al. (Havre et al., 2002) and Byron and Wattenberg (Byron and Wattenberg, 2008) and references therein) and Sankey diagrams (Riehmann et al., 2005) or flow maps (Phan et al., 2005). Related combinations of stacked bar charts and Sankey diagrams were published by Fry (Fry, 2004, section 4.6) and by Rosvall and Bergstrom (Rosvall and Bergstrom, 2008). The approach is also related to the representation of "level set trees" as one-dimensional functions by Klemelä (Klemelä, 2004), although level set trees are not identical to contour trees.
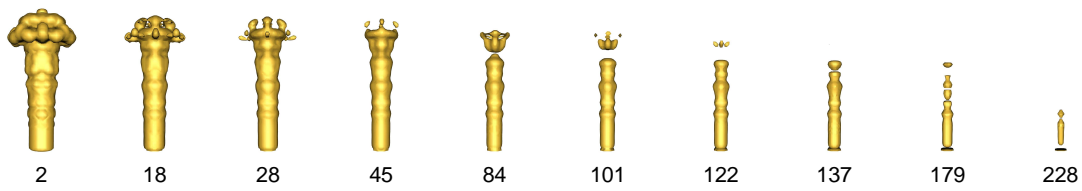
Figure 3: Isosurfaces of the fuel data set for the isovalues indicated in Figure 2.

In this work, we extend the approach of Kraus to the case of uncertain contour trees by combining the visualization of multiple contour trees in a single image. Uncertainty visualization has been an established topic in scientific visualization for several years (Johnson and Sanderson, 2003). A survey of this research was published by Griethe and Schumann (Griethe and Schumann, 2006). One of the most common techniques to visualize uncertainty is the utilization of free graphical variables, in particular color. However, the simultaneous use of color to visualize data and its uncertainty at the same time is problematic. For example, Kardos et al. (Kardos et al., 2006) found that the utilization of saturation to visualize uncertainty is rather ineffective if hue is employed to visualize the actual data. Differences between graphs are usually determined by graph matching techniques and are also often visualized by color coding as discussed, for example, by Delugach and de Moor (Delugach and de Moor, 2005).

Grayscale morphology was published by Sternberg (Sternberg, 1986) and is a well-established image processing technique, in particular for segmentation of medical data. In this context, grayscale morphology is therefore more suitable to determine the uncertainty of parts of the contour tree than simplification methods for contour trees, which were suggested, for example, by Carr et al. (Carr et al., 2004) and Pascucci et al. (Pascucci et al., 2004). Moreover, grayscale morphology allows us to reveal additional topological structures that might have been hidden by data errors.

3 MORPHOLOGICAL IMAGE PROCESSING

We employ opening and closing operators of grayscale morphology (with a flat structuring function) to compute two versions of the data set, which contain uncertain and certain topological structures, respectively. The degree of uncertainty can be controlled by the number of applications of these morphological operators. In this work, however, we only show results for at most one application of each operator.
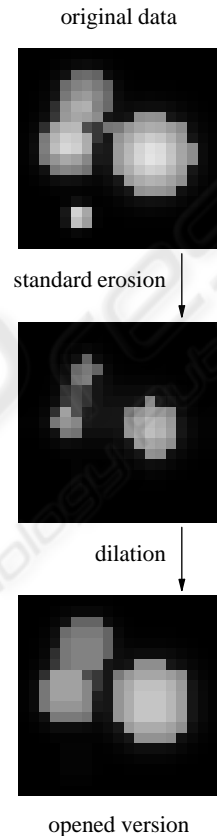
original data

standard erosion

dilation

opened version

Figure 4: Illustration of a standard opening operation (erosion followed by dilation).

3.1 Modified Opening

To reveal uncertain topological structures that are obscured by noise, we apply a new variant of the opening operator.

The standard opening is illustrated in Figure 4 and starts with an erosion, which is applied to each voxel v:

$$f_v \leftarrow \min_{w \in N(v)} \{f_w\}$$

where f_v is the data value of voxel v and $N(v)$ is the set of the 26 neighbors of v and v itself. This erosion is followed by a dilation:

$$f_v \leftarrow \max_{w \in N(v)} \{f_w\}$$

original data

modified erosion / dilation

dilation / erosion

opened version
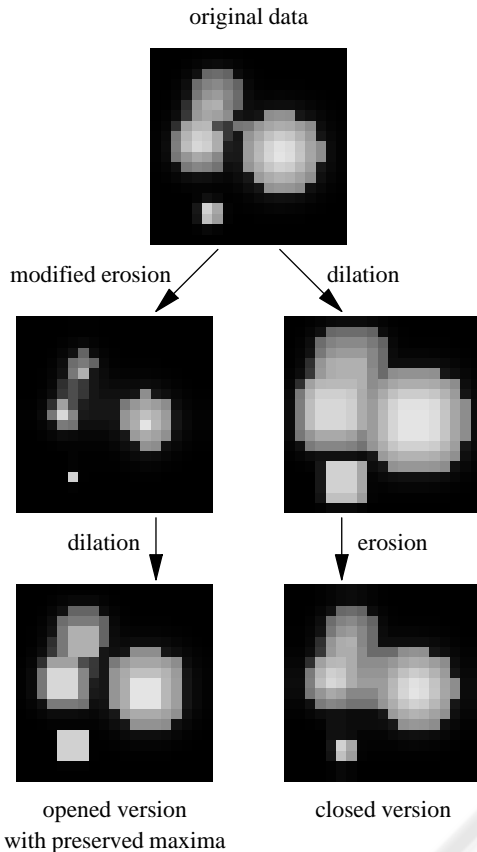with preserved maxima

closed version

Figure 5: Opening with preservation of maxima *(left)* and closing *(right)*.

Almost separated peaks and plateaus in the data set will be separated by the opening operator. Additionally, small separations tend to be widened. Thus, isosurface components are often split into multiple separated components by the opening operator. In terms of the contour tree, this corresponds to additional edges. Therefore, topological structures that were obscured by noise or other errors can be revealed by the opening operator.

However, the standard opening operator also removes peaks from the data as shown in Figure 4. Since these peaks correspond to small isosurface components, the opening can also prune edges from the contour tree. There are several reasons why this removal of edges is undesirable in the context of this work. First of all, small isosurface components might be removed by the standard opening operator and joined with a nearby component by the closing operator; thus, they vanish in both cases even though they exist in the original data. This contradicts the idea of representing the range of uncertainty by only two versions of the data. Furthermore, the merging and rendering of the two contour trees computed from

the opened and closed volumes can be significantly simplified if the "closed tree" (the contour tree computed from the closed data set) is a pruned version of the "opened tree" (the contour tree computed from the opened data set); i.e., if no edges of the latter are removed.

Due to these considerations, we propose a new variant of the opening operator that tries to preserve arbitrarily small peaks. While the dilation is unchanged, the modified erosion is:

$$f_v \leftarrow \begin{cases} f_v & \text{if } f_v = \max\limits_{w \in N(v)} \{f_w\} \\ \min\limits_{w \in N(v)} \{f_w\} & \text{otherwise} \end{cases}$$

This erosion operator is illustrated in Figure 5. It makes sure that the data values of local maxima are preserved by the corresponding opening operator. While it might still remove some very small isosurface components, it cannot remove components that a larger than about one voxel.

3.2 Closing

The standard closing consists of a dilation, i.e.,

$$f_v \leftarrow \max_{w \in N(v)} \{f_w\},$$

followed by an erosion, i.e.,

$$f_v \leftarrow \min_{w \in N(v)} \{f_w\}.$$

The closing joins peaks and plateaus that are close to each other as illustrated in Figure 5. Thus, isosurface components are often joined and their total number is reduced. The remaining isosurface components are therefore considered particularly stable or "certain." In terms of the contour tree, this corresponds to a pruning of the tree. In fact, our algorithm assumes that the closed tree is a pruned version of the opened tree. Therefore, it is unnecessary to preserve local extrema in the way we proposed for the opening operator.

3.3 Variants

By modifying the neighborhood $N(v)$ or applying the dilation and erosion operators multiple times, the level of uncertainty can be adjusted to specific requirements.

It should also be noted that the employed visualization of the contour tree (Kraus, 2010) is a conservative approximation in the sense that it never shows a separation between two isosurface components unless they are clearly separated in the volume data, i.e., only rather stable components are visualized. If this kind

of conservative visualization is employed, the closing might not be necessary but the original data can be used instead.

4 MERGING MULTIPLE CONTOUR TREES

After different versions of the original data set have been compute by morphological image processing (see Section 3), an approximation to the contour tree is computed for each version of the data set as described in (Kraus, 2010). Then, these contour trees are merged into one tree and the uncertainty of their nodes is determined.

While it is possible to merge any number of contour trees, we will focus on the case of just two trees: the "opened tree," which is based on the morphologically opened data set, and the "closed tree," which is based on the morphologically closed data set. Furthermore, we will assume that the closed tree is a pruned version of the opened tree; i.e., it lacks some (uncertain) nodes of the opened tree but it has no additional nodes.

The approximate computation of the contour tree described in (Kraus, 2010) partitions the whole data range into uniform intervals. For each interval, it determines the voxels with data values in the interval and computes connected groups of these voxels, which are used to approximate connected components of isosurfaces. These are the nodes of the computed contour tree. Moreover, overlaps between connected components of neighboring intervals are recorded in order to construct edges of the contour tree.

In this work, we also record the largest overlap of each connected component in the opened data set with the connected components in the closed data set for the same data interval. If no such overlap exists for a specific connected component of the opened data set then this component is removed from the processing.

The recorded overlaps allow us to match the nodes of the two trees instead of matching their tree structures. Specifically, we distinguish the following cases for each node of the closed tree:

1. No node of the opened tree is matched to a specific node of the closed tree: This contradicts our assumption that the opened tree contains all nodes of the closed tree. Nonetheless the node of the closed tree is included in the merged tree since this does not result in any ambiguities.

2. Exactly one node of the opened tree is matched to the node of the closed tree: In this case the two nodes are merged into one; in particular, all recorded overlaps with nodes of neighboring data intervals are inherited from the two nodes.

3. More than one node of the opened tree is matched to the node of the closed tree: In this case, the nodes of the opened tree are included in the merged tree but the node of the closed tree is not. Recorded overlaps with the latter node are ignored since they are ambiguous.

Merging nodes consists mainly of merging the lists of edges to other nodes of neighboring data intervals. Moreover, references to merged nodes have to be adjusted. The main advantage of this approach is that the resulting merged tree can be visualized in a similar way as described in (Kraus, 2010).

5 VISUALIZING MERGED CONTOUR TREES

Our visualization of the merged contour tree is based on a visualization technique for contour trees of error-free data, which was presented in (Kraus, 2010). This method computes x coordinates of the nodes based on the data intervals of the corresponding connected components. The y coordinates are computed by first sorting the nodes of each data interval in order to minimize the number of crossing edges, in particular of edges that include at least one relatively large connected component. Then, partial sums of the surface area of the connected components are computed to determine the actual y coordinates of the nodes.

The resulting grid of vertices is used to draw separating lines between nodes that are not connected by edges, which correspond to recorded overlaps between connected components as discussed in Section 4. More details and several examples of this visualization technique are presented in (Kraus, 2010).

In order to adapt this visualization to the merged contour tree, we distinguish between separating lines that mark uncertain structures and all others. To classify a separating line, the two connected components that are separated by the line are considered. If they are both part of the "opened tree" (see Section 4) *and* the primary overlaps of both components refer to the same connected component of the "closed tree" then the separating line is considered uncertain since it is an internal separation within the same certain component of the closed tree.

It should be noted that arbitrarily many components of the opened tree can be associated with a single component of the closed tree. Moreover, these uncertain separations can also extend over several data intervals and thus form arbitrarily deep tree structures.

Table 1: Timings for computing and rendering the visualizations in Figure 6. All visualizations decompose the data range into 200 intervals. The fuel data set and the silicium data set are trilinearly upsampled versions of the publicly available data sets (Bartz, 2005; Levoy, 2001).

data set	size	time in seconds			
		original	original + closed	original + opened	closed + opened
fuel	$127 \times 127 \times 127$	5	110	112	79
silicium	$195 \times 67 \times 67$	16	188	219	197
CT head	$256 \times 256 \times 113$	86	384	470	452

The lines between uncertain structures are rendered in a different color than the other separating lines. The former color should be closer to the background color in order to convey the idea of a weaker or less certain separation. In this work, however, bright red is used in order to emphasize these lines. Examples of the visualization are presented in the next section.

6 RESULTS

We present results for three publicly available data sets (Levoy, 2001; Bartz, 2005). Figure 6 shows a comparison between the original contour tree visualization (Kraus, 2010) for these data sets in the second row and the proposed generalization of this visualization in the third, fourth, and fifth row. The differences between pairs of contour trees are marked by red lines in order to emphasize them.

The third row depicts the differences between the contour trees of the original data and the contour trees of the morphologically closed data. Since the closing operation can join separated isosurface components, some parts of the original separating lines are no longer valid. These parts are marked in red in the visualization. In most cases only the ends of the separating lines are affected. There are, however, also some cases (in particular in complex data sets) where a separation vanishes for all relevant isovalues. Since a single closing operation will only join hardly separated components, the red lines indicate uncertain separations.

In the fourth row the differences between the contour trees of the original data and the contour trees of the morphologically opened data is shown. The opening operation can separate connected isosurface components; thus, additional separating lines are introduced, which are marked in red. These lines usually extend lines of the original visualization to a larger range of isovalues. However, there are also cases of additional structures, which are only revealed by the opening operation. The opening operation can sepa-

rate components, which are connected in the original data. Thus, the additional separating lines are only conjectured, i.e., they are less certain than the separations in the original visualization.

For the sake of completeness, the fifth row combines the contour trees of the morphologically closed and opened data set. Again, the differences are marked in red.

Our prototypical implementation was tested on a PC equipped with 2 GB RAM and two 3.6 GHz Pentium 4 CPUs (only one was used to run the program). Timings for the three data sets depicted in Figure 6 are summarized in Table 1. The computational costs of merging two contour trees appear to be rather high; however, no attempts have been made to optimize the code.

7 CONCLUSIONS AND FUTURE WORK

By employing grayscale morphology we can compute multiple versions of a data set, which include either more or less certain separations of isosurface components than the original data set. Using these versions of the data, the proposed visualization of multiple contour trees in a single image allows us to visually distinguish the more certain parts of a contour tree from the less certain parts. Thus, it enables us to visualize uncertain structures in contour trees. Apart from this interpretation, our proposed visualization can also be considered a preview of the effects of grayscale morphological filters.

This work demonstrates that uncertainty visualization is feasible even for very large graphs. We achieved this goal by matching the objects represented by the nodes of two graphs (i.e., measuring the overlap of isosurface components) instead of matching the abstract structure of the two graphs, which would introduce additional ambiguities and uncertainties.

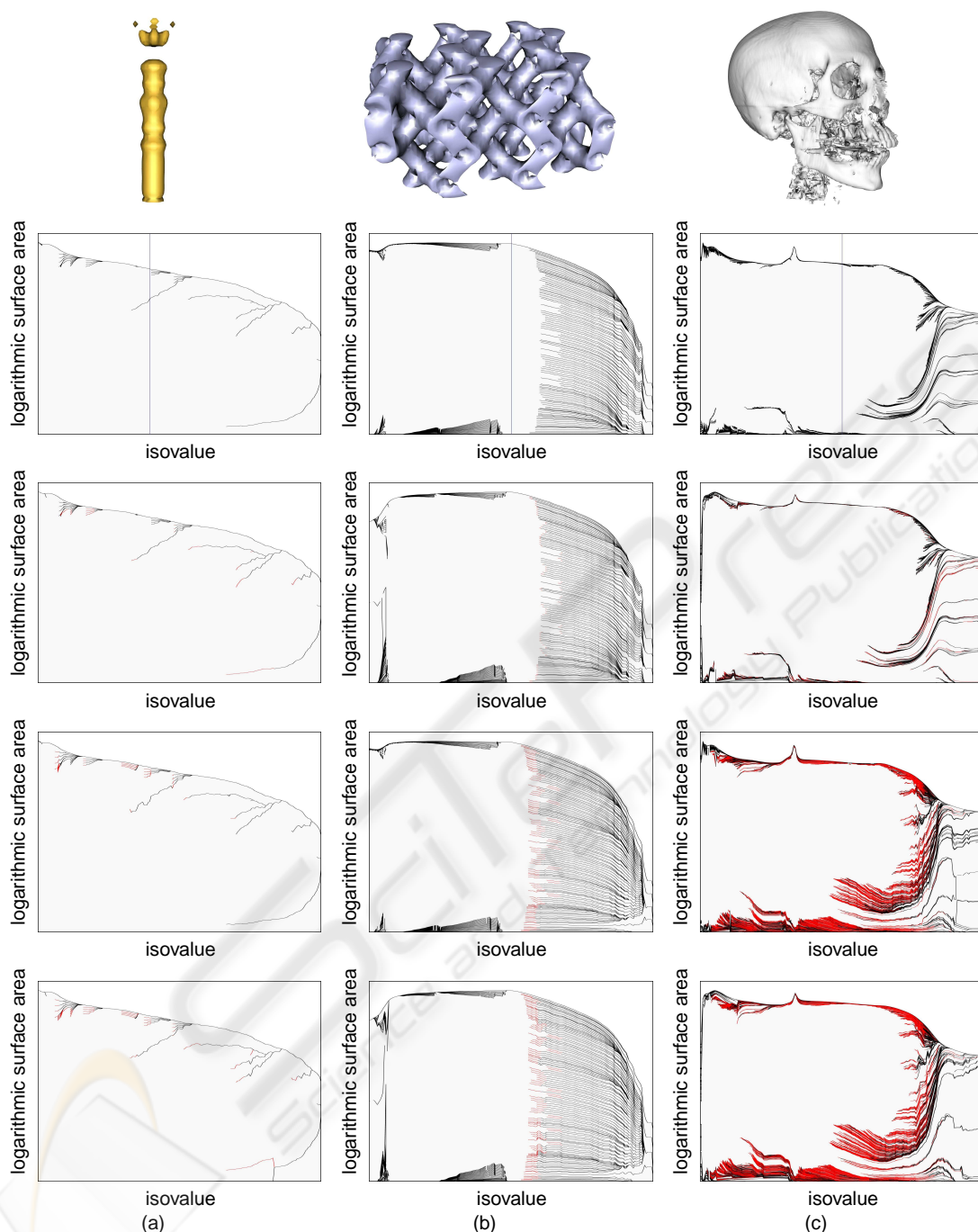Future work includes performance optimizations, the adjustment of the visualization to the actual de-

Figure 6: Visualizations of contour trees in plots of the logarithmic surface area for (a) the fuel data set, (b) the silicium data set, and (c) the CT head data set. From top to bottom: an isosurface from the data set (first row), the standard contour tree visualization with a vertical bar indicating the isovalue corresponding to the isosurface (second row), differences to the morphologically closed data set marked in red (third row), differences to the morphologically opened data set marked in red (fourth row), and differences between the closed and the opened data set marked in red (fifth row).

gree of uncertainty of the data, and the computation and visualization of the degree of uncertainty. Further plans include generalizations to different image processing operations, and the integration of alternative

computations of contour trees.

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