LEARNING AND PREDICTION BASED ON A RELATIONAL HIDDEN MARKOV MODEL

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Abstract: In this paper we show a novel method on how the well-established *hidden markov model* and the *relational markov model* can be combined to the *relational hidden markov model* to solve currently unrecognized challenging problems of the original models. Our presented methods allows for prediction on different granularity level depending on the validity of the underlying observations. We demonstrate the use of this new method based on a spatio-temporal qualitative representation and validate the approach in the *RoboCupSoccer* multi-agent environment.

1 INTRODUCTION

Prediction is one of the most fundamental tasks to be accomplished in order to provide (pro-)active assistance. The areas of application vary from electronic markets (e.g. (Jabbour and Maldonado, 2007)) to autonomous robotics (e.g. (Ball and Wyeth, 2003)), adaptive user interfaces (e.g. (Anderson et al., 2002)), expert systems and ambient intelligence (e.g. (Das et al., 2003)). Although the fundamental task seems to share many properties, the specific requirements differ significantly. Especially in domains that are based on physically grounded sensor data, the prediction task is required to handle incomplete as well as noisy data. Additionally, the training data that is required to learn the prediction model is often sparse with respect to different environment settings. Consequently it tends to lead to substandard prediction results in domains with limited and/or non-representative training data. In this paper we present an approach that addresses this problem by the use of taxonomic hierarchies of attribute values that allow for prediction on different levels of granularity in dependency of the quality of the underlying training data of the prediction model. Our model combines the idea of the *relational markov model* (RMM) introduced by (Anderson et al., 2002) and the well established *hidden markov model* (HMM) (overview in (Rabiner, 1989)). For the purpose of evaluation we apply the new developed *relational hidden markov model* (RHMM) to the *RoboCup*-domain.

The *RoboCup*-domain is widely used as a reference domain for prediction methods (see e.g.,(Ball and Wyeth, 2003; Marín et al., 2005)). Prediction in this domain is used to either support cooperative agents by selecting or adapting their next action to avoid interference with other agents' behaviors or to help competitive agents by choosing their most efficient behavior. We use the *RHMM* to predict upcoming events by qualitative tempo-spatial evidences. The advantage of a qualitative spatial representation is that it allows a flexible and intuitive taxonomic specification of attribute hierarchies. Taxonomies provide a human view to the problem domain (and therefore support the process of modeling the domain properties) by arranging the domain-parts to categories, e.g. apples and pears belong to the category fruits.

Early probabilistic methods e.g., hidden markov models (see overview in (Rabiner, 1989)) were based on a limited representation but combined with efficient learning and prediction methods that allowed to handle both incomplete and noisy information. The selection of an appropriate prediction method does not only depend on the well-balanced ratio between expressiveness, efficiency and learning but also on the specific requirements like the ability to handle prediction at different levels of granularity with respect to the given validity (e.g., the training data).

2 RELATED WORK

Prediction based on probabilistic inference has gained strong interest in recent years. Different probabilis-

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tic methods have been studied/proposed ranging from *static* and *dynamic belief networks* e.g., *Bayesian Networks* in (Albrecht et al., 1998) based on the *conditional independence assumption* to different *markov models* based on the *markov independence assumption* (at different orders) (see (Anderson et al., 2002) for a short overview). Earlier work strongly focused on different types of *hidden Markov models (HMM)* (Rabiner, 1989). Although successful in different domain and applications they imposed strong limitations and restrictions with regard to the representation but also provided efficient learning and inference methods. More recent research has shown that the expressiveness of the representation can be extended in different directions with limited cutting backs with respect to efficiency and precision (e.g. (Fine and Singer, 1998), (Kersting et al., 2006)).

In contrast to these previous approaches we focused our investigation on the problem of sparse training data and the problem of inference on different levels of granularity. One approach that considers an extended representation and addresses these problems is the *relational markov model* (Anderson et al., 2002). This approach uses domain specific information (similarity of states) to provide a faster learning rate and therefore the ability to deal with sparse reference data. In contrast to the *hidden markov model*, this approach does not provide a sensor model to represent the uncertainty of perceptions.

To provide probabilistic inference with the usage of a relational structure, several methods have been researched, e.g. *probabilistic relational models* (PRM) (D'Ambrosio et al., 2003) and *dynamic PRMs* (DPRM) (Sanghai et al., 2003). These methods are similar to bayessian networks in the way that their nodes can depend to a freely specifiable amount of parents, other than the restricted structure of a *HMM* for example. This leads to a very complex structure and an enormous inference effort. Due to this the inference is done via an approximative inference method, a *Rao-Blackwellized* particle filter (Murphy and Russel, 2001).

The concept of attributes like in *RMM* has also been applied to a kind of *HMM*: The logical HMM (*LoHMM*, s. (Kersting et al., 2006)). In this case the states of the *LoHMM* are capable of containing variables and therefore variable-bindings to provide relational inference in another meaning than *RMM*, to express a dependency of states over time rather than a relation over granularity levels like this work does.

Instead of representing different levels of granularity over states (like *RMM* resp. this work does) the Hierarchical HMM (*HHMM*) offers a hierarchy over several *HMMs* (s. (Fine and Singer, 1998)).

3 LEARNING AND PREDICTION BASED ON RHMM

Before introducing the relational hidden markov model, we give a brief description on *HMMs* and *RMMs* that build the basis for this approach.

3.1 Prerequisites

The **HMM** (s. (Rabiner, 1989)) can be characterized as a double stochastic process. The *HMM* is defined as the tuple $HMM =< S,E,A,B,\pi >$, where π is the initial state distribution. S is defined as a set of hidden states, E denotes the set of possible evidences (visible states). The transition probabilities A are given by a quadratic matrix $A = |S|x|S|$ and the two-dimensional matrix B describes the emission probability of evidences in dependency to the states with $B = |S|x|E|$. A well-known application of the HMM is the calculation of the most probable (hidden) state transitions based on a sequence of observations and to forecast a probability distribution (prediction model).

The **RMM** uses in contrast to the HMM a taxonomy of states to improve the inference process of a markov chain by *smoothing*. Furthermore, the RMM depends only on state transitions (therefore no emission probability / sensor model is given). State transitions trained with limited or no reference data will be approximated by considering nearby (more abstract) states' data depending on the taxonomy. It has been shown in (Anderson et al., 2002) that this approximation leads to better inference quality. Formally, the RMM is defined as a tuple $RMM =< \mathcal{D},\mathcal{R},A,\pi >$, where $\mathcal{D}$ describes the set of taxonomic relations (in Anderson et al. they are denoted as *domains*), $\mathcal{R}$ is a set of predicate-attribute relations with $Predicate(Attribute_1, Attribute_2, ...)$ that specifies a set of states by instantiating corresponding to the leaves of the taxonomy / domain of the attributes.

To provide an explicit model of the sensory uncertainty and the ability to deal with sparse reference data the *relational markov model* and the *hidden markov model* will be combined. The proposed *RHMM*-method will provide flexible inference on different levels of granularity. Similar to the *HMM* the *RHMM* separates hidden and visible states and each state is represented by a relation.

3.2 Structure

The *relational hidden markov model* is defined as a tuple $RHMM =< \mathcal{D},\mathcal{R},\mathcal{E},A,B,\pi >$ with the set of

all domains $\mathcal{D}$, the set of all hidden relations $\mathcal{R}$, the set of all visible relations $\mathcal{E}$, the transition matrix A, the sensor matrix B and the initial distribution π.

To describe the domain-specific similarities we define the hidden states of the *RHMM* as a set of relations $\mathcal{R}$ and the visible states (evidences) as a set of relations $\mathcal{E}$, with each relation containing a set of attributes. The attribute values and the similarities[1] between them are specified by a set of domains $\mathcal{D}$, one domain $D \in \mathcal{D}$ for each attribute. Therefore we define a domain as a hierarchical structure specifying the different levels of granularity analogous to the *RMM*s. A similarity between different values of an attribute is expressed by summing them up to one value on a more abstract granularity level within the domain. The visible and hidden states are handled in the same way. To define a set of relations we specify a function called $leaves(\delta)$, gathering all leaf nodes from a given node δ in the corresponding domain. By use of this function we define a relation $R(d_1,\cdots,d_k)$ by it's containing ground relations (following the pattern of definition from((Anderson et al., 2002)) as:

$$R(d_1,\cdots,d_k) = \{ \quad R(\delta_1,\cdots,\delta_k) \in \mathcal{S} | \delta_i \in leaves(d_i). \; \forall i(1 \leq i \leq k)\} \tag{1}$$

To specify how abstractions of predicates are build from the domains we omitted using all possible abstractions (like in *RMM*s) due to a high computational effort. Instead we build abstractions by abstracting all attributes at the same time. Therefore we define a function $depth(d)$ returning the depth of a node in a domain from its most abstract root. For example the depth of the most abstract value of a domain is zero. Furthermore, the boolean function $min(d_1,d_2)$ is only fulfilled if the difference between the *depth* of the parameters is minimal in the corresponding domains. The abstraction set $\mathcal{G}(s)$ of a ground predicate $s = R(\delta_1,\cdots,\delta_k)$ is defined as:

$$\mathcal{G}(s) = \{ \quad R(d_1,\cdots,d_k) \subseteq \mathcal{R} | d_i \in nodes(D_i) \wedge \delta_i \in leaves(d_i) \wedge min(depth(d_i), depth(d_j)). \; \forall i \forall j(1 \leq i,j \leq k)\} \tag{2}$$

3.3 Inference

Basically the inference in a *RHMM* is a combination of the inference in *RMM*s and the inference in *HMM*s. To determine the probability for a state transition $a_{i,j}$ from state s_j at time $t-1$ to state s_i at time t we consider all more abstract state transitions $a_{\alpha,\beta}$ of the requested one (similar to *relational markov models*). Therefore, α and β specify a relation on a more abstract granularity level:

$$\begin{aligned} a_{i,j} &= P(q_t = s_i | q_{t-1} = s_j) \\ &= \textstyle\sum_{\alpha \in \mathcal{G}(q_t)} \sum_{\beta \in \mathcal{G}(q_{t-1})} \lambda_{\alpha,\beta} a_{\alpha,\beta} P(q_t|\beta) \end{aligned} \tag{3}$$

$a_{\alpha,\beta}$ determines the transition probability of a more abstract state transition by including all contained transition probabilities in the calculation of the given transition probability as follows:

$$a_{\alpha,\beta} = \sum_{s_i \in \alpha} P(s_i|\alpha) \sum_{s_j \in \beta} o_{i,j} \tag{4}$$

$o_{i,j}$ represents the originally trained state transition probability. To include more similar state transitions stronger than less similar state transition we used the proposed mixture function of the *RMM-Rank*-method.

$$\lambda_{\alpha,\beta} \propto (\frac{n_{\alpha,\beta}}{10})^{rank(\alpha)+rank(\beta)} \tag{5}$$

The rank function is defined as $rank(R(d_1,\cdots,d_k) = 1 + \sum_{i=1}^{k} depth(d_i)$. Lambda is chosen that $\sum_{\alpha,\beta} \lambda_{\alpha,\beta} = 1$. $n_{\alpha,\beta}$ is the amount of state transitions from a predicate α to a predicate β. Analogical to $a_{i,j}$ we determine the emission probabilities $b_{i,j}$ on a different set of predicates, domains and attributes. For inference in the *RHMM* the *FORWARD-Algorithm* (known from *HMM*s) is used:

$$P(Q_{t+1}|e_{1:t+1}) = \quad \alpha P(e_{t+1}|Q_{t+1}) \textstyle\sum_{q_t \in Q_t} P(Q_{t+1}|q_t) P(q_t|e_{1:t}) \tag{6}$$

Whereby α is a factor that ensures that the resulting state distribution sums up to one. $P(e_{t+1}|Q_{t+1})$ represents the sensor model determined by $b_{i,j}$. $P(Q_{t+1}|q_t)$ represents the transition model, determined by $a_{i,j}$.

To compute the approximatively probability of a state/relation on a higher granularity level after inference the containing states on the lowest granularity level can simply be combined using the following equation:

$$P(R_t \in \mathcal{R}) = \sum_{s_i \in R_t} P(s_i) \tag{7}$$

Like in HMMs it is necessary that the states on all granularity levels are disjunct. The set $\{s_1,\cdots,s_k\} \in R_t$ specifies the ground states of the relation R_t with k ground states as given in eqn. 1.

[1] In terms of taxonomic distance.

For training we assumed hidden but not invisible states to perform a simple maximum likelihood estimation for training the *RHMM* (s. eqn. 8). Therefore the model will be trained by determining the relative frequency of the state transitions and recognized evidences in the given states.

$$a_{i,j} = \frac{\sum_t N(S_{t+1} = j, S_t = i)}{\sum_t N(S_t = i)} \quad (8)$$

Where N counts the state transitions of the parameters.

4 EVALUATION

To evaluate the *RHMM* prediction performance and accuracy we applied this model to the RoboCup multi-agent domain. Therefore the task was to predict the next action of an opponent soccer player, e.g. pass(left,near). For evaluation we show first how to represent the spatio-temporal RoboCup environment in the *RHMM* and then discuss the results in prediction and training.

4.1 Domain Representation

To represent the essential features of the environment, two attribute domains have been specified: The distance and the direction to represent relative coordinates. Figure 1 illustrates the distance and direction domains, e.g., the distance domain with five separate values on the finest granularity level. On the next more abstract level these four states are combined to three states, e.g. the distance values *Near* and *Middle* are combined to the value *AnyNear*.

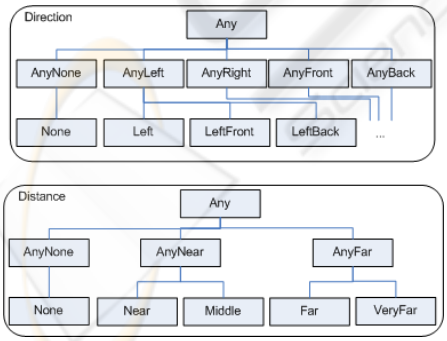

Figure 1: Illustration of the distance and direction domains without the "none" states.

The direction domain is specified like the distance domain on three separate levels of granularity. On the finest granularity level 13 separate states are distinguished. Each attribute domain has a value *None* to represent features that cannot be determined, e. g. unseen objects.

For the hidden states we created two predicates, $dribble(direction, distance)$ and $pass(direction, distance)$ leading to 130 hypotheses on the finest granularity level, 30 hypotheses on the next granularity level and two hypotheses on the most abstract granularity level to distinguish. For the visible states we experimented with different features of the environment, e.g. the relative position of the nearest teammate, the relative position of the next opponent and more. A simple heuristical method to determine the best position on the field regarding the distance to the goal, the distance to the ball owner, a negative influence of near opponent players and a positive influence of near teammates turned out to be most suitable after a short period of tests with different teams. The visible states therefore represent the relative position to the best heuristically determined position on the field. Therefore one predicate represents the evidence, $evidence(direction, distance)$, with 65 states to distinguish.

After the discretisation of the states, the time also needs to be discretised into time-slices. Therefore the actions' time-intervals represent time-slices with a dynamic length. To get consistent time-slices for each point in time, the ball-leaders action is regarded to be the reference time-interval. Each action is recognized by a symbolic method based on (Miene, 2004). The evidence will be perceived after each action ends respectively each time an action starts. Based on these snapshots (evidence/action pairs) the model will be trained.

4.2 Prediction Accuracy

The *RHMM* has been evaluated in the *Simulation League 3D RoboCupSoccer* domain on the data of 20 games of the team *Virtual Werder 3D*. By the fact that the reference data is off-line available a leave-one-out cross-validation has been performed to ensure a high degree of accuracy.

First we tested how the *RHMM* assesses the 130 different hypotheses depending on the amount of available training sequences (354 Sequences have been gathered during the 20 games). To generate comparable results we performed this test with exactly the same data with the *HMM* and a simple symbolic method (called *SIMPLE*) setting the probability of the hypothesis to 100% if it occurs in the trained data, 0% if not. For all these three methods we measured the amount of wrong hypotheses assessed higher than the right one, for example if the third most

likely hypothesis was the right one, the error is two. This error scale is represented by the y-axis in Fig. 2. The x-axis shows the amount of trained samples.

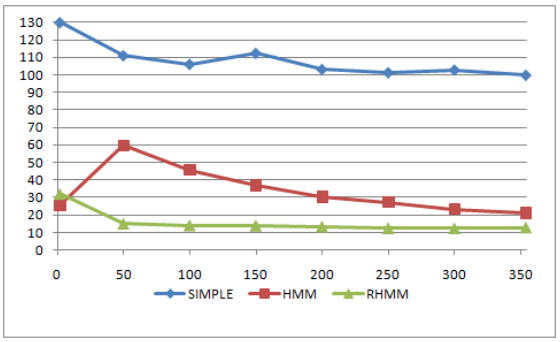

Figure 2: Comparing the inference accuracy of HMM, SIMPLE and RHMM method by the hypothesis rank test.

Figure 2 shows the results indicating that the *RHMM* predominantly outperforms the *HMM* and *SIMPLE* method. Especially the *SIMPLE* method seems to be an inappropriate method for action prediction in such an uncertain environment. The simple method predicted the right action in an average after $107,32$ wrong hypotheses. Basically the inability of representing beliefs of certain evidences seems to be responsible for the enormous amount of errors. Also the *RHMM* predominantly outperforms the traditional *HMM* with an average error of $13,74$ to $37,05$ in evaluating the hypotheses, especially with a low amount of reference data. This shows that considering domain-specific information in *RHMM* leads to an positive influence on the prediction accuracy.

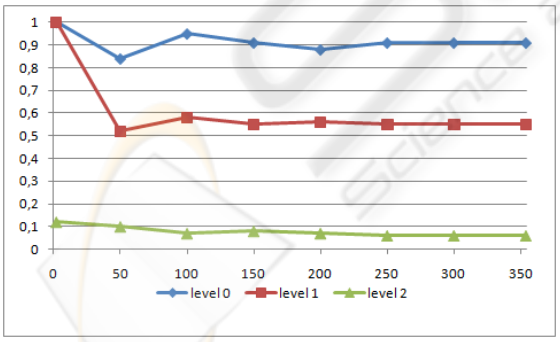

Figure 3: Inference accuracy of the RHMM method on different granularity levels with 130;30;2 states to distinguish.

As a great aspect of the given representation, the ability to perform inference on different levels of granularity, we were able to test the inference on the given three levels by their accuracy. On the y-axis we see the error-factor, the amount of wrong predicted states to the overall amount of predictable states. The x-axis represents the amount of trained samples. In figure 3 we see an expected behavior, the more abstract the level of granularity is the less errors occur. On the finest granularity level the average error is relatively high with 89% but this value does not consider that maybe not the right hypothesis has been chosen but a very similar hypotheses to the right one. This assumption can be confirmed by looking at the more abstract granularity levels. On the next abstraction level the average error is 59% selecting between 30 hypotheses and on the most abstract level the error could be reduced to an average of 7%. Additionally we can think about another way to interpret the results: By the probability that a previously wrong predicted state has been predicted right at random on a higher granularity level. Therefore from level 0 to level 1 the average error of 89% should be reduced (randomly) to $89\% - 89\% \cdot \frac{1}{30} \approx 86\%$ (without regarding any proper method to combine the states). The average error of the *RHMM* method with the given taxonomy therefore is 59%, $\approx 68,6\%$ lower than expected. The randomly reduced error rate from level 0 to level 2 is $89\% - 89\% \cdot \frac{1}{2} = 44,5\%$, but with the *RHMM* method the error rate is only 7%.

Further the ability to infer on different granularity levels offers to specify a minimal certainty for the prediction and to perform inference on a dynamic granularity level. By adopting the level of granularity a hypothesis can be searched accessing the given certainty. So instead of defining a fixed granularity level a minimum certainty is used to automatically determine an ideal granularity level.

4.3 Complexity and Performance

The inference complexity of the *RHMM* approach can be reduced to the complexity of a *HMM* by precalculating the model parameters considering the relational dependencies. The precalculation effort on the other hand is highly dependent to the complexity of the model structure, especially the amount of attributes for a relation and the attributes' domain complexity. To indicate how complex the precalculations and the inference it-self are, the following table shows the used time on an *Athlon 2400 XP-M* for the presented domain in milliseconds:

Test	Time Ø	Tests	σ	Min	Max
Training	68,01	1000	0,15	50	80
Precalc.	704,23	1000	0,24	680	730
Inf. w/ prec.	2,99	14900	0,03	0	10
Inf. w/o prec.	272,82	14900	0,04	260	430

Figure 4: *RHMM-RANK* inference without precalculations.

Fig. 4 shows the inference measurements with and without precalculations and the corresponding time to determine the precalculation values. The table shows that the precalculations for the given model structure can be done in a very short period of time ($704,23ms$) and decrease the used inference time significantly (from $272,8ms$ to $2,99ms$). Further, the complexity of the inference method is linear dependent to the amount of evidences if the look-up-table has been precalculated.

5 DISCUSSION

The tests also revealed a seldom and not preferable property of the *RHMM*: An over-generalization in some cases. If only a very small amount of reference data is available for one state but a very large amount of data is available for another but very similar state, the state with the few reference data will be mostly neglected during inference. This is not always preferable, cause the few data could be a more appropriate basis for the inference of this state. However, the training data in the *RoboCup* domain was relatively good distributed.
Overall the *RHMM* could outperform the *HMM* in the meaning of prediction accuracy by the assumption that the domain could be represented by the described relational properties and on the requirement of a higher training effort. However, the inference effort could be reduced to the *HMM* ones by precalculation during the training phase.

6 CONCLUSIONS AND FUTURE WORK

In this work we presented the *relational hidden markov model* based on the well-established *HMM* and the *RMM* and showed it's application for spatio-temporal reasoning. The presented method lead to a significant increased inference accuracy with a minimal increased calculation effort. Additionally, the ability for inference on different and dynamic levels of granularity provides better control over the results of the prediction process. In the presented domain the *RHMM* could predominantly outperform the well-known *HMM* in inference accuracy at the costs of a more complex inference or training mechanism.

Other known inference problems from *HMMs*, e.g. the *viterby algorithm* should be further investigated in the context of *RHMM*. Also the application to undiscretized states / observations can become in the focus of further research. The applicability of this method to other domains should also be regarded.

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