TOWARDS RECOMMENDER SYSTEMS BASED ON KALMAN FILTERS

A New Approach by State Space Modelling

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Abstract: This position proposes an original approach based on a new formulation of a recommender system. This formulation uses state space description for users and web resources. Then states and parameters are predicted and estimated with two stages algorithms of a Kalman filter. In this paper, we give the main theoretical results of this original approach.

1 INTRODUCTION

In Web-based services of dynamic content, recommender systems face the difficulty of identifying new items and providing recommendations for users.

Personalized recommendation has become a desirable feature of Web sites to improve customer satisfaction and customer retention. Recommendation involves a process of gathering information about site visitors, managing the content assets, analyzing current and past user interactive behaviour, and, based on the analysis, delivering the right content to each visitor.

Recommendation methods can be distinguished into three different approaches : rule-based filtering, content based filtering and collaborative filtering. Collaborative filtering (CF) is one of the most successful and widely used recommender system technology. CF analyzes users ratings to recognize commonalities between users on the basis of their historical ratings, and then generates new recommendations based on like-minded users' preferences.

The main idea of this paper is to propose an alternative way for recommender systems. Our work is based on the following assumption: we consider Users and Web resources as a dynamic system described in a state space. This dynamic system can be modelled by techniques coming from control system methods. The obtained state space is defined by state variables which are related to the users. We consider that the states of the users (by states, we understand « what are the resources they want to see in the next step ») are measured by the grades given to one resource by the users.

In this paper, we are going to present the effectiveness of Kalman filtering based approach for recommendation. After a short introduction, we will detail the backgrounds of this approach i.e. state space description and Kalman filter. Then, we expose the applied methodology. Our conclusion will give some guidelines for future works.

2 BACKGROUNDS

This part is devoted to the presentation of the theoretical backgrounds of the used techniques.

2.1 State Space Modelling

A state space representation is a mathematical model of a physical system having a set of input, output and state variables related by first-order differential equations. Inputs, outputs and states are expressed as

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vectors and the differential and algebraic equations are written in matrix form. The state space representation provides a compact way to analyze systems with multiple inputs and outputs. The state variables are the smallest possible subset of system variables that can represent the entire state of the system at any given time. Considering the standard formulation of a state space system, with the following terms :

x_k : state vector at time k

y_k : measurement vector at time k

i_k : inputs vector

A, B, E and C are matrices with the appropriate dimensions. This state space description can be seen in three manners:

- we consider that the whole dynamic system (users and web resources) can be described by a linear non singular system :

$$\begin{cases} x_{k+1} = Ax_k + Bi_k \\ y_k = Cx_k \end{cases} \tag{1.a}$$

- to introduce interactions between internal components, we will consider that the system could be described by linear singular system :

$$\begin{cases} Ex_{k+1} = Ax_k + Bi_k \\ y_k = Cx_k \end{cases} \tag{1.b}$$

- to take into account the complexity of the interactions, we can consider a non linear system

$$\begin{cases} x_{k+1} = f\left(x_k, A\left(x_k\right), B, i_k\right) \\ y_k = Cx_k \end{cases} \tag{1.c}$$

2.2 Kalman Filtering

The Kalman filter addresses the general problem of estimating the state of a system described by equations as (1.a) to (1.c). We need to introduce random variables w_k and v_k which represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions.

The Kalman filter estimates a process by using a feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements.

The equations for the Kalman filter are of two groups :

- time update equations which are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step.
- measurement update equations which are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

The time update equations can be seen as predictor equations, while the measurement update equations can be thought of as corrector equations. The measurement update equations are responsible for the feedback—i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. After each time and measurement update pair, the process is repeated with the previous a posteriori estimates used to project or predict the new a priori estimates. This recursive nature is one of the very main features of the Kalman filter

3 OUR ORIGINAL APPROACH

3.1 State Space Description

Our approach will be based on the following representation of the system represented by Users and Resources (which will be called items). Our idea will be to introduce time (represented by k) as follows:

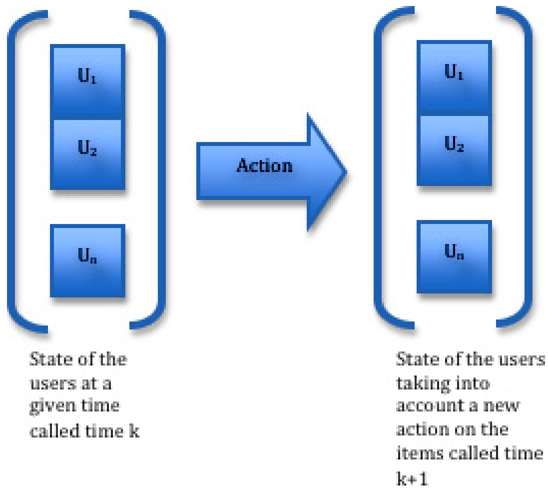

Figure 1: State representation.

Knowing this state representation, we can see the global system as a block scheme as presented in Figure 2:

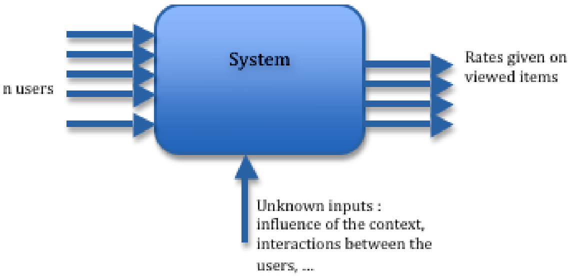

Figure 2: Block scheme.

To be in a most general case (including the influence of the context, unmodelled events, unknown inputs, interactions between users, …), we propose to work on singular state space system. This assumption will give us the possibility to take into account the interaction between users (the interactions will be users to users at time k, users at time k and users at time k-i). We consider that:

u_k^i : vector containing the users at time k – state vector with i varying from 1 to n, the number of users

$item_k^j$: vector containing the resources at time k – measurement vector with j varying from 1 to l, the number of available resources.

$$\begin{cases} E_{k+1}\begin{bmatrix} u_{k+1}^1 \\ u_{k+1}^2 \\ \cdots \\ u_{k+1}^n \end{bmatrix} = M_k \begin{bmatrix} u_k^1 \\ u_k^2 \\ \cdots \\ u_k^n \end{bmatrix} + \varepsilon_k \\ \begin{bmatrix} item_k^1 \\ item_k^2 \\ \cdots \\ item_k^l \end{bmatrix} = H_k \begin{bmatrix} u_k^1 \\ u_k^2 \\ \cdots \\ u_k^n \end{bmatrix} + \varsigma_k \end{cases} \tag{2}$$

H_k: measurement matrix with $l \times n$ dimensions

ε_k: state noise – we assume that this term will represent the lack of knowledge and/or the disturbances due to the model

ζ_k: measurement noise – we assume it introduces disturbances due to the evaluation process (grades given by the users)

The following assumptions :

- users are the state variables and Kalman filtering will predict and estimate the future states

- web resources are the measurements linked to states by matrix H_k which contain the past history

- under observability (Observability is a measure for how well internal states of a system can be inferred by knowledge of its external outputs) conditions

3.2 System Transformations

From (Gantmatcher, 1959), there are two non singular matrices P and Q such as system () is equivalent to the following one :

$$\begin{cases} \begin{bmatrix} u_{k+1}^1 \\ u_{k+1}^2 \\ \cdots \\ u_{k+1}^n \end{bmatrix}_1 = M_k^1 \begin{bmatrix} u_k^1 \\ u_k^2 \\ \cdots \\ u_k^{n_1} \end{bmatrix}_1 + \varepsilon_k^1 \\ \overline{N}\begin{bmatrix} u_{k+1}^1 \\ u_{k+1}^2 \\ \cdots \\ u_{k+1}^n \end{bmatrix}_2 = \begin{bmatrix} u_k^1 \\ u_k^2 \\ \cdots \\ u_k^n \end{bmatrix}_2 + \varepsilon_k^2 \\ y_k = H_k^1 \begin{bmatrix} u_k^1 \\ u_k^2 \\ \cdots \\ u_k^{n_1} \end{bmatrix}_1 + H_k^2 \begin{bmatrix} u_k^1 \\ u_k^2 \\ \cdots \\ u_k^{n_1} \end{bmatrix}_2 + \varsigma_k \end{cases} \tag{3}$$

Using these matrix transformations, we propose the state canonical form which decomposes the system in subsystems. We are going to link these subsystems to communities of users.

3.3 Estimation Procedure

The estimation procedure is based on a 2 stages filter (bootstrap algorithm):

- estimation of M_k : parameters estimation is done using a weighed least square algorithm :

$$\begin{bmatrix} item_k^1 \\ item_k^2 \\ \dots \\ item_k^l \end{bmatrix} = \Phi_k^T \Theta_k + \varepsilon_k \quad (4)$$

Φ_k: vector containing states of the system

Θ_k: matrix containing parameters

Estimation algorithm:

$$\begin{cases} \hat{\Theta}_{k+1} = \hat{\Theta}_k + M_{k+1}\Phi_k \left(y_{k+1} - \Phi_k^T \hat{\Theta}_k \right) \\ M_{k+1} = \left(\lambda M_k^{-1} + \Phi_k \Phi_k^T \right)^{-1} \end{cases} \quad (5)$$

State estimation – Kalman filter

State estimation by Kalman filtering will act using the following structure:

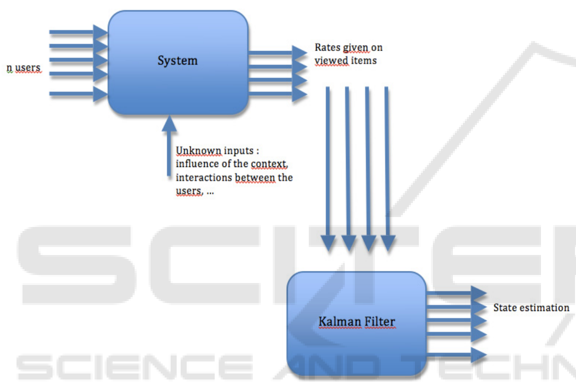

Figure 3: Kalman filtering implementation.

This stage is devoted to state prediction and estimation using the following relations:

$$\begin{cases} \begin{bmatrix} \hat{u}_{k+1/k}^1 \\ \hat{u}_{k+1/k}^2 \\ \dots \\ \hat{u}_{k+1/k}^n \end{bmatrix} = M_k \begin{bmatrix} \hat{u}_{k/k}^1 \\ \hat{u}_{k/k}^2 \\ \dots \\ \hat{u}_{k/k}^n \end{bmatrix} \\ P_{k+1/k} = M_k P_{k/k} M_k^T + Q_k \\ \begin{bmatrix} \hat{u}_{k+1/k+1}^1 \\ \hat{u}_{k+1/k+1}^2 \\ \dots \\ \hat{u}_{k+1/k+1}^n \end{bmatrix} = \begin{bmatrix} \hat{u}_{k+1/k}^1 \\ \hat{u}_{k+1/k}^2 \\ \dots \\ \hat{u}_{k+1/k}^n \end{bmatrix} + K_{k+1} \left(\begin{bmatrix} item_{k+1}^1 \\ item_{k+1}^2 \\ \dots \\ item_{k+1}^l \end{bmatrix} - H_k \begin{bmatrix} \hat{u}_{k+1/k}^1 \\ \hat{u}_{k+1/k}^2 \\ \dots \\ \hat{u}_{k+1/k}^n \end{bmatrix} \right) \\ K_{k+1} = P_{k+1/k} H_{k+1}^T \left(H_{k+1} P_{k+1/k} H_{k+1}^T + R_{k+1} \right)^{-1} \\ P_{k+1/k+1} = \left(I - K_{k+1} H_{k+1} \right) P_{k+1/k} \end{cases} \quad (6)$$

3.4 Expected Results

Using these algorithms, we are going to compute predictions and estimations of the states of the system i.e. the predicted and estimated ratings given by the users to the items. Moreover, this approach will take into account interactions between subsets of users and will be an alternative way in user's recommendations. Thus, if we can estimate ratings given by the users, we can use them as an indicator.

4 CONCLUSIONS

In this position, we propose the theoretical backgrounds for implementing state spaces modelling to describe the dynamical behaviours of the systems composed by users, web resources and their past. This new original formulation describing relationships between states (users) and measurements (resources) will give us the opportunity to show the internal subsystems (communities), the cross links between these subsystems, the dynamical behaviour of this kind of complex system. Then, having all these informations, and choosing the appropriate model (linear, singular, non linear), we will use the performances of Kalman filtering to estimate and to predict the future states of the system.

REFERENCES

B. D. O Anderson, J. B. Moore, Optimal filtering, Englewood Cliffs, *Prentice Hall*, 1979

M. Boutayeb, S. Nowakowski, M. Darouach, A failure detection and accommodation method for singular systems with unknown parameters and states Gant-matcher, *The theory of matrices*, Vol. 1 and 2, 1959

B. Mobasher and S. S. Anand, editors. Intelligent Techniques for Web Personalization, volume 3169 of Lecture Notes in Artificial Intelligence. *Springer-Verlag*, 2005.

O. Nasraoui. World Wide Web personalization. In J. Wang, editor, Encyclopedia of Data Warehousing and Mining, pages 1235–1241. *Idea Group*, 2005

R. Guttman, A. Moukas, and P. Maes. Agent-mediated electronic commerce: A survey. *Knowledge Engineering Review*, 13(3), June 1998

M. Balabanovic and Y. Shohan. Fab: Content-based, collaborative recommendation. *Communications of the ACM*, 40, 1997

R. Burke. Hybrid systems for personalized recommend-dations. In B. Mobasher and S. S. Anand, editors, *Intelligent Techniques for Web Personalization. Springer-V*erlag, 2005

A. Das, M. Datar, A. Garg, and S. Rajaram. Google news personalization: scalable online collaborative filtering. *In Proceedings of the International World Wide Web Conference*, 2007

J. B. Schafer, K. J., and J. Riedl. Recommender Systems in e-Commerce. In *Proceedings of the ACM Conference on Electronic Commerce*, 1999

T. Kailath Linear systems, Prentice Hall, Information and System sciences series, 1980