THE OBSERVABILITY PROBLEM IN TRAFFIC MODELS
An Algebraic Step by Step Method

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Abstract: This article deals with the problem of observability of traffic networks, understanding as such the problem of identifying if a set of available flow measurements is sufficient to estimate the remaining flows in the network, OD-pair or link flows. An algebraic method for solving the observability problems is given. Specifically, a step by step procedure allowing updating the information once each item of information (OD-pair or link flow) becomes available. The method is illustrated by its application to a simple network. The results show that the proposed method provide useful information on which OD pair or link flows are informative on other OD pair and link flows, and that the method is applicable to large networks due to its numerical robustness and stability.

1 INTRODUCTION

There are some situations in practice in which one needs to know the state of a traffic network by measuring a subset of flows and, based on this information, predicting other flows, which are not free to take arbitrary values but must be subject to constraints, imposed by the networks topology, to be in agreement with the measured flows. In this area, some Artificial Intelligence techniques, such as Bayesian networks or Neural networks, have been used (see (Castillo et al., 2008c; Yin et al., 2002; Ledoux, (1997; Smith and Demetsky, 1994), for example). However, before using these techniques it is important to discuss the observability problem, which consists of identifying if a set of available (measured) flows is sufficient to calculate other given subset of flows.

Observability analysis is a previous step to state estimation. It addresses the question: do we have enough measurements to estimate the state of a system? Observability techniques are essential in many fields of knowledge, and in particular in traffic prediction.

Some examples of observability problems in traffic networks are:

1. Determine if a subset of available traffic flows is sufficient to obtain the values of another subset of traffic flows.
2. Obtain a minimum subset of observations that allow the knowledge of other given subset of flows or the observability of all flows in the network.
3. Identify observable flows, given a set of observed flows (partial observability).

Though the problem of observability can be stated in a general context, as done in (Castillo et al., 2007), who discuss the problem of observability of linear system of equations and inequalities, most of existing publications relate to particular fields (see, for example, the survey provided by (Abur and Expósito, 2004), which includes applications to power systems).

Observability techniques can be classified as:

1. *Algebraic.* These techniques consider the algebraic relations between the flows and operate them algebraically to draw observability conclusions (see (Monticelli and Wu, 1985a; Monticelli and Wu, 1985b), (Monticelli, 2000), (Abur and Expósito, 2004), (Gou and Abur, 2000; Gou and Abur, 2001), (Castillo et al., 2005), (Castillo et al., 2006), (Castillo et al., 2008a), (Castillo et al., 2008b)).
2. *Topological.* These techniques consider only topological and/or qualitative relations between flows to derive observability results (see (Clements and Wollenberg, 1975); (Krumpholz et al., 1980), (Nucera and Gilles, 1991) and (Castillo et al., 2007; Castillo et al., 2008b)).

Since all these techniques are based on the mathematical properties of the systems of equations and

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have the same structure for traffic problems, these approaches, which have already been applied to “physical” networks, are equally applicable to traffic networks.

This paper is focused on algebraic methods, which can be used with two aims in mind: (a) obtaining the exact algebraic relation among different flows, such that some can be calculated when other are known, and (b) obtain observability information, that is, determine which flows can and which flows cannot be calculated when a subset of flows is known, but without seeking the corresponding formulas which allow us the calculations. The second aim is simpler and requires less effort than the first one.

This paper deals with observability problems, assuming that the matrix relating link and OD flows is given. However, though the solution of the system of equations implied by OD-link relations is clearly dependent on the particular selection of the choice probabilities or proportions of the users selecting different paths, the observability problem is not in general. Of course, there are many especial cases in which some linear equations can become linear combinations of other, but what we are really interested in is in structural linear dependencies and not accidental dependencies. Thus, fixing the probabilities (unless they are fixed to zero values) is not a problem. In fact, they should be fixed at random with the aim of avoiding accidental dependencies (with zero probability in practice). Another different matter is to fix some probabilities to zero or not zero. This means incorporating or deleting paths, and then, the observability problem is clearly dependent on these selections.

Note that the technique developed is applicable not only for observing total link flows, but also for disaggregated link flows, by origin, destinations, or any type of disaggregation as that associated with plate scanning. However, for the sake of simplicity we illustrate the method with total link flows.

This paper is organized as follows. In section 2 the observability problem is stated and the algebraic method is given. Section 3 illustrates the proposed method by their application to a simple network. Finally, in Section 4 some conclusions are given.

2 THE OBSERVABILITY PROBLEM

Consider a traffic network $(\mathcal{N},\mathcal{A})$ where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ is the set of links. Let v_a be the flow of link a, p_{ik} the probability of a user to choose path k of the OD-pair i, t_i the OD-pair flow i, δ^i_{ak} the incidence matrix, that is, $\delta^i_{ak}=1$ if link a belongs to path k of OD-pair i, and 0, otherwise.

Then, for compatibility of OD-pair and link flows, i.e., for the conservation law to be satisfied, one must have:

$$v_a=\sum_i\left(\sum_k p_{ik}\delta^i_{ak}\right)t_i=\sum_i f_{ai}t_i, \tag{1}$$

where $\mathbf{F}$ is a matrix with elements f_{ai} defined as

$$f_{ai}=\sum_k p_{ik}\delta^i_{ak};\quad \sum_k p_{ik}=1;\quad \forall i. \tag{2}$$

The system of equations (1) in matrix form becomes

$$\mathbf{V}=\mathbf{F}\mathbf{T}, \tag{3}$$

where $\mathbf{V}$ and $\mathbf{T}$ are the column matrices of link and OD-pair flows with dimensions $m\times 1$ and $n\times 1$, respectively.

It is important to note that given the OD flows t_i, Equation (3) allows us to calculate the link flows v_a. Thus, given the topology of the network and the OD path flows, the OD flows are the minimum number of data items needed to determine the remaining flows of the network. In fact, enumerating paths can be avoided if the matrix $\mathbf{F}$, which gives the proportions of OD flows travelling through each link, is known. However, since OD flows are not normally known, each of them needs to be replaced by link flows that are practically observable. As we will see, the algebraic approach proceeds by replacing the OD flows with the observed links, until all of them have been replaced.

Therefore, it is assumed that a subset $\mathbf{T}_1$ of $\mathbf{T}$ and a subset $\mathbf{V}_1$ of $\mathbf{V}$ are observed (known) and the complementary subsets $\mathbf{T}_0$ of $\mathbf{T}$ and $\mathbf{V}_0$ of $\mathbf{V}$ are not.

Then, the system (3) can be partitioned as

$$\begin{pmatrix}\mathbf{V}_0\\ ---\\ \mathbf{V}_1\end{pmatrix}=\begin{pmatrix}F_{00} & | & F_{01}\\ --- & + & ---\\ F_{10} & | & F_{11}\end{pmatrix}\begin{pmatrix}\mathbf{T}_0\\ ---\\ \mathbf{T}_1\end{pmatrix}, \tag{4}$$

In order to join (write together) the unknown flow variables $\mathbf{T}_0$ and $\mathbf{V}_0$, the system (4) can be written in the alternative equivalent form

$$\mathbf{D}=\begin{pmatrix}-F_{01}\mathbf{T}_1\\ -----\\ -F_{11}\mathbf{T}_1+\mathbf{V}_1\end{pmatrix}=B\mathbf{z}=\begin{pmatrix}F_{00} & | & -\mathbf{I}_p\\ --- & + & ---\\ F_{10} & | & \mathbf{0}\end{pmatrix}\begin{pmatrix}\mathbf{T}_0\\ -----\\ \mathbf{V}_0\end{pmatrix}, \tag{5}$$

a system where the unknowns appear on the right hand side and the observations on its left hand side. The coefficient matrix is B, the independent term column matrix is $\mathbf{D}$ and $\mathbf{z}$ contains the unknown flows $\mathbf{T}_0$ and $\mathbf{V}_0$.

We finally note that for illustrative purposes and for the sake of facilitating the understanding of this example, the OD-pair and link flows have been distinguished as two different items, but from a mathematical point of view they are undistinguishable. In other words, the data and the unknown sets can contain any subset of variables.

2.1 The Algebraic Approach: A Step-by-step Procedure

The step-by-step procedure allows us to discover what happens, in terms of observability, as some new items of information (OD-pair or link flows) become available.

The rationale for the algorithm below is basically to express the observable flows in terms of the actually observed flows. That is, to transfer "columns to rows" and vice versa in matrix **F**. If all variables can be expressed as linear combinations of measurements, the network is observable; otherwise, it is not. The actual operations are based on the orthogonal transformation algorithm reported in (Castillo et al., 2000; Castillo et al., 2002). The proposed algorithm provides two sets and one matrix of interest:

1. Set $\mathcal{C}$ of cardinality n, whose elements are c_j, contains the list of a minimum set of required measurements to attain observability of all variables. These measurements are denominated *essential measurements or basic measurements*.
2. Set $\mathcal{B}$ of cardinality m, whose elements are b_i, contains the list of *redundant measurements* for observability purposes, that is, even if the measurements are lost, the network remains observable.
3. Matrix $\mathbf{F}$[1] of dimension $m \times n$ contains the coefficients of the linear combinations of the redundant measurements in terms of the required (essential) ones.

Algorithm 1 (Basic observability procedure).

INPUT. *The set of links $\mathcal{A}$, the set of OD-pairs, two disjoint subsets $\mathcal{B}$ and $\mathcal{C}$ of the set $\mathcal{H}$ of all flows (OD-pair and link flows) such that $\mathcal{H} = \mathcal{B} \cup \mathcal{C}$, $\mathcal{B} \cap \mathcal{C} = \emptyset$ and $|\mathcal{C}| = n$, an initial matrix* **F** *giving the flows in $\mathcal{B}$ in terms of the flows in $\mathcal{C}$.*

OUTPUT. *A transformed matrix* $\mathbf{F}^*$ *associated with the transformed $\mathcal{B}$ and $\mathcal{C}$ sets.*

Step 1: Choose a pivot. *Choose an unobservable element in $\mathcal{B}$, that is, a row i of matrix* **F** *that we called α, and an element in $\mathcal{C}$, that is a column j of the same matrix, called β, such that the corresponding value $f_{\alpha\beta} \neq 0$, and go to Step 2. If such an element $f_{\alpha\beta}$ is null, stop the process informing on the impossibility of exchanging the supplied elements b_α and c_β.*

[1] Matrix **F** can be easily obtained from the network topology. For example, it can be the matrix **F** in (3) or any other resulting after manipulation of this one by exchanging the variables in $\mathcal{B}$ and $\mathcal{C}$.

Step 2: Pivoting. *Perform the pivoting process, that is, calculate the transformed matrix* $\mathbf{F}^*$ *using the following transformation, being α and β the i row and j column, respectively, chosen for pivoting in Step 1,*

$$f^*_{ij} = \begin{cases} f_{ij} - \dfrac{f_{\alpha j}}{f_{\alpha\beta}} f_{i\beta} & \text{if } i \neq \alpha;\ j \neq \beta \\ -\dfrac{f_{\alpha j}}{f_{\alpha\beta}} & \text{if } i = \alpha;\ j \neq \beta \\ \dfrac{f_{i\beta}}{f_{\alpha\beta}} & \text{if } i \neq \alpha;\ j = \beta \\ \dfrac{1}{f_{\alpha\beta}} & \text{if } i = \alpha;\ j = \beta \end{cases} \tag{6}$$

Next, exchange the flow c_β in terms of the flow b_α and all other flows in $\mathcal{C}$ at this stage. In other words, incorporate the flow in position α of **B** *into position β of list* **C**, *and the flow in position β of* **C** *into position α of list* **B**.

Once replaced c_β in all equations associated with the system

$$B = FC,$$

we obtain the new matrix F^* such that

$$B^* = F^* C^*,$$

where the asterisk refers to the new situation, i.e. after the interchange of b_α and c_β has been done.

Note that the systems of equations $\mathbf{B} = \mathbf{FC}$ *and* $\mathbf{B}^* = \mathbf{F}^*\mathbf{C}^*$ *are equivalent in the sense that they have the same solutions.*

Step 3: Update the list of essential and redundant flows. *Return matrix* $\mathbf{F}^*$ *and the updated flow subsets $\mathcal{B}^*$ and $\mathcal{C}^*$. If a non-boldfaced flow in* **B** *(row of* $\mathbf{F}^*$*) has null coefficients in all its columns associated with non-observable basic flows (columns of* $\mathbf{F}^*$*), the flow is observable, and then its name is boldfaced, that is, added to the set of observable flows.*

Since it appears to be more convenient and informative to update the flow knowledge as soon as each unit of information is obtained, an algorithm for this step-by-step process is given below. It uses Algorithm 1 as its main tool.

Algorithm 2 (Observability updating procedure).

INPUT. *A list $\mathcal{D}$ of flows to be observed, the initial matrix* **F**, *and a partition $\mathcal{B}$ and $\mathcal{C}$ of all the flow variables $\mathcal{H}$ (OD-pair and link flows) such that $\mathcal{H} = \mathcal{B} \cup \mathcal{C}$, $\mathcal{B} \cap \mathcal{C} = \emptyset$ and $|\mathcal{C}| = n$.*

OUTPUT. *The updated matrix* $\mathbf{F}^*$ *and sets $\mathcal{B}^*$ and $\mathcal{C}^*$, together with the sets $\overline{\mathcal{B}}$ and $\overline{\mathcal{C}}$ of all flows that become known in $\mathcal{B}$ and $\mathcal{C}$, respectively, due to the observed flows in each step.*

Step 0: Initialization step. *Initialize the sets* $\overline{\mathcal{B}}$ *and* $\overline{\mathcal{C}}$ *of known flows in* $\mathcal{B}$ *and* $\mathcal{C}$*, respectively, to empty sets.*

Repeat the following steps for each flow d_r *in the list* $\mathcal{D}$*.*

Step 1: Update observability matrix. *If the flow* d_r *is already in set* $\mathcal{C}$*, i.e.* d_r *coincides with some* $c_\beta \in \mathcal{C}$*, simply add* d_r *to list* $\overline{\mathcal{C}}$*. Otherwise, it must coincide with some* $b_\alpha \in \mathcal{B}$*, and then use Algorithm 1 to incorporate the flow variable* $d_r \equiv b_\alpha$ *to set* $\mathcal{C}$*. To this end, select a flow* c_β *of* $\mathcal{C}$ *not in* $\overline{\mathcal{C}}$ *to be exchanged with* d_r*. If this is not possible because there is no non-null element* $f_{\alpha j}$*, inform on this impossibility of observing* d_r *(it must already have a fixed value, which can be calculated in terms of already observed flows) increase r in one unit and continue in Step 1. Otherwise, update matrix* $\mathbf{F}$ *to* $\mathbf{F}^*$*, exchange flows* c_β *and* d_r *in sets* $\mathcal{B}$ *and* $\mathcal{C}$*, respectively, using Algorithm 1 and add* d_r *to list* $\overline{\mathcal{C}}$*.*

Step 2: Identify all known flows in set $\mathcal{B}$**.** *Find the known flows* $b_k \in \mathcal{B}$*, i.e. the rows of* $\mathbf{F}$ *such that the* f_{kj} *are null for all j associated with the unknown flows in* $\mathcal{C}$*, and add them to set* $\overline{\mathcal{B}}$*.*

Step 3: Return observability information. *Return matrix* $\mathbf{F}^*$*, the sets* $\mathcal{B}^*$ *and* $\mathcal{C}^*$*, and the subsets of known flows* $\overline{\mathcal{B}}$ *and* $\overline{\mathcal{C}}$*, increase r in one unit and continue in Step 1.*

Both algorithms 1 and 2 return matrices $\mathbf{F}^*$, which give the symbolic relation between the flows in $\mathcal{B}$ and those in $\mathcal{C}$, that is, the linear formulas that permit the flows in $\mathcal{B}$ to be written in terms of those in $\mathcal{C}$. Note that once calculated, they can be used many times (for observations taken at different days, hours, etc.).

Note that the technique in Algorithms 1 and 2 does not require the actual values of the observed flows. This is an important advantage from a practical point of view because one is not subject to observation errors that in quite a few cases can and certainly do lead to the incompatibility of the system of equations (3).

Since this method works using algebraic operations with real numbers, it is subject to rounding errors. In particular, testing for zero flows must be replaced by testing for small numbers, which can give numerical problems for large networks.

With respect to use the second algorithm with a given list $\mathcal{D}$ of observable flows, it is convenient to study the rank associated with the rows of $\mathcal{D}$, to avoid observing redundant flows, that is, a set of flows which contains the same information as a proper subset of it. This check permits the unnecessary observations, i.e., those observations that are linear combinations of other observations, to be eliminated.

2.2 Computational Issues

In this section we deal with some computational issues.

Since the proposed method is a pivoting process analogous to the Gauss elimination, all the gained experience for Gauss methods is applicable to the proposed method. This implies that it is applicable to very large networks.

For some particular cases of the choice probabilities and network topologies, the system can become ill-conditioned, and then, the standard methods used to solve this problem can be applied. In particular one can use the different well known partial and complete pivot strategies, which are very useful.

The complexity of algorithm 2 is similar to the complexity of inverting a matrix. If one has n essential variables, the number of operations required for obtaining a minimum set of observed links for observability of the whole system is $(3n-1)n^2$, which gives a clear idea of how the size of the problem affects the computational time.

3 EXAMPLE OF APPLICATION

In this section we illustrate the proposed method by its application to a simple network in order to be able to show the results in form of tables of reasonable size. Though this method was tested using other networks, as the Nguyen-Dupuis network (13 nodes, 38 links, 8 OD pairs, 50 routes), and the real Ciudad Real network (102 nodes, 218 links, 72 OD pairs, 179 routes), for example, and similar results were obtained. Since the algebraic techniques used are very well known from the point of view of their numerical robustness and stability, it can be said that they behave very well for very large networks.

The simple network consists of 9 nodes and 18 links, as shown in Figure 1. We use an example of bidirectional flow, i.e., we assume the existence of symmetric links, i.e. any pair of nodes i and j is connected in both directions by links ℓ_{ij} and ℓ_{ji}, respectively.

We have assumed the following OD-pairs flows (elements of the matrix $\mathbf{T}$):

$$\{t_1, t_2, t_3, t_4, t_5, t_6\} \equiv \{t_{14}, t_{24}, t_{34}, t_{41}, t_{42}, t_{43}\},$$

where the subindices refer to the OD-pair or the node numbers, respectively, and the paths (given in terms

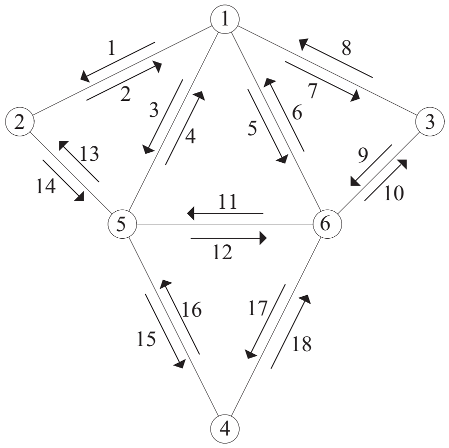

Figure 1: The simple network, showing the nodes and links.

of nodes):

OD-pair $1-4$:	$\{1,2,5,4\},\{1,2,5,6,4\},\{1,5,4\},$ $\{1,5,6,4\},\{1,6,4\},\{1,6,5,4\},$ $\{1,3,6,4\},\{1,3,6,5,4\},$
OD-pair $2-4$:	$\{2,5,4\},\{2,5,6,4\},$
OD-pair $3-4$:	$\{3,6,5,4\},\{3,6,4\},$
OD-pair $4-1$:	$\{4,5,2,1\},\{4,6,5,2,1\},\{4,5,1\},$ $\{4,6,5,1\},\{4,6,1\},\{4,5,6,1\},$ $\{4,6,3,1\},\{4,5,6,3,1\},$
OD-pair $4-2$:	$\{4,5,2\},\{4,6,5,2\},$
OD-pair $4-3$:	$\{4,5,6,3\},\{4,6,3\}.$

To illustrate the step-by-step method we explain our input data and give a detailed explanation of the different steps in Algorithm 2 below.

Example 1 (The step-by-step approach) .
INPUT. It consists of the following list $\mathcal{D}$ of flows to be observed (only link flows are observed in this example):

$$\mathcal{D} \equiv \{v_1, v_8, v_{10}, v_{11}, v_{12}, v_{15}\},$$

the initial matrix **F**, which has been obtained using (1), and is given in Table 1, iteration 0, and as partition of the flow variables $\mathcal{H}$ we take $\mathcal{B}$ as the set of link flows, and $\mathcal{C}$ as the set of OD-pair flows, respectively.

Step 0: Initialization step. We initialize the sets of known flows to empty sets, that is, $\overline{\mathcal{B}} = \emptyset$ and $\overline{\mathcal{C}} = \emptyset$. We show the initial sets $\mathcal{B}$ and $\mathcal{C}$ and their updated versions $\mathcal{B}^*$ and $\mathcal{C}^*$ in the first rows and columns, respectively, of the tables, and we distinguish the sets $\overline{\mathcal{B}}$ and $\overline{\mathcal{C}}$ with boldfaced letters.

With all this, we have the initial observability information in Table 1, iteration 0. Note that in this and the following tables, the matrix **F** and the sets $\mathcal{B}$, $\mathcal{C}$, $\overline{\mathcal{B}}$ and $\overline{\mathcal{C}}$ are shown.

Next, we repeat the following steps for each flow d_r in the list $\mathcal{D}$, starting with flow v_1.

Step 1: Update observability matrix. Since the flow v_1 is not in set $\mathcal{C}$ but is in set $\mathcal{B}$, Algorithm 1 is used to incorporate the flow variable v_1 to set **C**. To this end, we select the flow $c_\beta \equiv t_1$ of $\mathcal{C}$ to be exchanged with v_1, and update matrix **F** to $\mathbf{F}^*$ and add v_1 to list $\overline{\mathcal{C}}$ (boldfaced) (see the right part of Table 1).

Step 2: Identify all known flows in set $\mathcal{B}$. At this stage, there are known flows $b_k \in \mathcal{B}$, i.e. there are rows of matrix **F** with null values in the columns of unknown flows in $\mathcal{C}$, $\{t_2, t_3, t_4, t_5, t_6\}$. So, the flows $\{t_1, v_3, v_5, v_7\}$ are added to $\overline{\mathcal{B}}$.

Step 3: Return observability information. We produce this information in the right part of Table 1.

Since repeating all the steps for all the iterations would be too reiterative and space consuming, we assume that we are at the beginning of iteration 4 (see Table 3), i.e., when we will observe v_{12}, and continue with Steps 1 to 3, as follows.

Step 1: Update observability matrix. Since the flow v_{12} is not in set $\mathcal{C}$ but is in set $\mathcal{B}$, Algorithm 1 is used to incorporate the flow variable v_{12} to set **C**. To this end, we select the flow $c_\beta \equiv t_2$ of $\mathcal{C}$ to be exchanged with v_{12}. Note that this is not the only option, because there are other non-null $f_{\alpha j}$ values associated with the unobserved flows in $\mathcal{C}$, such as v_{14} or v_{15}. Then, we update matrix **F** to $\mathbf{F}^*$ and add v_{12} to list $\overline{\mathcal{C}}$ (we boldface it, as shown in Table 3), iteration 5.

Step 2: Identify all known flows in set $\mathcal{B}$. At this stage, there are flows $b_k \in \mathcal{B}$ which become known, i.e. there are rows of the matrix **F** with null values in the column of t_3, the only unknown flow in $\mathcal{C}$. In this iteration, v_{14} is the only new known flow added to $\overline{\mathcal{B}}$.

Step 3: Return observability information. We return the information in Table 3, iteration 5.

Finally, in the next iteration all the flows become known (see that in Table 4 all the flows are boldfaced).

Note also that the coefficients in this table permit

Table 1: Initial **F** matrix and **F** after observing $\{v_1\}$.

	Iteration 0							Iteration 1					
	t_1	t_2	t_3	t_4	t_5	t_6		$\mathbf{v_1}$	t_2	t_3	t_4	t_5	t_6
v_1	0.3	0.0	0.0	0.0	0.0	0.0	$\mathbf{t_1}$	4.0	0.0	0.0	0.0	0.0	0.0
v_2	0.0	0.0	0.0	0.3	0.0	0.0	v_2	0.0	0.0	0.0	0.3	0.0	0.0
v_3	0.3	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_3}$	1.0	0.0	0.0	0.0	0.0	0.0
v_4	0.0	0.0	0.0	0.3	0.0	0.0	v_4	0.0	0.0	0.0	0.3	0.0	0.0
v_5	0.3	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_5}$	1.0	0.0	0.0	0.0	0.0	0.0
v_6	0.0	0.0	0.0	0.3	0.0	0.0	v_6	0.0	0.0	0.0	0.3	0.0	0.0
v_7	0.3	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_7}$	1.0	0.0	0.0	0.0	0.0	0.0
v_8	0.0	0.0	0.0	0.3	0.0	0.0	v_8	0.0	0.0	0.0	0.3	0.0	0.0
v_9	0.3	0.0	0.7	0.0	0.0	0.0	v_9	1.0	0.0	0.7	0.0	0.0	0.0
v_{10}	0.0	0.0	0.0	0.3	0.0	1.0	v_{10}	0.0	0.0	0.0	0.3	0.0	1.0
v_{11}	0.3	0.0	0.3	0.3	0.5	0.0	v_{11}	1.0	0.0	0.3	0.3	0.5	0.0
v_{12}	0.3	0.3	0.0	0.3	0.0	0.5	v_{12}	1.0	0.3	0.0	0.3	0.0	0.5
v_{13}	0.0	0.0	0.0	0.3	1.0	0.0	v_{13}	0.0	0.0	0.0	0.3	1.0	0.0
v_{14}	0.3	0.7	0.0	0.0	0.0	0.0	v_{14}	1.0	0.7	0.0	0.0	0.0	0.0
v_{15}	0.5	0.3	0.3	0.0	0.0	0.0	v_{15}	2.0	0.3	0.3	0.0	0.0	0.0
v_{16}	0.0	0.0	0.0	0.5	0.5	0.5	v_{16}	0.0	0.0	0.0	0.5	0.5	0.5
v_{17}	0.5	0.3	0.3	0.0	0.0	0.0	v_{17}	2.0	0.3	0.3	0.0	0.0	0.0
v_{18}	0.0	0.0	0.0	0.5	0.5	0.5	v_{18}	0.0	0.0	0.0	0.5	0.5	0.5

Table 2: **F** after observing $\{v_1, v_8\}$ and $\{v_1, v_8, v_{10}\}$.

	Iteration 2							Iteration 3					
	$\mathbf{v_1}$	t_2	t_3	$\mathbf{v_8}$	t_5	t_6		$\mathbf{v_1}$	t_2	t_3	$\mathbf{v_8}$	t_5	$\mathbf{v_{10}}$
$\mathbf{t_1}$	4.0	0.0	0.0	0.0	0.0	0.0	$\mathbf{t_1}$	4.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{v_2}$	0.0	0.0	0.0	1.0	0.0	0.0	$\mathbf{v_2}$	0.0	0.0	0.0	1.0	0.0	0.0
$\mathbf{v_3}$	1.0	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_3}$	1.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{v_4}$	0.0	0.0	0.0	1.0	0.0	0.0	$\mathbf{v_4}$	0.0	0.0	0.0	1.0	0.0	0.0
$\mathbf{v_5}$	1.0	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_5}$	1.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{v_6}$	0.0	0.0	0.0	1.0	0.0	0.0	$\mathbf{v_6}$	0.0	0.0	0.0	1.0	0.0	0.0
$\mathbf{v_7}$	1.0	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_7}$	1.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{t_4}$	0.0	0.0	0.0	4.0	0.0	0.0	$\mathbf{t_4}$	0.0	0.0	0.0	4.0	0.0	0.0
v_9	1.0	0.0	0.7	0.0	0.0	0.0	v_9	1.0	0.0	0.7	0.0	0.0	0.0
v_{10}	0.0	0.0	0.0	1.0	0.0	1.0	$\mathbf{t_6}$	0.0	0.0	0.0	-1.0	0.0	1.0
v_{11}	1.0	0.0	0.3	1.0	0.5	0.0	v_{11}	1.0	0.0	0.3	1.0	0.5	0.0
v_{12}	1.0	0.3	0.0	1.0	0.0	0.5	v_{12}	1.0	0.3	0.0	0.5	0.0	0.5
v_{13}	0.0	0.0	0.0	1.0	1.0	0.0	v_{13}	0.0	0.0	0.0	1.0	1.0	0.0
v_{14}	1.0	0.7	0.0	0.0	0.0	0.0	v_{14}	1.0	0.7	0.0	0.0	0.0	0.0
v_{15}	2.0	0.3	0.3	0.0	0.0	0.0	v_{15}	2.0	0.3	0.3	0.0	0.0	0.0
v_{16}	0.0	0.0	0.0	2.0	0.5	0.5	v_{16}	0.0	0.0	0.0	1.5	0.5	0.5
v_{17}	2.0	0.3	0.3	0.0	0.0	0.0	v_{17}	2.0	0.3	0.3	0.0	0.0	0.0
v_{18}	0.0	0.0	0.0	2.0	0.5	0.5	v_{18}	0.0	0.0	0.0	1.5	0.5	0.5

obtaining all the flows of $\mathcal{B}$ in terms of the observed flows, that now are all the flows in $\mathcal{C}$:

$$\begin{array}{cccccccc}
t_1 & = & 4v_1 & & & & & \\
v_2 & = & & & & v_8 & & \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
v_9 & = & -v_1 & -2v_{12} & 2v_{15} & v_8 & & v_{10} \\
t_6 & = & & & & -v_8 & & v_{10} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
v_{17} & = & & & v_{15} & & & \\
v_{18} & = & & v_{12} & -v_{15} & & v_{11} &
\end{array}$$

Next, we illustrate the general problems stated in the introduction with some particular cases.

Table 3: **F** after observing $\{v_1, v_8, v_{10}, v_{11}\}$ and $\{v_1, v_8, v_{10}, v_{11}, v_{12}\}$.

	Iteration 4							Iteration 5					
	$\mathbf{v_1}$	t_2	t_3	$\mathbf{v_8}$	$\mathbf{v_{11}}$	$\mathbf{v_{10}}$		$\mathbf{v_1}$	$\mathbf{v_{12}}$	t_3	$\mathbf{v_8}$	$\mathbf{v_{11}}$	$\mathbf{v_{10}}$
$\mathbf{t_1}$	4.0	0.0	0.0	0.0	0.0	0.0	$\mathbf{t_1}$	4.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{v_2}$	0.0	0.0	0.0	1.0	0.0	0.0	$\mathbf{v_2}$	0.0	0.0	0.0	1.0	0.0	0.0
$\mathbf{v_3}$	1.0	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_3}$	1.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{v_4}$	0.0	0.0	0.0	1.0	0.0	0.0	$\mathbf{v_4}$	0.0	0.0	0.0	1.0	0.0	0.0
$\mathbf{v_5}$	1.0	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_5}$	1.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{v_6}$	0.0	0.0	0.0	1.0	0.0	0.0	$\mathbf{v_6}$	0.0	0.0	0.0	1.0	0.0	0.0
$\mathbf{v_7}$	1.0	0.0	0.0	0.0	0.0	0.0	$\mathbf{v_7}$	1.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{t_4}$	0.0	0.0	0.0	4.0	0.0	0.0	$\mathbf{t_4}$	0.0	0.0	0.0	4.0	0.0	0.0
v_9	1.0	0.0	0.7	0.0	0.0	0.0	v_9	1.0	0.0	0.7	0.0	0.0	0.0
$\mathbf{t_6}$	0.0	0.0	0.0	-1.0	0.0	1.0	$\mathbf{t_6}$	0.0	0.0	0.0	-1.0	0.0	1.0
t_5	-2.0	0.0	-0.7	-2.0	2.0	0.0	t_5	-2.0	0.0	-0.7	-2.0	2.0	0.0
v_{12}	1.0	0.3	0.0	0.5	0.0	0.5	$\mathbf{t_2}$	-3.0	3.0	0.0	-1.5	0.0	-1.5
v_{13}	-2.0	0.0	-0.7	-1.0	2.0	0.0	v_{13}	-2.0	0.0	-0.7	-1.0	2.0	0.0
v_{14}	1.0	0.7	0.0	0.0	0.0	0.0	$\mathbf{v_{14}}$	-1.0	2.0	0.0	-1.0	0.0	-1.0
v_{15}	2.0	0.3	0.3	0.0	0.0	0.0	v_{15}	1.0	1.0	0.3	-0.5	0.0	-0.5
v_{16}	-1.0	0.0	-0.3	0.5	1.0	0.5	v_{16}	-1.0	0.0	-0.3	0.5	1.0	0.5
v_{17}	2.0	0.3	0.3	0.0	0.0	0.0	v_{17}	1.0	1.0	0.3	-0.5	0.0	-0.5
v_{18}	-1.0	0.0	-0.3	0.5	1.0	0.5	v_{18}	-1.0	0.0	-0.3	0.5	1.0	0.5

Problem 1. Determine if the subset of link flows $\mathcal{B} \equiv \{v_1, v_8\}$ is sufficient to determine the subset of traffic flows $\mathcal{G} \equiv \{v_2, v_3, v_4, v_5, v_6, v_7\}$.

Solution. The answer to this problem is positive, because supplying the given set $\mathcal{B}$ as the input list to Algorithm 2, it returns what it is in Table 2, iteration 2, where we can see that all the flows in the given set $\mathcal{G}$ are boldfaced, i.e. the flows in $\mathcal{G}$ are observable.

Moreover, if we compare these results with Figure 1, we are able to deduce that the firsts eight links of the network (upper area) are lineal combinations. So, any pair of links of this set $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ allows us to know the flow of all these links.

Problem 2. Determine a minimum set of observations (OD-pair and/or link flows) that allow observability of the network.

Solution. Since the rank of matrix **F** in Table 3 is $n = 6$, then, the minimum subset must contain 6 flows. Thus, after applying Algorithm 2 with $\mathcal{D} \equiv \{v_1, v_8, v_{10}, v_{11}, v_{12}, v_{18}\}$ we obtain Iteration 6, Table 4, where all observable flows are boldfaced, i.e., those whose coefficients of unobserved essential variables are null (this means that the linear combinations can be calculated, because all nonnull terms are known).

Problem 3. Identify observable flows, given the set of observed flows $\{v_1, v_8, v_{10}, v_{11}\}$.

Solution. In Iteration 4 of Table 3, after incorporating all observed flows, it can be seen that $t_1, t_4, t_6, v_2, v_3, v_4, v_5, v_6$ and v_7 have a zero in the

Table 4: **F** after observing $\{v_1, v_8, v_{10}, v_{11}, v_{12}, v_{18}\}$.

	Iteration 6					
	v_1	v_{12}	v_{15}	v_8	v_{11}	v_{10}
t_1	4.0	0.0	0.0	0.0	0.0	0.0
v_2	0.0	0.0	0.0	1.0	0.0	0.0
v_3	1.0	0.0	0.0	0.0	0.0	0.0
v_4	0.0	0.0	0.0	1.0	0.0	0.0
v_5	1.0	0.0	0.0	0.0	0.0	0.0
v_6	0.0	0.0	0.0	1.0	0.0	0.0
v_7	1.0	0.0	0.0	0.0	0.0	0.0
t_4	0.0	0.0	0.0	4.0	0.0	0.0
v_9	-1.0	-2.0	2.0	1.0	0.0	1.0
t_6	0.0	0.0	0.0	-1.0	0.0	1.0
t_5	0.0	2.0	-2.0	-3.0	2.0	-1.0
t_2	-3.0	3.0	0.0	-1.5	0.0	-1.5
v_{13}	0.0	2.0	-2.0	-2.0	2.0	-1.0
v_{14}	-1.0	2.0	0.0	-1.0	0.0	-1.0
t_3	-3.0	-3.0	3.0	1.5	0.0	1.5
v_{16}	0.0	1.0	-1.0	0.0	1.0	0.0
v_{17}	0.0	0.0	1.0	0.0	0.0	0.0
v_{18}	0.0	1.0	-1.0	0.0	1.0	0.0

column positions of t_2 and t_3, which are the only unknown OD flows in set $\mathcal{C}$.

4 CONCLUSIONS

The problem of exact observability of traffic flows can be dealt with in a simple form even for large networks. To solve this problem an algebraic step-by-step method is given. This method allows determining the set of observable flows and permits updating the observability information every time we have a new item of information, so that a detailed observability analysis can be done at each step of the process.

The illustration of the proposed methodology using a simple network shows that the proposed method is efficient and practically valid.

Finally, some suggestions for future work are the application of topological techniques to traffic networks, and developing an alternative method sharing the advantages of both observability techniques algebraic and topological.

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