

# "RAIN FALL" PARTICLE MODEL FOR SHAPE RECOVERY AND IMAGE SEGMENTATION

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**Abstract:** This paper studies the problem of shape recovery and image segmentation with examples related to medical imaging. Our purpose is to explore an alternative physics based image segmentation model in comparison with parametric intensive methods such as active contour or level set approaches. The proposed model can offer a more computational efficient approach. As an early attempt, a novel segmentation method based on physically motivated particle system is presented, analyzed and integrated for 2D and 3D applications. Different from previous particle based segmentation method, our proposed approach is governed physically by fluid dynamic model. Additionally a novel "rain fall" model is presented as an alternative paradigm for shape reconstruction and image segmentation when working with complex 2D and 3D medical images. In this paper, an overview of fluid mechanical model and fluid particle simulation process is presented as well. Segmentation results on 2D images and shape recovery of 3D images are presented followed by discussions and conclusions.

## 1 INTRODUCTION

Deformable Models for image segmentation and shape recovery have been attracting considerable attentions in the past decades (Ajit, 1996). Classical methods such as SNAKE (Michael, 1987) have gained popularity in various aspects of computer vision, computer graphics and image analysis. Their main features can be summarized as follows: a) the methodology is analogous to the way that elastic physical objects respond to the applied forces in the physical world, hence the established model is very natural and intuitive; b) the behaviour of geometrical shape is constrained by the forces which can be defined based on the features of the image. c) due to their physically based nature, deformable models can offer a dynamic simulation framework on which real time computation can be carried out and adaptively tuned.

Deformable Particle system based segmentation approaches are developed as modeling tools of deformable model (Andrei C. 2004) (Herng-Hua, 2008). They are motivated by both deformable image segmentation method and particle based graphical technique. Here more physical constraints can be incorporated into the approach, since the external image feature based forces and the internal smoothing forces

are all modeled as real physical forces ,e.g. electro static force (Andrei C. 2004) or charged fluid force (Herng-Hua, 2008).

This paper presents an early attempt to explore an alternative solution based on fluid particles for shape recovery and image segmentation problem. It also presents results on both 2D image segmentation and 3D shape recovery. The paper is organized as follows: Section 2 presents an overview of the fluid particle model and the application to medical image segmentation and shape recovery. Section 3 shows the results for both 2D and 3D images along with discussions. Section 4 summarizes the contributions and discusses the potential benefits and possible extensions in the future.

## 2 FLUID PARTICLE MODEL FOR IMAGE SEGMENTATION

In this paper, we consider the particles to form a fluid system where the internal forces among particles are governed by hydrodynamics laws. Based on the image features, the external image forces can be defined analogously to gravity or viscous forces which would affect the movements of the particles.

## 2.1 Fluid Particle Hydrodynamics

Fluid Dynamics is governed by continuous differential equations. For example, (Andrei, 2006) implemented two partial differential equations to describe the model. In our model, fluid system is discretized into particles, where a computational tool is needed in order to calculate the continuous formulations in a discretized fashion. SPH (Smoothed Particle Hydrodynamics) is a computational tool which was first proposed in astronomy for the simulation of clusters (Joe J. 1992), and widely used in simulating fluid and gas models. The main functional of SPH is to approximate continuous function values and carry out continuous function calculations such as gradient or Laplacian on discretized particles or elements. This can be illustrated by following equations:

$$\langle A(x) \rangle = \sum_j \omega_h(|x - x_j|) \frac{m_j}{\rho_j} A_j \quad (1)$$

$$\langle \nabla A(x) \rangle = \sum_j \nabla \omega_h(|x - x_j|) \frac{m_j}{\rho_j} A_j \quad (2)$$

$$\langle \Delta A(x) \rangle = \sum_j \Delta \omega_h(|x - x_j|) \frac{m_j}{\rho_j} A_j \quad (3)$$

Where the sign  $\langle \rangle$  denotes SPH approximation (M.H. Everts. 2004).  $A(x)$  is a continuous function,  $\langle A(x) \rangle$  is its SPH approximation on discretized particles.  $A_j$  is the function value on the  $j$ -th particle.  $x$  denotes the current particle of interest,  $x_j$  denotes its neighbor particle  $j$ .  $m_j$  and  $\rho_j$  are the mass and density of the  $j$ -th particle.  $\omega_h$  is a weighting kernel function.

$$\omega(r, h) = \begin{cases} \frac{315}{64\pi h^9} & r < h, \\ 0 & r > h. \end{cases} \quad (4)$$

One form of the weighting kernels is described in equation (4) which is also utilized in this paper. In (4)  $r$  is the distance between particles and  $h$  is a pre-set cut-off distance.

The motion of the fluid particles is governed by the following equation (Clayton, 1975):

$$\dot{v} = \frac{f_{pressure}}{\rho} + \frac{f_{viscous}}{\rho} + \frac{f_{external}}{\rho} \quad (5)$$

Where  $\rho$  is density,  $v$  is the velocity thus  $\dot{v}$  is the acceleration. force created due to the change in the pressure can be calculated as the negative pressure differences  $f_{pressure} = -\nabla P$ , Here pressure  $P$  is computed as  $P = k((\frac{P}{\rho_0})^\gamma - 1)$ , where  $\rho_0$  is the standard density of the fluid under the standard atmosphere pressure,  $k$  is a constant through which we can modify the incompressibility of the fluid,  $\gamma$  is a constant which also affects the compressibility. Low value of  $\gamma$  models

the fluid particles to be more compressible. The acceleration can be directly computed as  $\frac{f_{pressure}}{\rho} = -\langle \frac{\nabla P}{\rho} \rangle$ , the term  $\frac{\nabla P}{\rho}$  can be further calculated as  $\frac{\nabla P}{\rho} = \nabla(\frac{P}{\rho}) + \frac{P}{\rho^2} \nabla \rho$ . Using the above definitions and derivatives we can calculate the acceleration caused by pressure as  $\frac{f_{pressure}(P_i)}{\rho_i} = \sum_j \nabla \omega_{ij} m_j (\frac{P_j}{\rho_j^2}) + \frac{P_i}{\rho_i^2}$ . Where  $\omega_{ij}$  is the weight value between the  $i$ -th and  $j$ -th particle,  $\rho_i$  is calculated also using SPH approximation as  $\rho_i = \sum_j \omega_{ij} m_j$ . Acceleration caused by viscosity effect is defined as  $\frac{f_{viscous}}{\rho} = \mu \Delta v$ , Which is calculated using SPH approximation as  $\frac{f_{viscous}(x_i)}{\rho_i} = \mu \langle \Delta v \rangle_i = \mu \sum_j \Delta \omega_{ij} \frac{m_j}{\rho_j} (v_j - v_i)$ . The motion of the fluid particles can now be defined by integrating the equation along the streamline of particles. The fluid simulation is achieved by applying an integration approximation (e.g. Euler's method) to calculate the velocity and position of each fluid particle (6).

$$\begin{cases} v_i(t + \Delta t) = v_i(t) + \dot{v}_i \Delta t, \\ P_i(t + \Delta t) = P_i(t) + v_i(t) \Delta t. \end{cases} \quad (6)$$

where  $P_i(t)$  is the position of the  $i$ -th fluid particle at the time instance  $t$ .

## 2.2 Application of Fluid Particle to Image Segmentation

In order to apply fluid particles for image segmentation, we need to define and incorporate the external image force  $f_{external}$  in equation 5. For instance when dealing with binary images, we can obtain the gradient map of the simple binary image of which the pixels have the value equal to 255 in the edge region and 0 in the rest regions. Then we can incorporate the image pixel values to establish an external force field. Equation 7 models the external image force as proportional to the gradient values and distance between fluid particles and pixel grids.

$$\begin{aligned} f_{image} &= \sum_j^N \omega(r, h) (P_{pixel_j} - P_{particle_i}) U_{jth-pixel} \eta \\ f_{external} &= f_{image} - \beta v_i(t) \end{aligned} \quad (7)$$

Where  $P_{particle_i}$  is the position of the current particle of interest;  $P_{pixel_j}$  is the position of the  $j$ -th pixel.  $U_{jth-pixel}$  is the gradient value of that pixel;  $\eta$  is an adjustable coefficient.  $N$  is the number of pixels in the image.  $-\beta v_i(t)$  is the damping force which consumes and minimizes the kinetic energy of the  $i$ th particle,  $\beta$  is an adjustable damping coefficient. We apply the localization weighting function in equation (4) to select pixels which are closer to the fluid particle of interest within a pre-defined adjustable cut-off radius  $h$ .

By applying the image force  $f_{image}$  in equation (7) to each fluid particle, the fluid flow would gradually be attracted to the boundaries of the object in the image and finally reside along the boundaries.

There are several tunable parameters in the fluid model (such as the initial velocity of the particles, cut-off radius of the weighting function and the parameters in calculation of internal fluid forces), which need to be selected for a particular fluid-flow simulation. One approach can be to first adjust the parameters of the fluid particles in the absence of the real image until an initial smooth laminar flow is obtained; then by defining the initial positions of the fluid particles, we can accomplish the segmentation. As it will be seen in our experimental studies, the fluid particles are initialized in the image plane along one side for segmenting the simple 2D binary image.

### 2.3 "Rain Fall" Model

In general, when applying deformable model based segmentation to 2D images (e.g. classical SNAKE algorithm), the computation is carried out in the image plane. In this paper, our approach for segmenting complex 2D images and recovering 3D shapes is based on the physical notion of the "Rain Fall", where the fluid particles outside the image plane would drop down onto the image. Computationally this can be achieved by initializing the fluid particles outside the image plane (or the image space for 3D image), where the fluid particles will "drop" down onto the plane/space to do the segmentation. The above notion and the follow-up segmentation process is analogous to the phenomenon of rain pouring down to a plane or a cavity. Figure 1(a) to 1(c) illustrate the conceptual model of the "Rain fall" model. This approach can be considered as a segmentation method initiated in 3D space.

When the fluid particles reached the image plane, they would stop falling and start to flow under the influence of the image forces as discussed in equation (6) figure 1(c). This process is analogous to the natural phenomenon where the rain fall down to the ground and flow influenced by the terrain topography to form some paths. In our approach, the "terrain topography" are the image features such as object boundaries.

## 3 EXPERIMENTAL STUDIES

This section presents initial experimental studies exploring the method of this paper. The results are presented in a range from 2D binary image, 2D vessel

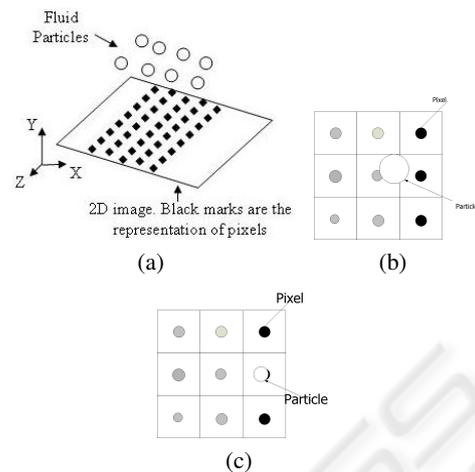


Figure 1: illustration of "Rain Fall" model, the black marks are the pixels in the image, while the white circles represent the fluid particles. The "Rain" particles are initialized outside the image plane. The pixels with darker color represents larger pixel value. (a) Initial step of "Rain Fall", The particles are initialized outside the image plane; (b) Snapshot of a single particle when dropping onto the image. Each particle has been assigned an initial velocity vector whose direction vertically points down to the image plane; (c) Snapshot of a single particle when it reaches the image plane and is attracted by a pixel with larger pixel values. The particle moved toward the pixel and finally resides on it.

image to 3D image. In order to demonstrate the segmentation, we show series of results including the initial stage, intermediate stage and the final stage.

### 3.1 Results on 2D Images

Figure 2(a) to 2(c) illustrate the segmentation process of fluid particles toward 2D binary image. For the 2D binary image, the fluid particles are initialized along the left side of the image. The constant  $\gamma$  for computing pressure force in equation 5 is set to be 2, the constant  $\mu$  for computing viscous force in equation 5 is set to be 7, the cut-off distance  $h$  in equation (4) is set to be twice of the pixel spacing of the target image. Each fluid particle is assigned an initial velocity of 1 milli-meter per second, so they can start to move at the beginning. The direction of the initial velocity vector points to the boundary. Figure 2(a) shows the fluid particles start to move under the attraction of the boundary. Figure 2(b) shows the fluid particles go through the boundary, this is due to the remaining kinetic energy. Figure 2(c) shows that finally the fluid particles reside on the boundary. (i.e. the boundary acts like a valley toward which the fluid flows). Figure 3(a) and 3(b) show the result of the method working on vascular structure 2D image. In

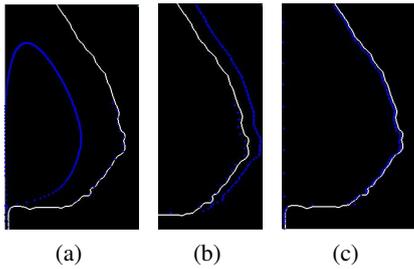


Figure 2: Segmentation results for 2D binary image. In this application fluid particles are initialized on the left side of the image plane (a) Beginning step. Each particle is assigned an initial velocity vector whose direction points to the right boundary. Influenced by the image forces, particles are attracted toward the boundary; (b) Propagating step, particles are oscillating around the boundary; (c) Final step, particles reside along the boundary.

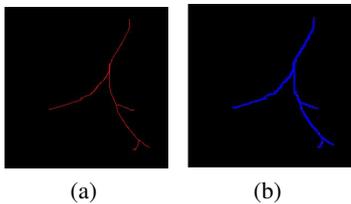


Figure 3: Segmentation result for 2D vessel image with bifurcation. The fluid particles are initialized within the image plane, (a) Original image; (b) Segmented Image.

order to segment the bifurcation along the path, we initialize two fluid particles streamlines on both sides of the image. The parameter setup is the same as the experiment in figure(2). Since the process steps are similar to Figure(2), we only show the original image in Figure 3(a) and the final result in Figure 3(b).

However when dealing with 2D image with complex vascular structures and background noise, the simple streamlines initialization is not sufficient. For example if there are many bifurcations along each vascular structure in the vessel image and particles are only initialized within the image plane, they will be only attracted by outer vascular structures and fail to segment the inner vessels. As a result we implement our proposed "Rain fall" model. Figure 4(a) to 4(d) demonstrate the results on 2D image with complex vessel structures. Figure 4(a) is the original image. In this application, Thresholdfilter is applied to take out some of the background noise in the original image. The image after thresholding is shown in figure 4(b). The fluid particle parameters which are used in this experiment are similar to the above 2D experiments, however we initialize the fluid particles such that the initial positions of them are set to be on top of the image and they are arranged evenly distributed (during the rain fall, the rain particles are assumed to be dis-

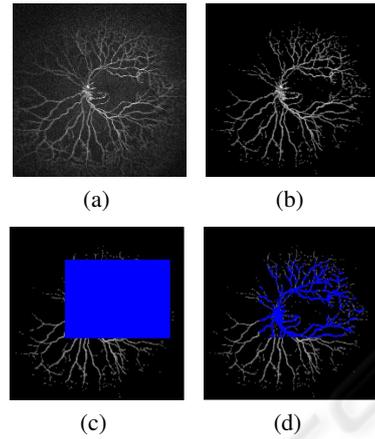


Figure 4: Segmentation result for 2D complex vessel image. The "Rain Fall" model is implemented, (a) Original 2D vessel image; (b) Thresholded 2D vessel image; (c) Initialized Particles. Particles are arranged to be evenly distributed, which form a square array on top of the image. The spacing of the particles are twice of the image pixels(thus the particles are too close to be recognized); (d) Segmented Image.

tributed evenly). The direction of the initial velocity vector is set to point toward the image plane. When the segmentation starts, the fluid particles will move toward the image or "drop" down to the image. We assign each particle an equal initial velocity which is 1 milli-meter per second to start the simulation. If the particles hit the image plane like "rain fall", they will stop falling and move under the influence of the fluid mechanics and the attraction of the image forces.

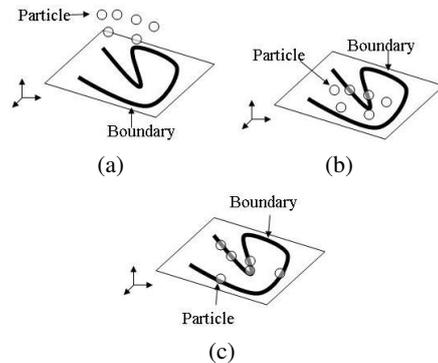


Figure 5: Conceptual diagram for the "Rain Fall" model, (a)Initialization stage of "Rain Fall" model;(b) Segmentation stage1 (Particles reach the image plane) of "Rain Fall" model; (c) Segmentation stage2 (Particles are attracted by the boundary in the image) of "Rain Fall" model.

Figure 5(a) to 5(c) explains the concept idea. In the practical application we initialize the fluid particles so that they are placed in another 2D plane just on top of the image, as shown in figure 4(c). We can

obtain the final segmented image, as illustrated in figure 4(d).

### 3.2 3D Shape Recovery

Since the fluid particles in "Rain Fall" model are falling from the outside of the 2D image plane, they have 3 degrees of freedom of motion when compared with the standard 2D particle based models. As a result, it is possible to take advantage of this feature and extend the model to work on the 3D image data. Analogous to the extraction of boundaries in 2D image, in 3D image we can establish the external image forces by calculating the spatial gradient of the voxels(volume pixel), then substitute the magnitude of the spatial gradient of each voxel for the pixel gradient value  $U_{jth-pixel}$  in equation 7. Since voxels which have large magnitude of spatial gradient are the ones lying on the surface of the object in the image, when the fluid particles drop toward the object, they will be attracted by the surface voxels, as a result the object shape can be recovered by the fluid particles. Following this idea, an application of "Rain Fall" model on shape recovery of 3D image is developed.

Figure 6(a) to Figure 6(d) illustrate the results. Figure 6(a) is the original 3D vascular structure data obtained by MRI, which is visualized using OpenGL graphic rendering engine. In order to carry out the shape recovery, we need to initialize the fluid particles properly so that the entire 3D object can be inside the motion range of the particles. Analogously to the initialization of "Rain Fall" model on segmenting 2D images, We can initialize The fluid particles to stay within other planes. However since the target now is a 3D object, we initialize the particles such that they form the 6 boundary planes of a bounding cubic space where the 3D object is located figure 6(b). Thus the target object is entirely covered by the particles. Then each particle is assigned an initial velocity vector having the same magnitude and the direction pointing vertically toward the target 3D object. Other parameter settings are similar to 2D application. When the "Rain Fall" simulation starts, some of the fluid particles are attracted by the image forces generated by the surface voxels of the image object and move toward the surface. When some of the particles reach the surface, they will eventually stop moving and stay on the surface, others will keep moving and eventually fall outside the cubic space. This is illustrated in figure 6(c). The final recovery result is displayed in figure 6(d).

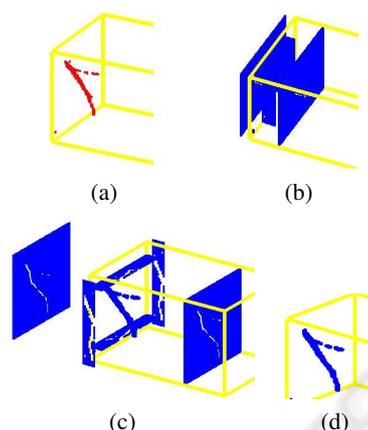


Figure 6: 3D shape recovery, (a) Original 3D vessel image with the bounding volume; (b) Initialized Particle plains (The particle plains are the bounding plane of the bounding volume). The particles start to move; (c) after fluid flow. Some particles are attracted by and move toward the surface voxels, finally they reside on the surface; (d) Recovered 3D Shape.

### 3.3 Discussion

Several novel features of the fluid particles "Rain Fall" model are worth to mention. One is the ability to segment complex structures in 2D images. Traditional deformable model such as SNAKE (Ajit, 1996) and electric particles method (Herng-Hua, 2008) can only deal with convex or simple concave object in 2D image, since these methods only work inside the image plane. For the "rain fall" model, the fluid particles are initialized outside the image plane and "fall" down to the plane to do the segmentation, thus it has more degrees of freedom and can be applied to segment complex 2D images such as vascular images. Another feature is the ability to recover 3D shapes. Since particles can move in 3D space, they also can be attracted by some external force field within certain range of space. As a result, when being attracted by the surface voxels of 3D object, the "Rain Fall" particles recover the 3D shape. However the current "Rain Fall" model is still sensitive to background noise and the segmentation result can be affected. For example in figure 4(d), we can see that there are some unconnected particles inside the vascular structures. This is due to the corruption of the vascular pixels by noise. As an example to deal with the problem, we implement k-nearest neighbourhood checking to double check the pixel labels. This is illustrated in figure 7(a) where the  $i$ th pixel is corrupted by the noise. After checking its 8-nearest neighbourhood pixels, there are more than 2 pixels belonging to the vascular structure, illustrated as Neighbour pixel 1 and 2. In this case

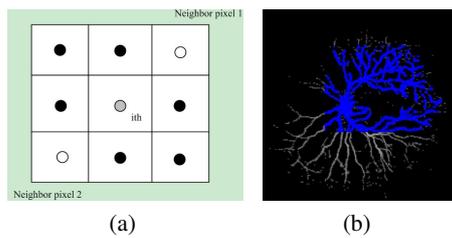


Figure 7: 8-nearest neighborhood checking, (a) Concept diagram; (b) After neighborhood checking.

the *ith* pixel should be labeled as vascular pixel. The resulting image is shown in figure 7(b). Compared with figure 4(d), the vascular structure has less unconnected regions in the vascular structures. Apparently other anti-noise post-processing methods can be implemented as well.

#### 4 CONCLUSIONS AND FUTURE WORK

In this paper, we recreate a fluid particle based image segmentation and shape recovery method which belongs to the deformable model category particularly the particle based deformable model. Different from the existing particle based method, we applied fluid mechanical model through using SPH (Smoothed Particle Hydrodynamics) to compute the internal constraining forces among particles. Upon minimizing the kinetic energy of the fluid particle system in terms of the internal fluid forces and the external image forces, image segmentation can be achieved. In order to complete the image segmentation, we explored the initialization of the fluid particles and developed the "Rain fall" model as to segment complex structures in the image. Finally we tried to segment the complicated vessel image. Upon using threshold filter as the pre-processor and 8-nearest neighbourhood checking as the post-processor, the segmented vessel image is good in connectivity and smoothness. Finally we extended the "Rain Fall" model to recover 3D object shape. As pointed out in the paper, our method can have a better potential for segmenting complex structures such as non convex vascular structure compared with the existing methods due to the higher degrees of motion freedom of the particles. We also extend our work to 3D shape recovery. More advanced anti-noise pre-processing methods are required such as vesselness diffusion enhancement filter (Rashindra, 2006) in the future study. We can collect voxels inside the volume of the segmented object as oppose to only the boundaries, which can be used in the point-based ren-

dering of deformable objects in our VR(Virtual Reality) training project.

#### REFERENCES

- Ajit Singh, Dmitry Goldgof, Demetri Terzopoulos. (1996). Deformable models in Medical image Analysis. In *Medical image analysis 1996*. Elsevier Science Press.
- Michael Kass, Andrew Witkin, Demetri Terzopoulos. (1987). Snake: Active Contour Models. In *International journal of Computer Vision*, 321-331. Kluwer Academic Publishers.
- Andrei C. Jalba, Michael H.F. Wilkinson, Jos B.T.M. Roerdink. (2004). CPM: A Deformable Model for Shape Recovery and Segmentation Based on Charged Particles. In *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 26, No. 10, October 2004. IEEE Computer Society.
- Herng-Hua Chang and Daniel J. Valentino. (2008). An Electrostatic Deformable Model for Medical Image Segmentation. In *Comput Med Imaging Graph. 2008 January* ; 32(1): 2235..
- Andrei C. Jalba and Jos b.T.M. Roerdink. (2006). A Physically-Motivated Deformable Model Based on Fluid Dynamics In *ECCV 2006, Part 1, LNCS 3951*, pp. 496-507, 2006 Springer-Verlag Berlin Heidelberg
- Rashindra Manniesing, Max A. Viergever, Wiro J. Niessen. (2006). Vessel enhancing diffusion A scale space representation of vessel structures. In *Medical Image Analysis 10 (2006) 815-825*. Elsevier Science
- Joe J. Monaghan. (1992). Smoothed Particle hydrodynamics. In *Annu. Rev. Astron. Physics*, 30:543, 1992.
- Joe J. Monaghan. (2005). Smoothed Particle hydrodynamics. In *Reports on progress in Physics*, 68:1703-1759, 2005.
- M. Miller, R. Keiser, A. Nealen, M. Pauly, M. Gross and M. Alexa. (2004). Point Based Animation of Elastic, Plastic and Melting Objects. In *Eurographics/ACM SIGGRAPH Symposium on Computer Animation (2004)* Eurographics Association
- M.H. Everts H. Bekker A.C. Jalba J.B.T.M. Roerdink. (2006). Particle Based Image Segmentation with Simulated Annealing.
- T. Deschamps, P. Schwartz, D. Trebotich, P. Colella, D. Saloner, R. Malladi. (2004). Vessel segmentation and blood flow simulation using Level-Sets and Embedded Boundary methods. In *International Congress Series 1268 (2004) 75 80* Elsevier Science
- Clayton T. Crowe, John A. Roberson. (1975). *Engineering Fluid Mechanics*. Houghton Mifflin Co., 1975