## DYNAMIC AND EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION FOR SENSOR SELECTION IN SENSOR NETWORKS FOR TARGET TRACKING

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Abstract: When large sensor networks are applied to the task of target tracking, it is necessary to successively identify subsets of sensors that are most useful at each time instant. Such a task involves simultaneously maximizing target detection accuracy and minimizing querying cost, addressed in this paper by the application of multi-objective evolutionary algorithms (MOEAs). NSGA-II, a well-known MOEA, is demonstrated to be successful in obtaining diverse solutions (trade-off points), when compared to a "weighted sum" approach that combines both objectives into a single cost function. We also explore an improvement, *LS-DNSGA*, which incorporates periodic local search into the algorithm, and outperforms standard NSGA-II on the sensor selection problem.

## **1 INTRODUCTION**

When a large sensor network is used for target tracking, it is not practical to constantly query all sensors, due to practical constraints on computation, sensing range, communication bandwidth, and energy consumption. The number of sensors to be queried (for high tracking accuracy) may itself vary, and querying strategies are desirable that allow the data to determine which sensors should be queried at successive instants. This paper addresses the task of successively identifying subsets of sensors while simultaneously addressing two conflicting objectives: cost and tracking accuracy.

To illustrate our approach, we use a simple formulation of communication cost, which increases with the number of sensors and the communication distance (from a sensor to the decision-maker's location). Maximizing (predicted) tracking accuracy is considered equivalent to minimizing mean squared error (MSE), and addressed in our approach by minimizing the Posterior Cramer-Rao Lower Bound (PCRLB) which provides a theoretical performance limit of any estimator for a nonlinear filtering problem. In earlier work (Zuo et al., 2006), we have been successful in using the posterior CRLB for sensor selection in tracking problems.

A straightforward approach often adopted in the engineering literature is to attempt to optimize a linear combination of different objective functions. Exploring different linear combinations yields solutions corresponding to different tradeoff points. However, this approach has serious deficiencies, and is outperformed by multi-objective evolutionary algorithms (MOEAs) that simultaneously evolve multiple candidate solutions, as discussed in (Deb, 2001), where the *Non-dominated Sorting Genetic Algorithm-II (NSGA-II)* is proposed; other MOEA variations include SPEA2, PAES and EMOCA (Rajagopalan et al., 2005). In this paper, we have successfully applied an improvement of NSGA-II that incorporates local search.

## 2 MODELS AND OBJECTIVES

This section describes the target motion model, sensor model, and the two objectives to be optimized.

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#### 2.1 Target Motion Model

Each target is assumed to be moving in a 2-D Cartesian coordinate plane according to a dynamic white noise acceleration model (Bar-Shalom et al., 2001):

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{v}_k \tag{1}$$

where constant **F** models the state kinematics, and  $\mathbf{x}_k = [x_k \dot{x}_k y_k \dot{y}_k]^T$  defines the target state at time *k*. Here,  $x_k$  and  $y_k$  denote the target position and,  $\dot{x}_k$  and  $\dot{y}_k$  denote the target velocities.  $\mathbf{v}_k$  is a white Gaussian noise with covariance matrix **Q**.

#### 2.2 Sensor Measurement Model

We assume that homogeneous bearing only sensors are randomly deployed in a 2D Cartesian coordinate plane. The *fusion center* has knowledge about individual sensors (e.g., positions and measurement accuracy). When queried, a sensor communicates its estimate of the target state to the fusion center. The measurement model is given by:

$$\boldsymbol{\theta}_{k}^{j} = h(\mathbf{x}_{k}) + \mathbf{w}_{k}^{j} = tan^{-1} \left( \frac{y_{k} - y^{s_{j}}}{x_{k} - x^{s_{j}}} \right) + \mathbf{w}_{k}^{j} \qquad (2)$$

where  $\theta_k^j$  is the original measurement from sensor *j* with additive white Gaussian noise  $\mathbf{w}_k^j$ , whose variance is parameterized as **R**.

#### 2.3 Objectives

Communication cost is computed as follows:

$$Cost = \sum_{i=1}^{n} (C_0 + C_1 \sqrt{(X_{ch} - X_i^2) + (Y_{ch} - Y_i^2)}) \quad (3)$$

Here,  $(X_{ch}, Y_{ch})$  and  $(X_i, Y_i)$  denote the coordinates of cluster head and  $i^{th}$  queried sensor, respectively. Cost increases with the number of queried sensors.

Tracking accuracy is estimated using PCRLB on the estimation error of the target position, summing up the position bound along each axis at time k+1:

$$C_{k+1} = J_{k+1}^{-1}(1,1) + J_{k+1}^{-1}(3,3)$$
(4)

where  $J_{k+1}^{-1}(1,1)$  and  $J_{k+1}^{-1}(3,3)$  are bounds on the MSE corresponding to  $x_{k+1}$  and  $y_{k+1}$  respectively. Particle filters are used to calculate PCRLB as well as to estimate the target state. We use the recursive approach in (Tichavsky et al., 1998) to calculate  $J_k$ , i.e., sequential FIM, as follows:

$$J_{k+1} = D_k^{22} - D_k^{21} (J_k + D_k^{11})^{-1} D_k^{12}$$
(5)

where

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$$D_k^{11} = E\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \log p(\mathbf{x}_{k+1}|\mathbf{x}_k)\}$$
(6)

$$D_k^{12} = E\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_{k+1}}\log p(\mathbf{x}_{k+1}|\mathbf{x}_k)\}$$
(7)

$$D_k^{21} = E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_k} \log p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} = (D_k^{12})^T$$
(8)

$$D_{k}^{22} = E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_{k+1}}\log p(\mathbf{x}_{k+1}|\mathbf{x}_{k})\} + E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_{k+1}}\log p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})\}$$
(9)

## 3 MULTI-OBJECTIVE OPTIMIZATION

A vector  $x_1$  dominates  $x_2$  if  $x_1$  is not worse than  $x_2$  in any objective, and  $x_1$  is strictly better than  $x_2$  in some objective. A decision vector  $x_1$  is **Pareto-optimal** if no vector  $x_2$  dominates  $x_1$ . MOO problems require algorithms that find a well distributed set of Paretooptimal solutions with least computational expense.

The "Weighted Sum" (Wtd.) approach in addressing MOO problems optimizes a weighted sum of the individual objective functions, obtaining different tradeoff solutions by using different weights; unfortunately, this approach has well-known failings (Deb, 2001). In its application to the problem of interest in this paper, we attempt to minimize  $\mathbf{F} = (\mathbf{w})\text{Cost} + (1-\mathbf{w})\text{PCRLB}$ , for different values of  $\mathbf{w}$ , first by restricting the number of sensors chosen (in each step) to 2, and later allowing this number to vary.

MOEAs provide great promise since they can simultaneously evolve multiple solutions exploring regions of the Pareto set. Currently, the most widely used MOEA is NSGA-II (Deb, 2001), which we have applied to the sensor selection problem using PCRLB and communication cost as the two objectives to be minimized. We have also formulated a new variant, "Local Search based Dynamic NSGA-II" (LS-DNSGA), described in Table 1, which has provided better results on the problem under consideration. The newly introduced features of LS-DNSGA are:

(1) **Population Seeding.** At each time step (except the first), the initial population is seeded with 80% of the final non-dominated solutions obtained at the end of previous time step. This mechanism helps since target positions generally do not vary much in consecutive time steps.

(2) **Local Search.** An iterative improvement procedure is applied periodically to solutions from the best non-dominated solution front, minimizing  $\mathbf{F} = w\text{Cost} + (1 - w)\text{PCRLB}$  with

$$W = \frac{\frac{(MaxCost-Cost)}{(MaxCost-MinCost)}}{\frac{(MaxCost-Cost)}{(MaxCost-Cost)} + \frac{(MaxPCRLB-PCRLB)}{(MaxPCRLB-MinPCRLB)}}$$

where, *MinCost, MaxCost* and *MinPCRLB, Max-PCRLB* represent minimum and maximum Cost and

PCRLB values in the entire population. Local search is carried out by repeated successive mutation for each bit in the candidate solution, accepting the mutation if and only if it decreases F.

Table 1: LS-DNSGA(Local Search based Dynamic Nondominated Sorting Genetic Algorithm), executed at each time step when sensors are to be selected.



## **4** SIMULATION RESULTS

Several series of simulations were carried out to evaluate the proposed algorithm. The first set, discussed in subsection 4.1, compares LS-DNSGA with a weighted summation approach, a single objective approach, and with NSGA-II. The single objective approach is applied in two ways: a) Exhaustive search approach searching only sensor subsets of size 2. b) Single objective GA capable of finding any number of sensor subsets. In subsection 4.2 LS-DNSGA is applied and two sensor selection schemes namely, fixed number of sensors (2) and fixed cost strategy, are investigated. Finally, subsection 4.3 compares LS-DNSGA and NSGA-II on larger sensor networks. The appendix contains details of various parameters involved in system models, sensor selection schemes and cost computation.

#### 4.1 Experiment I

In these simulations, we compare the LS-DNSGA with a single objective approach and standard NSGA-II. We consider the target tracking problem, with total number of sensors 16, for 15 time steps.

For LS-DNSGA, we chose a population size of 40, maximum number of generations being 100, and mutation probability of 0.08.

For the weighted sum method, 11 different values  $\mathbf{w} \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$  are considered for illustrative purposes; obviously, more solutions can be obtained by choosing more values of textbfw. An exhaustive search with 2 sensor selection scheme, and a single objective weighted sum GA procedure were employed.

Average performance was computed over 51 trials using different initial seed values. We computed the  $26^{th}$  attainment surface, representing median performance. In figure 1 median non-dominated sets obtained for LS-DNSGA are plotted along with the median performances for different weights considered for the weighted sum approaches. We separately compared (a) LS-DNSGA and the exhaustive approach both with the restriction of 2 sensors, and (b) LS-DNSGA with single objective GA with the number of sensors allowed to vary.

The non-dominated sets shown in Figure 1 (space limitations restrict us to only a few time steps) result in the following observations:

- The weighted sum approach using exhaustive search finds solutions that only cover a very small region as compared to LS-DNSGA.
- The weighted sum GA approach is also unable to cover the entire non-dominated set satisfactorily.
- For all time steps, the weighted sum approach solutions are mostly dominated, and never better than those obtained using LS-DNSGA.
- Better results are obtained if the number of sensors is allowed to vary.

For a 2-objective problem, the hypervolume represents the sum of the areas enclosed within the hypercubes formed by the points on non-dominated front and a chosen nadir point. Since the objectives are to be minimized, larger hypervolumes are desirable, representing better spread and quality of solutions. We compare LS-DNSGA(with and without population seeding) with NSGA-II (with and without population seeding), measuring the average hypervolume (Padhye et al., 2009) over 51 runs, using the nadir point (700, 250), based on maximum cost and maximum PCRLB values obtained over all time steps. Figure 2 shows that LS-DNSGA is the best performer in reaching steady state hypervolume fastest without showing any oscillations, whereas the other algorithms converged more slowly and sometimes exhibited oscillatory behavior. For all algorithms other than LS-DNSGA, for time steps 1 and 2, hypervolume starts at a higher value and keeps falling with-



Figure 1: Plots for median attainment surface curves for LS-DNSGA along with median performances of weighted summation approaches for different Ws over first 4 time steps. Plots indicate that LS-DNSGA is able to find a well distributed set of solutions but the weighted sum approaches can only find few points on the non-dominated set.

out reaching a constant value; this is known to occur when few, newer and less converged, solutions replace better but less diverse population members (Padhye, 2009), (Laumanns et al., 2002).

The improvements in results obtained by LS-DNSGA are due to the combined effect of local search and population seeding mechanisms. If the Pareto front is not changing very rapidly then doing a local search on seeded solutions can lead to finding of new Pareto front quickly. Although local search increases the number of function evaluations in each generation, our simulations showed that LS-DNSGA was often able to reach steady state requiring fewer function evaluations than other algorithms. Due to space limitations, hypervolume curves are shown for only a few time steps; the authors can be contacted for more details.

#### 4.2 Experiment II

Simulations were also carried out to compare the querying strategies with fixed cost vs. fixed number of sensors. Three sets of experiments are performed, with 51 runs for each set. First, the weighted sum approach was applied and a single objective function  $\mathbf{F}$ ,

was minimized for 11 different values  $\mathbf{w} \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$ . Exhaustive search was conducted with exactly two sensors to be chosen for querying in each time step. Median Cost and PCRLB for  $\mathbf{w} = 0.0, 1.0$ , based on 51 runs, are listed in Table 2. We then applied LS-DNSGA with exactly 2 sensors and having minimum PCRLB is selected for querying - this is the 'two sensor strategy'. Separately, LS-DNSGA was applied, selecting the solution which has minimum PCRLB and querying Cost less than 300 units is chosen- this is the 'fixed cost strategy'. 51 runs of were executed with population size 50, maximum generations 100, and mutation probability 0.07, and the  $26^{th}$  attainment surfaces were computed.

For the two-sensor strategy, solutions (with two sensors) were located on the median non-dominated front, and the two solutions corresponding to minimum Cost and minimum PCRLB are referred to as 'left' and 'right' respectively, and listed in Table 3. By comparing these extreme solutions ('left' and 'right') with the weighted sum approach solutions (corresponding to w=0.0 and w=1.0), we can evaluate whether LS-DNSGA succeeds in find the extrema. In Table 4 we list the extreme solutions on the entire me-



Figure 2: Hypervolume Curves for LS-DNSGA with and without seeding, and NSGA-II with and without Seeding for time steps 1 - 4, under Experiment I, indicating fast and accurate convergence of LS-DNSGA, whereas others are slow in convergence and their performance levels do not match LS-DNSGA.

Table 2: Optimal Solutions (rounded-off to two decimal places) obtained by the weighted sum approach using exhaustive search (only considering 2 sensors) for w values 0.0 and 1.0

	w=0.0	w=1.0
Т	[Cost, PCRLB]	[Cost, PCRLB]
1	9.41, 0.32	9.41, 0.33
2	9.41, 1.21	9.41, 1.23
3	20.36, 1.71	9.41, 4.33
4	20.36, 1.89	9.41, 13.39
5	45.89, 1.59	9.41, 32.22
6	77.85, 2.75	9.41, 61.13
7	70.71, 2.41	9.41, 104.24
8	100.25, 2.20	9.41, 160.77
9	102.61, 2.58	9.41, 240.91
10	100.25, 0.51	9.41, 346.99
11	98.51, 6.16	9.41, 485.67
12	84.21 <mark>,</mark> 23.08	9.41, 662.66
13	84.2 <mark>1</mark> , 47.34	9.41, 869.66
14	82.05, 75.76	9.41, 1110.85
15	82.05, 111.06	9.41, 1401.24

dian non-dominated front found by LS-DNSGA using fixed cost strategy.

From Tables 2 and 3 we observe that values under  $left^*$  column are comparable with those under **w=1.0**. Similarly,  $right^*$  column is comparable with **w=0.0**.

Table	3:	Left	and	Right	extreme	LS-DN	SGA	for
woser	isorg	uery*.	Rou	nded-of	f to two d	lecimal p	places.	

Т	<i>Left</i> *[Cost, PCRLB]	Right*[Cost, PCRLB]
1	9.41, 0.38	9.41, 0.24
2	9.41, 1.75	9.41, 0.96
3	9.41, 3.83	20.41, 1.67
4	9.41, 7.94	20.41, 1.78
5	9.41, 7.37	46.04, 1.44
6	9.41, 10.16	70.97, 2.91
7	9.41, 11.03	80.50, 2.01
8	9.41, 11.03	78.14, 2.57
9	9.41, 12.87	103.00, 2.33
10	9.41, 16.41	103.00, 0.06
11	9.41, 11.03	98.88, 5.08
12	9.41, 41.73	84.52, 20.77
13	9.41, 62.20	84.52, 44.58
14	9.41, 102.56	82.34, 73.23
15	9.41, 143.86	82.34, 107.11

In other words, solutions obtained by LS-DNSGA are non-dominated (or non-inferior) when compared to those obtained by the weighted summation approach. Moreover, for the first few time steps, the sensor pair that minimizes Cost also minimizes PCRLB, perhaps due to the positioning of sensors in the field and initial position of particle. The extreme solutions for cost constrained LS-DNSGA (with multiple sensor selec-

Т	Left <sup>**</sup>	Right <sup>**</sup>					
	[Cost, PCRLB]	[Cost, PCRLB]					
1	3.39, 5.86	520.56, 0.25					
2	3.39, 17.01	520.56, 0.63					
3	3.39, 5.74	520.56, 0.97					
4	3.39, 7.68	520.56, 0.70					
5	3.39, 7.47	520.56, 0.70					
6	3.39, 8.39	520.56, 0.87					
7	3.39, 10.04	520.56, 0.78					
8	3.39, 9.77	520.56, 0.73					
9	3.39, 10.32	520.56, 1.07					
10	3.39, 12.76	520.56, 0.12					
11	3.39, 10.48	520.56, 3.40					
12	3.39, 25.19	520.56, 10.20					
13	3.39, 42.96	520.56, 19.32					
14	3.39, 63.93	520.56, 30.51					
15	3.39, 103.12	520.56, 45.44					

Table 4: **Left** and **Right** extreme optimal points obtained by LS-DNSGA for *fixedcoststrategy*\*\*. Rounded-off to two decimal places.

tion), are 1 sensor (min. cost 3.3888 units) and 16 sensors (max. Cost 520.562), computed separately, which correspond to points on the true Pareto optimal front.

The M.S.E. in position is plotted against time in Figure 3 for various strategies. The corresponding average costs over the time steps are shown in Figure 4, demonstrating that LS-DNSGA fixed cost (or multiple sensor) strategies show better performance over other strategies for most of the time steps (some variations in performance can be attributed to noise). The single objective approach with w=1.0, implying minimization of cost only, shows the worst performance. Comparing figures 3 and 4 it can be seen that increased average cost corresponds to a lower MSE. Further, increasing cost from 300 to 400 units does not yield a distinguishable improvement in MSE as compared to change from 200 to 300 units.

The non-dominated solutions obtained for one simulation and time step 6 are shown in Figure 5. The optimal point by single objective approach (78.9, 2.7) with variable number of sensors has lower PCRLB than the extreme right solution on the non-dominated solution with fixed number (2) of sensors (32.3, 5.0). Also, a point on the Pareto front with 3 sensors (72.3, 2.7) dominates the single objective optimal point, eliminating the latter from the Pareto front, which leads to the selection of point (32.3, 5.0) as the one with two sensors and minimum PCRLB value for querying with multi-objective approach. This shows why choosing a solution with fixed number of sensors from the entire non-dominated front may lead to



Figure 3: Comparison of MSE for LS-DNSGA with fixed number (2) of sensors, LS-DNSGA with Cost Constrained, and Single Objective Approaches with different weights.



Figure 4: Illustrating the Performance in terms of Average Cost of LS-DNSGA with fixed number (2) of sensors, LS-DNSGA with Cost Constrained, and Single Objective Approaches with different weights.

poor performance, and why the two-sensor strategy performs worse than the weighted sum approach with w=0.0 (or 0.5).

#### 4.3 Experiment III

Further experiments were conducted to compare LS-DNSGA and NSGA-II with varying numbers (30, 40 and 50) of sensors. Hypervolume results are presented, averaged over 51 runs, for 15 time steps in Table 5, summarized using "+", "-" and "o" respec-

Time Step:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
30 Sn.	-	0	+	0	0	0	0	0	0	0	0	0	0	0	0
40 Sn.	+	1	0	+	0	0	0	0	0	0	+	0	0	0	0
50 Sn.	-	+	+	+	+	+	+	+	0	0	0	+	+	+	+

Table 5: LS-DNSGA performance for different sensors over 15 time steps, cost based.

Comparison for 16 Sensors, 6th time step



Figure 5: Illustration why 2 sensor strategy in multiobjective can perform poorly.

tively to indicate that LS-DNSGA performance was better, worse, and almost similar compared to NSGA-II. Thus, + under time step 2 in third row in table 5 indicates that LS-DNSGA performed significantly better in terms of hypervolume over NSGA-II for time step 2. Similarly o in first row under time step 2 indicates both algorithms performed similarly.

These results show that as the number of sensors increases, the relative advantage of LS-DNSGA over NSGA-II increases. Even when the hypervolumes obtained were similar, we observed that LS-DNSGA invariably reached the steady state hypervolume requiring significantly fewer generations. Seeding and local search appear to be most helpful in later time steps, and when the number of sensors is large. Additionally, a single objective approach was also applied to minimize Cost and PCRLB, separately, and these solutions were found to be often dominated by extreme solutions obtained by the multi-objective approach.

## 5 CONCLUSIONS

We have addressed the task of selecting subsets of sensors (for target tracking) as a multi-objective optimization problem, simultaneously minimizing communication cost and PCRLB (providing a bound on the estimated MSE). A well-known evolutionary multi-objective algorithm, NSGA-II, was applied to

this problem, along with a new variant, LS-DNSGA, proposed in this paper. LS-DNSGA was compared against the weighted summation approach and showed superior performance (faster and accurate convergence) over NSGA-II, based on median attainment surfaces and average hypervolume. The evolutionary approach was successful in finding a welldistributed set of tradeoff solutions providing better decision making as compared to the weighted sum approach. Two strategies, fixed cost and fixed number of sensors, were tested and former was found to be more useful. The proposed algorithm, LS-DNSGA, was found to perform well on large sensor networks as compared to NSGA-II, and single objective approach, highlighting the usefulness of local search mechanism. Howevwer, local search becomes computationally more expensive when population size increases; future work will explore a probabilistic local search mechanism.

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