

# A COMPARISON OF FOUR UNSUPERVISED CLUSTERING ALGORITHMS FOR SEGMENTING BRAIN TISSUE IN MULTI-SPECTRAL MR DATA

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**Abstract:** The effects of atrophy and diffusion of the boundary between grey and white matter, common in elder individuals, represents a difficult problem for segmentation, not observed in healthy younger adults. The aim of this study is to evaluate four well-known unsupervised clustering algorithms in brain tissue segmentation using MR scans with atrophies and lesions. The brain is segmented into 3 different types: white matter, grey matter and CSF. We used four MR sequences: T1W, T2W, T2\*W and FLAIR to classify each voxel in the image. No spatial information was used. The algorithms tested were  $k$ -means, EM (Gaussian mixture), MVQ (minimum variance quantisation) and Mean Shift. The datasets were acquired from an aged cohort ( $> 70$  years). The resulting segmentations were quantitatively compared to expertly collected ground truth on 12 datasets, using the Dice coefficient as an overlap measure. The classification algorithms could be ranked in the following order: MVQ,  $k$ -means, EM and MeanShift from best to worst. The MVQ algorithm did best of all with over a .9 Dice overlap on CSF, and over .8 on white matter.

## 1 INTRODUCTION

Regional cerebral atrophy is a well known feature of neurodegenerative diseases such as Alzheimer's disease. Quantitative measures of regional atrophy based on volume can serve as an aid to diagnosis for the neurologist for such diseases although presently these approaches are not suitable for routine clinical diagnosis (Head et al., 2005; Lehtovirta et al., 1995; Busatto et al., 2003; Good et al., 2001; Miller et al., 1980; Burton et al., 2002).

Previous automated techniques typically involve generating a binary mask of the tissue types involved (normally white matter, grey matter and cerebral spinal fluid), e.g. (Crum, 2007), but can also generate an estimate of the fraction of each tissue in every voxel, e.g. (Thacker and Jackson, 2001a). The assignment of discrete labels to each voxel can be tackled in a variety of ways falling into two broad categories — supervised and unsupervised. The former includes a

learning step whereby many examples are provided to the algorithm. In an unsupervised setting there is no such step. For a supervised example in the context of grey and white matter segmentation, see (Vrooman et al., 2007) where spatial coordinates that are known to correspond to white matter or grey matter are used as initial seed points to seed a kNN ( $k$ -nearest neighbour) classifier.

Semi-supervised approaches are also used. In (Murgasovaa, 2009) the authors generate a probabilistic atlas and use it to drive the evolution of the EM algorithm.

On the fully unsupervised side, many papers use  $k$ -means on either single images or pairs of images, for example (Blatter et al., 1995). (Pham et al., 2000) gives an overview of all well known segmentation techniques used in grey matter white matter segmentation up until 2000.

The SFC Brain Imaging Research Centre at the University of Edinburgh has developed a multispec-

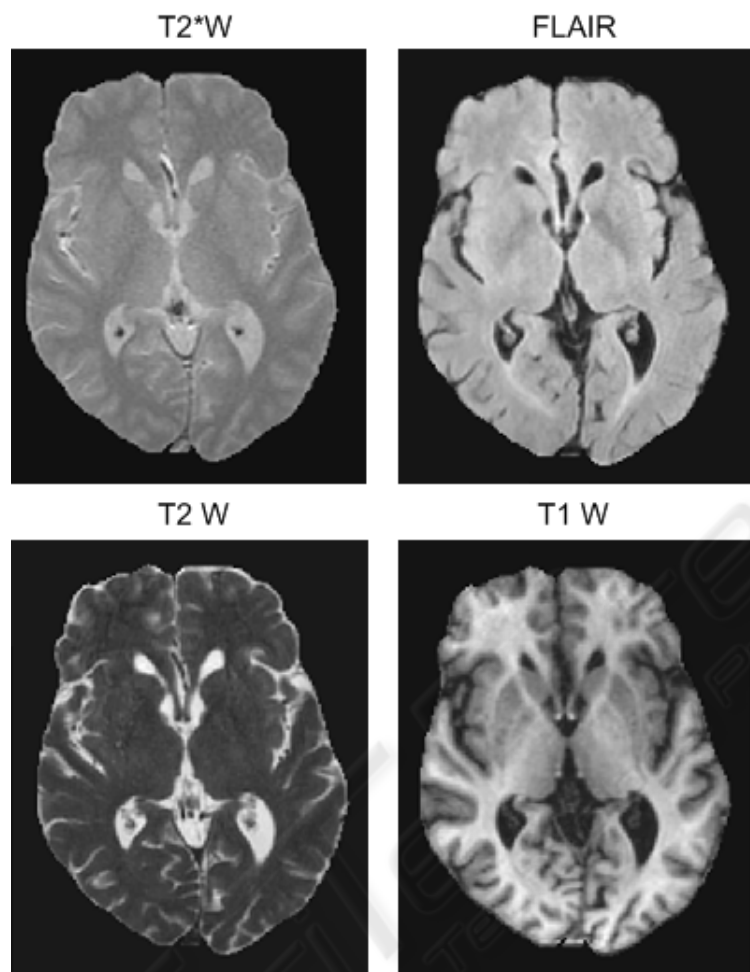


Figure 1: T2\* W, FLAIR, T2 W, T1 W, axial slice of coregistered sequences from a subject after brain extraction.

tral approach for segmenting tissue types in normal and abnormal brains, including white matter lesions (WMLs), based on mapping two magnetic resonance (MR) structural sequences in the red/green colour space and using minimum variance quantisation (MVQ) to cluster and quantify the tissues. The current study builds on that work.

For a relevant (albeit dated) comparison between a supervised and an unsupervised clustering techniques (neural networks) see (Hall et al., 1992). Due to the advantages that unsupervised techniques have for achieving an automatic segmentation process in reduced number of steps, we decided to evaluate the performance of four well-known unsupervised clustering algorithms *k*-means (MacQueen et al., 1966), EM (Gaussian mixture) (Dempster et al., 1977), MVQ (minimum variance quantization) (Xiang and Joy, 1994) and Mean Shift (Comaniciu and Meer, 2002).

It is known that the use of two or more se-

quences increases the information at each voxel and aids the automated classification (Thacker and Jackson, 2001a). This paper addresses the problem of fully automated detection of white matter/grey matter and CSF from a multispectral image (we used T1 weighted, T2 weighted, T2\* weighted and FLAIR), in an unsupervised setting.

The structure of the paper is as follows. Section 2 details the acquisition parameters of the scans in question. Section 3 describes the actions performed on the datasets before they were processed by the automated algorithms. Section 4 motivates the approach and graphically explores the problem. Section (5) describes how the algorithms were validated and how they operate. It addresses a subtle design choice relating to whether the data should be analysed on a per slice or volume basis. We then describe the clustering algorithms used, their merits and their disadvantages. Section (6) states numerically how each algorithm performed.

## 2 MATERIALS

We use 12 sets of structural MR sequences from the Disconnected Mind Study (<http://www.disconnectedmind.org.uk>), which aims to understand how and why some older people's cognitive function deteriorates more than others, and find associations between brain physiology and cognitive performance in a population of more than 1000 volunteers of the Lothian Birth Cohort 1936.

All MR images were rated for WML and atrophy using validated qualitative rating scales (Fazekas et al., 2002; Longstreth et al., 1996; Wahlund et al., 1990) by an experienced neuroradiologist (author JMW). Based on these ratings, the sample for the present work was selected so as to represent the full range of WMLs across a range of degrees of brain atrophy. The MR sequences in this cohort study were:

- A **T1-weighted** (T1W) brain volume was acquired ( $TR = 9ms$ ;  $TE = 4ms$ ; coronal acquisition). The 3D volume matrix size was  $156 \times 256 \times 256$ , with a voxel size of  $1.3 \times 1 \times 1mm^3$ .
- A **T2-weighted** (T2W) brain volume was acquired ( $TR = 11300ms$ ;  $TE = 100ms$ ; axial acquisition). The 3D volume matrix size was  $80 \times 256 \times 256$ , with a voxel size of  $2 \times 1 \times 1mm^3$ .
- A **T2\*-weighted** (T2\*W) brain volume was acquired ( $TR = 940ms$ ;  $TE = 15ms$ ; axial acquisition). The 3D volume matrix size was  $80 \times 256 \times 256$ , with a voxel size of  $2 \times 1 \times 1mm^3$ .
- A **FLAIR** brain volume was acquired ( $TR = 9000ms$ ;  $TE = 147ms$ ; axial acquisition). The 3D volume matrix size was  $80 \times 256 \times 256$ , with a voxel size of  $2 \times 1 \times 1mm^3$ .

Figure 2 shows an axial slice of an individual's brain with 3 sequences (T2\*W, FLAIR and T1W) fused into one image, with each one occupying a single colour channel (red, green and blue, respectively). Figure 1 shows the same slice, but as four separate images. As described below, the images have been rigidly co-registered and irrelevant material (such as the skull) has been removed.

## 3 PREPROCESSING

Each set of four structural sequences per subject were affinely registered to each other using the FSL application FLIRT (<http://www.fmrib.ox.ac.uk/fsl/flirt/>). FLIRT is an open source, command line linear registration tool maintained by the Analysis Group,

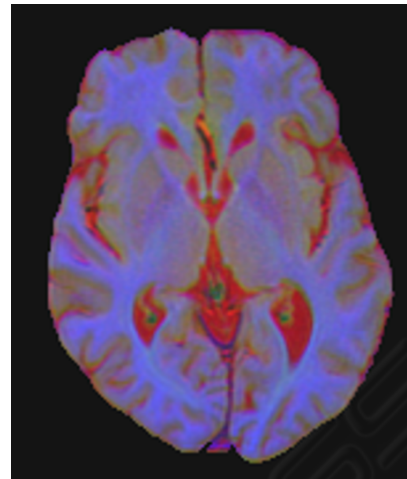


Figure 2: The T2\*W, FLAIR and T1W sequences fused in red, green and blue channels, respectively.

FMRIB, Oxford, UK. The volumes were resampled so that they were in the same space and contain the same number of images. The intensities were manually window-leveled in each volume so that the intensity ranges in all images were approximately the same. A mask of the brain was generated using the biomedical imaging software Analyze (<http://www.analyzedirect.com>), and in particular, its seeded region growing segmentation functionality, with stopping criteria dependant on thresholds. The masks were checked for errors and any errors were manually removed. The brain mask is used to zero all intensities outside the brain.

## 4 INVESTIGATION/VALIDITY

A precondition for the previous work is that a given voxel can be classified into one of the tissue types by determining the intensities at that voxel together with the intensity distribution across the entire image. A first step in determining the validity of this is to visualise the intensity distributions for a given image.

In our context this distribution manifests itself as a 4D joint histogram or scatter plot as shown in Figure 3.

It is evident that there are at least two obvious clusters which may correspond to different tissue types. Through experimentation these clusters have been determined to be grey matter/white matter (i.e. brain tissue) and CSF. In most projections the grey matter/white matter cluster is small in extent but in the T1W projections it has a large extent, with grey matter at one end and white matter at the other end. This highlights the fact that T1W is the best sequence

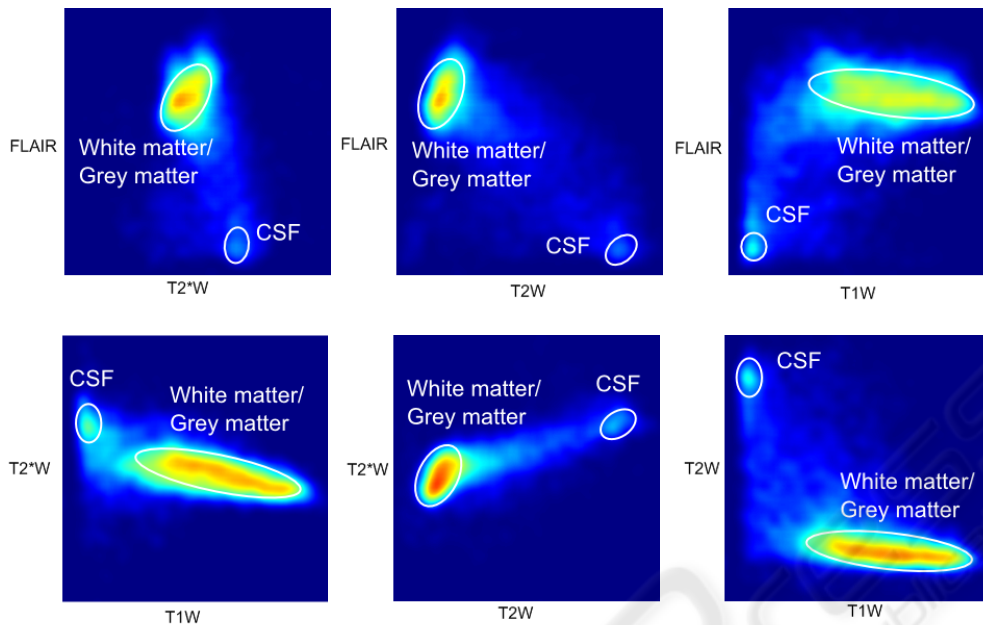


Figure 3: Joint histogram (or joint probability distribution) of the four sequences on a portion of the volume shown as six 2D projections. Each view is an average projection and so represents the marginal probability density for the different intensity types. The colour represents the probability/frequency at that point with dark blue representing low frequency and red representing high frequency, on a log scale. The projections have been smoothed. Sequence values are increasing from left to right and bottom to top. Contributions from the background (zero intensities) have been ignored and do not appear.

for segmenting grey from white matter because it provides the best contrast between them. It also highlights the fact that segmenting grey and white matter from each other using intensity information only is problematic because there is a continuum of intensity ranges between them, with no obvious boundary.

The CSF cluster has high intensity on a T2W scan and low intensity on T1W. The contrast between brain tissue and CSF is high in T2W, which again is a known result. In terms of volume, CSF represents a far smaller proportion of the brain than brain tissue and consequently this peak is not as pronounced as the other.

The clear separation of CSF and brain tissue suggests that an intensity based approach may work. Direct thresholding at predefined levels is unlikely to work because of the natural variability in MR images and because there may not be a threshold which separates them in any one sequence.

## 5 ALGORITHMIC DETAILS/JUSTIFICATION

A precondition for the previous work is that a given voxel can be classified by determining the intensi-

ties at that voxel together with the intensity distribution across the entire image. A fully automated clustering algorithm analyses the intensity distributions and partitions the intensity space into four clusters – background, CSF, grey matter and white matter. The boundaries between these clusters in feature space are known as feature space decision boundaries.

### 5.1 Validation Details

The algorithms were evaluated numerically by comparing the automatically generated segmentations to user collected segmentations which are considered to be true (ground truth). We used standard comparison metrics: true positive fraction (TPF), positive predictive value (PPV) and Dice coefficient. The TPF is the fraction of ground truth which was found (in terms of volume). The PPV is the fraction of the segmentation which is correct (in terms of volume). The Dice coefficient combines these two measures into one. In all cases a value of zero indicates a complete failure and one indicates a complete success. To be specific, the Dice metric is defined as:

$$Dice(X, Y) = \frac{2|X \cap Y|}{|X| + |Y|} \quad (1)$$

where  $X$  &  $Y$  are the ground truth and generated segmentations, respectively.

Presently, only the correctness of the CSF and WM segmentations have been evaluated. Intuitively CSF should be the easiest of the three tissue types to segment because it seems to be well localised in the joint histogram and well separated from the other tissue types. On the other hand it might be the hardest because it has a very high surface area to volume ratio in comparison to the other tissue types. Since errors are normally made at the boundary or surface of a region, those errors will represent a larger percentage of the total volume of the tissue and so have a larger effect on the Dice coefficient.

The ground truth was provided by the SFC Brain Imaging Research Centre. It was semi-automatically generated using a method developed in-house in the SFC Brain Imaging Research centre followed by manual editing by an experienced image analyst (author MVH). This presents an obvious bias towards the MVQ algorithm, and consequently the results presented with regard to the MVQ should be interpreted with caution.

## 5.2 Per Slice/Per Volume Analysis

The question arises as to whether the clustering process should proceed on a per volume or per slice basis. That is, should the clustering algorithms draw their samples from the entire volume or should they perform the clustering on each slice individually?

One advantage of the per-slice strategy is a relative immunity to magnetic field inhomogeneities at least in the axial direction. Obviously the extent of this advantage is tied to the severity of the magnetic field inhomogeneities, which in this experiment are not significant.

A disadvantage is that, although the total number of samples in the feature space is the same in every slice,  $128 \times 128$ , the number of samples in a given tissue type is highly variable between slices. In slices where a class does not appear in high frequency, there is an increased risk that it will be incorrectly classified.

Our investigation suggested that the per volume strategy significantly outperforms the other in both total segmentation accuracy and consistency. Consequently from this point on we will refer only to the results of the whole volume analysis.

## 5.3 Clustering Algorithms

### 5.3.1 *k*-Means

The *k*-means algorithm is an iterative clustering algorithm which, given a set of multidimensional data,

which it interprets as points in a Euclidean space, partitions it into a given number of clusters  $k$  such that the sum of the Euclidean distance squared from each point to its respective cluster center is approximately minimal. The *k*-means algorithm (MacQueen et al., 1966) can be applied directly to the intensities to acquire a partition. In this experiment, four clusters are

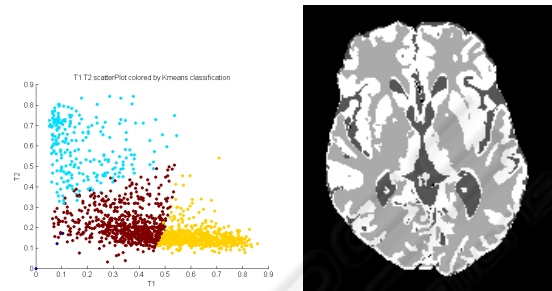


Figure 4: *k*-means intensity classifications over T1W & T2W. CSF, grey matter and white matter are coloured in blue, red and yellow, respectively.

requested and the *k*-means search is initialised with known approximate intensities for the different tissue types. As expected, the feature space decision boundaries on the resultant classifier are composed of straight hyper-planes in the 4-dimensional feature space. It is almost certainly the case that the optimal feature space decision boundaries, are not a collection of hyper-planes. Despite producing physically unrealistic feature space decision boundaries in this way, it has still performed well as shown in the results section, and has chosen to split the white-matter/grey matter distributions.

### 5.3.2 EM

The expectation-maximization (EM) algorithm finds a set of statistical model parameters which best explains a given set of observed data, in the presence of unobserved latent variables. It does this by determining the set of parameters which maximise the likelihood of the data. The EM algorithm was explained and given its name in (Dempster et al., 1977). It is an iterative method that alternates between an expectation stage, which calculates the distribution of the latent variables given the parameters of the model, and a maximisation stage, which determines the set of parameters of the model that maximises the expectation of the likelihood of the data. In the context of this paper, we seek to model the joint probability density function of the points within the brain as a function of their intensities in each sequence. The probabilistic model is deemed to be a mixture of a fixed number of Gaussian distributions, where the means, co-

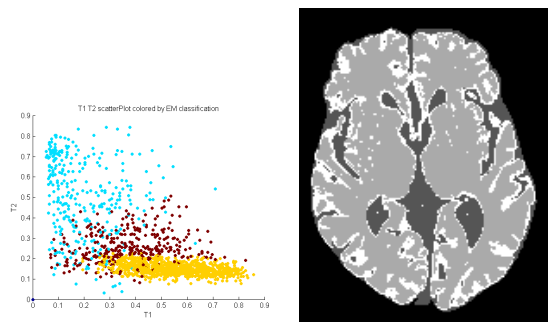


Figure 5: EM intensity classifications over T1W & T2W. CSF, grey matter and white matter are coloured in blue, red and yellow, respectively.

variances and weights of the Gaussians are allowed to vary. In this case, the latent variables determine which Gaussian distribution each sample belongs to. We initialise the EM algorithm with approximate parameters which have been determined *a priori* to relate to tissue types. Thus, once the EM algorithm has converged and the Gaussian mixture has been determined, we can directly assign a tissue type to each Gaussian distribution. We can then partition the image into tissue types by determining which of the Gaussians it is most likely a member of by using the maximum posterior probability. For this experiment we removed the background intensities from the set of voxels to be classified because this caused the EM algorithm to fail to converge. The minimum number of distributions for which the EM algorithm would give a sensible result was four. In practice, CSF was deemed to be the union of two found class types, whereas grey matter and white matter were represented by the other 2 classes.

The feature space decision boundaries produced by this clustering algorithm can be shown to be composed of quadratic surfaces, and so are, in general, curved. This can be seen in figure 5. Intuitively, these decision boundaries would be more like the optimal decision boundaries by virtue of the fact that they are curved. Despite this, it did not perform the best. In general it seemed to over-segment the white matter and under-segment grey matter. It also seemed to produce noisier segmentations.

### 5.3.3 MVQ

The minimum variance quantisation (MVQ) algorithm (Xiang and Joy, 1994) is used in image processing and compression for reducing the colour depth of an image, for example, from 65535 to 256. It produces noticeably better quality images by respecting the distributions of the colours in the image. More specifically it aims to minimise the variance (sum

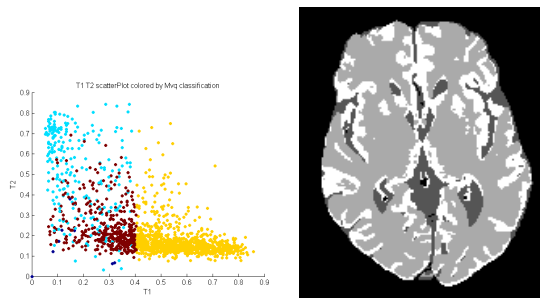


Figure 6: MVQ intensity classifications over T1W & T2W. CSF, grey matter and white matter are coloured in blue, red and yellow, respectively.

squared difference) between input image and the output image. The class of partitions it considers are box partitions (i.e. the partition surfaces are orthogonal hyper-planes). In common with the other clustering algorithms considered here, MVQ uses no spatial information. It can be interpreted as a clustering algorithm and this is how it is used in this experiment. We set the algorithm to quantise the number of colours in the fused image to four, and these colours are interpreted as the different tissue types. The algorithm can be generalised to  $N$  dimensions but typically (as in MATLAB's implementation) it is limited to three dimensions, (red, green, blue) and so we must ignore one of the channels (T2W).

This algorithm performed very well, although produced some physically unrealistic feature space decision boundaries.

### 5.3.4 Mean Shift

The mean shift algorithm (Comaniciu and Meer, 2002) is a non-parametric feature space analysis technique, which determines the gradient in feature space at any point. This can be used to find basins of attraction which partition the space, and consequently assign each point in the feature space to a cluster. Unlike the other algorithms the user does not supply the number of clusters as this is determined automatically. Instead the user supplies a radial basis function which is normally (as in this case) a spherically symmetric Gaussian distribution, centered at zero with a specified variance (or scale). Increasing the variance tends to decrease the number of clusters. For this experiment the scale parameter was tuned until the number of clusters was three or four.

Despite being one of the more elegant of the clustering algorithms, its slow runtime and inability to discriminate white matter from grey matter contributed to its early disqualification, and so it has not been quantitatively evaluated. Its slow runtime is accredited to its MATLAB implementation (the others

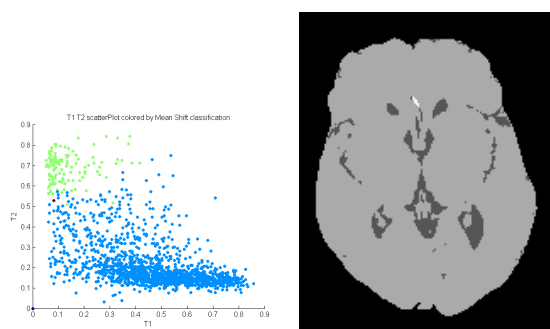


Figure 7: Mean shift intensity classifications over T1W & T2W. CSF and grey matter/white matter are coloured in green and blue, respectively.

are coded in C) and its inability to discriminate white matter from grey matter is likely due to the fact that they lie in the same attractive basin and so will never be split into different partitions. On the other hand, this clustering algorithm seemed to be very sensitive to small but definite intensity features, and may prove suited to the detection of pathologies such as WMLs — this has not yet been investigated.

## 6 RESULTS AND DISCUSSION

The quantitative results from 12 datasets can be seen in tables 1, 2 and 3. As explained above, the mean shift algorithm has not been quantitatively evaluated. Looking at the average Dice coefficient for CSF we see that MVQ performs best although the validation is biased towards it (see section 5.1). EM also has performed well, and has a number of desirable properties which the  $k$ -means or the MVQ lack, such as the ability to assign a probability of a voxel being in a given tissue classification, which can improve volume measurement accuracy, see (Thacker and Jackson, 2001b), amongst other advantages. Looking at the scores relating to white matter however, we see that  $k$ -means performs best of all, with 91% accuracy.

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The datasets and ground truth used in these experiments were collected as part of the Disconnected

Table 1:  $k$ -means: Dice coefficients for CSF and white matter for the  $k$ -means clustering algorithm. Confidence values calculated at 99% using student-t distribution. White matter ground truth is missing for datasets Sub1, Sub2 and Sub3 and so these do not appear.

$k$ -means		
Dataset	CSF	WM
Sub1	.72	
Sub2	.71	
Sub3	.73	
Z01	.72	.9
Z02	.72	.96
Z03	.73	.92
Z04	.81	.89
Z11	.73	.95
Z14	.74	.95
Z16	.78	.7
Z17	.77	.95
Z19	.7	.95
Average	.74 ± .03	.91 ± .1

Table 2: MVQ: Dice coefficients for CSF and white matter for the MVQ clustering algorithm. Confidence values calculated at 99% using student-t distribution. White matter ground truth is missing for datasets Sub1, Sub2 and Sub3 and so these do not appear.

MVQ		
Dataset	CSF	WM
Sub1	.86	
Sub2	.91	
Sub3	.91	
Z01	.91	.84
Z02	.92	.89
Z03	.87	.76
Z04	.91	.75
Z11	.91	.88
Z14	.9	.89
Z16	.85	.6
Z17	.92	.84
Z19	.9	.86
Average	.9 ± .02	.81 ± .11

Mind Project (<http://www.disconnectedmind.org.uk>). The Disconnected Mind Project is funded by Help the Aged (<http://www.helptheaged.org.uk/en-gb>) and the UK Medical Research Council. The SFC Brain Imaging Centre is part of the Scottish Imaging Network, a Platform for Scientific Excellence (SINAPSE, <http://www.sinapse.ac.uk>); author JMW is part funded by the SINAPSE collaboration.

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Table 3: EM: Dice coefficients for CSF and white matter for the EM clustering algorithm. Confidence values calculated at 99% using student-t distribution. White matter ground truth is missing for datasets Sub1, Sub2 and Sub3 and so these do not appear.

Dataset	EM	
	CSF	WM
Sub1	.86	
Sub2	.8	
Sub3	.73	
Z01	.7	.79
Z02	.68	.9
Z03	.75	.74
Z04	.76	.72
Z11	.83	.8
Z14	.78	.76
Z16	.81	.7
Z17	.76	.87
Z19	.71	.76
Average	.76 ± .04	.78 ± .08

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