

# FUZZY SET THEORY BASED STUDENT EVALUATION

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**Abstract:** The evaluation of students' learning achievements contains in several cases a lot of decisions that are based on the expertise and the opinion of the evaluator. Often this opinion is from nature vague and therefore this field is a good application area for fuzzy set theory based supporting methods and software implementations. In this paper, a new method called FUSBE (Fuzzy Set Theory Based Evaluation) is presented. It supports the scoring and grading of the students allowing the evaluator to express his or her judgment by the means of fuzzy sets that are later aggregated using fuzzy arithmetic. The method is transparent and easy-to-implement.

## 1 INTRODUCTION

The evaluation of student' assignments, homeworks, software, narrative answers, etc. when a fully automated scoring is not possible involves a lot of decisions that are from nature subjective and therefore usually the deviation between the marks or grades given by different evaluators and on different occasions for the same answers could be very high. The subjectivity can be reduced in several cases using standardized scoring criteria, specific examples of responses to the questions, or even sample software solutions but none of these approaches can solve all the problems. Besides, the more specific a guide is the more time consumable its learning and its application is. Furthermore, it is not always an applicable solution.

Another approach for dealing with subjectivism arises from the fuzzy set theory. The application of linguistic terms and related fuzzy sets is common to the human thinking and can result in decreasing the evaluation's sensitivity to "noisy" scoring data. In this case the relation between the linguistic terms and the traditional marks is established by the means of membership functions.

Starting from the early 1990s several ideas have been developed in order to find a better evaluation technique by the help of fuzzy techniques. Biswas (1994) proposed a particular (FEM) and a generalized (GFEM) method that were based on the vector representation of fuzzy membership functions and a special aggregation of the grades assigned to

each question of the student's answerscripts. Chen and Lee (1999) suggested a simple (CL) and a generalized (CLG) method that produced improvements by applying a finer resolution of the scoring interval and by including the possibility of weighting the four evaluation criteria. Nolan (1998) introduced a fuzzy classification model for supporting the grading of student writing samples in order to speed up and made more consistent the evaluation.

All the mentioned methods have their advantages and disadvantages that will be discussed in details in section 2 along with their short presentation. All of the methods contain heuristic elements and therefore there is always a possibility to develop new techniques that could bring advantages from one or more aspects.

In this paper, a new approach is suggested that tries to induce improvements by reducing the computational needs as well as by eliminating the summarization of the potential errors caused by the application of the similarity measure and quasi defuzzification at the evaluation of each question.

The rest of this paper is organized as follows. Section 2 contains the presentation and discussion of some well known methods followed by the introduction of the new technique in section 3.

## 2 FUZZY SET THEORY BASED EVALUATION METHODS

This section presents a short review of the basic ideas and key features of some student evaluation methods that apply elements of fuzzy set theory in order to facilitate the grading of the students' academic performance.

### 2.1 FEM and GFEM

The key idea of the Fuzzy Evaluation Method (FEM) (Biswas, 1994) is that each question in the student answerscript is evaluated independently with a discrete fuzzy set containing membership values for six uniformly distributed predefined points ( $X$ ) of the traditional percentage based evaluation scale [0,100].

$$X = \{0,20,40,60,80,100\} \quad (1)$$

The resulting fuzzy set is compared to all of the so called Standard Fuzzy Sets (SFSs). The SFSs are defined on the same universe of discourse [0,100] corresponding to the grading standard of the university. Each SFS corresponds to a traditional grade (e.g. Excellent). The comparison is made by the means of a similarity degree that is calculated by

$$S_i(\overline{E}_i, \overline{SFS}_j) = \frac{\overline{E}_i \cdot \overline{SFS}_j}{\max(\overline{E}_i \cdot \overline{E}_i, \overline{SFS}_j \cdot \overline{SFS}_j)}, \quad (2)$$

where the index  $i$  denotes the ordinal number of the question,  $\overline{E}_i$  is the vector containing the membership values of the evaluation and  $\overline{SFS}_j$  is the  $j^{\text{th}}$  standard fuzzy set, and “.” denotes the dot product. Further on, the degree corresponding to the SFS with maximum similarity will represent the evaluation of the actual question.

After processing all the questions a total score is determined by calculating the weighted average of the representative values (midpoints) of the fuzzy sets corresponding to the individual grades assigned to the questions by

$$TS = \frac{\sum_{i=1}^n (T(Q_i) \cdot P(g_i))}{100}, \quad (3)$$

where

$$\sum_{i=1}^n T(Q_i) = 100, \quad (4)$$

where the index  $i$  denotes the ordinal number of the question,  $n$  is the total number of questions,  $Q_i$  is the question,  $T(Q_i)$  is the weight of the question,  $g_i$  is the degree assigned to the question,  $P(g_i)$  is the representative value of the degree, and “.” symbolizes the dot product.

The Generalized Fuzzy Evaluation Method (GFEM) (Biswas, 1994) evaluates each answer from four different points of view, namely the accuracy of information, the adequate coverage, the conciseness, and the clear expression. The arithmetic mean of the midpoints of the fuzzy sets representing the four grades assigned will represent the evaluation of the given question expressed with marks between 0 and 100

$$E_i = \frac{\sum_{k=1}^4 P(g_{ik})}{4}, \quad (5)$$

where  $k$  identifies the point of view. One calculates the total score ( $TS$ ) as a weighted average of the individual marks

$$TS = \frac{\sum_{i=1}^n (T(Q_i) \cdot E_i)}{100}. \quad (6)$$

The applied weighting is the same as in the case of FEM.

The advantage of FEM and GFEM is their easy-to-understand and easy-to-implement character. Their disadvantage is that they determine separate grades for each question applying a rounding to the most similar grade, which introduces an error in each evaluation step. The error summarizes in course of the evaluation of the answerscript and at the end it can lead to a quite strange final result.

The use of the midpoints in the total score calculation is a quasi defuzzification before the final aggregation, which also can mislead the evaluation. Besides, the relation between the SFSs and the values of the midpoints is not defined clearly. However, the SFS based concept can soften the difference between the final scores given by independent evaluators owing to the feature that slightly differing evaluations can result in the same grade.

### 2.2 CL and CLG

The CL method proposed by Chen and Lee (1999) has several similar elements to FEM. However, they use a slightly different terminology. The method

defines a finer resolution of the scoring scale, which is in this case the interval [0,1] by using eleven so called satisfaction levels that are crisp similar to the traditional grade based evaluation. Here one uses an extended grade sheet for the evaluation's documentation, which contains for each question eleven cells that have to be filled in by the evaluator with values between 0 and 1. They describe in what amount the answer given by the student belongs to the predefined satisfaction levels. They can be considered also as membership values. After filling in the eleven cells of the current row a degree of satisfaction  $D(Q_i)$  is calculated for the current question  $Q_i$  by

$$D(Q_i) = \frac{\sum_{j=1}^{11} y_{ij} \cdot T(SL_j)}{\sum_{j=1}^{11} y_{ij}}, \quad (7)$$

where  $y_{ij}$  is the membership value assigned for the  $j^{\text{th}}$  satisfaction level  $SL_j$ , and  $T(SL_j)$  is the upper bound of the score interval corresponding to  $SL_j$ .

Finally, the total score of the student is calculated as a weighted average of the individual degrees of satisfaction

$$TS = \sum_{i=1}^n s_i \cdot D(Q_i), \quad (8)$$

where the weights have to satisfy the equation

$$\sum_{i=1}^n s_i = 100. \quad (9)$$

Chen and Lee also published in (Chen & Lee, 1999) a generalized version of their method (CLG). The applied approach is similar to GFEM; it uses the same four criteria for evaluation of each question from different points of view. Thus one calculates four degrees of satisfaction for each question. The overall mark  $P(Q_i)$  of the response is calculated as a weighted average of the four degrees of satisfaction

$$P(Q_i) = \frac{\sum_{k=1}^4 w_k \cdot D(Q_i, k)}{\sum_{k=1}^4 w_k}, \quad (10)$$

where  $w_k$  is the weight of the  $k^{\text{th}}$  criteria, and  $D(Q_i, k)$  is the degree of satisfaction of the  $k^{\text{th}}$  criteria. CLG determines the total score by substituting  $P(Q_i)$  for  $D(Q_i)$  in (8).

The CL and CLG methods are in several ways similar to the FEM-GFEM pair. They introduce improvements by a finer resolution of the scoring interval and by allowing the weighting of the four criteria. These modifications increase the computational need, however, this not a great problem owing to the fact that the methods are applicable in practice only when a software support is ensured.

### 2.3 Evaluation Based on Fuzzy Classification

Nolan (1998) reports the successful development, implementation and application of a fuzzy rule based model called Expert Fuzzy Classification System (EFCS). EFCS was developed in order to support the evaluation of fourth grade students' writing samples in case of narrative response exams. The system supports a well defined rating process aiming the reduction of the time needed for the evaluation as well as making the results more consistent.

The underlying rule base was created using the rules of the scoring guide applied in case of the traditional way of evaluation. The antecedent parts of the rules examine the existence of some skills like character recognition, text understanding, etc., which are represented by the input linguistic variables. The rules infer the measure of skills like reading comprehension, etc. that are represented by the consequent linguistic variables. An example rule is

IF understanding is *high*  
AND character-recognition is *strong*  
THEN reading-comprehension is *high*.

The resolution of the scoring universe is not high; the partitions usually consist of three fuzzy sets. The membership functions were developed based on the interval definitions given by a group of expert teacher graders.

In course of the evaluation the rater assigns one score for each dimension of the antecedent universe of discourse (input linguistic variables) and the system determines a final score using a Mamdani-type (Mamdani & Assilian, 1975) inference mechanism.

Although EFCS is an application specific system its concept easily can be used for evaluation tasks where there is available a clear defined rule system

(scoring guide) based on symbolic statements in the antecedent and consequent parts of the rules.

The advantage of EFCS is that it achieved both of the aims of its developer, namely the evaluation time reduction and the increase of the consistence of the grading given by different raters.

The drawback of EFCS is that it requires a tedious preparation work. The original system contained 200 rules and the participation of a group of expert grader was necessary for the determination of the fuzzy partitions.

### 3 FUZZY SET THEORY BASED EVALUATION

This section reports the development of a fuzzy set theory based evaluation model for student writing exams. The first subsection will describe the traditional approach applied in our institute. The proposed fuzzy solution for this task and the software based on it will be presented in the second subsection.

#### 3.1 The Traditional Approach

Although there is no standardized scoring guide in our institute usually the rating of the assignments with narrative responses happens as follows. The total number of marks for an assignment or group of consecutive assignments is 100. This number is divided between the questions of the assignment(s).

Table 1: Relation between scores and grades.

Score intervals	Grades
0 - 50	Unsatisfactory
51 - 60	Satisfactory
61 - 75	Average
76 - 85	Good
86 - 100	Excellent

Thus the lecturer that prepares the question sheet assigns marks between 1 and 25 to each question, viz. each sheet contains at least four questions.

Unlike the previously presented methods our institute does not use explicit weight number set, the significance of a question is expressed by the number of marks a student can achieve in case of a perfect response. The assignment of the actual number of marks is based on the expertise of the evaluator. At the end we calculate a total score calculated by summarizing the individual scores achieved in case of each question, and the final score

is mapped to a five-graded scale. The grades are “unsatisfactory”, “satisfactory”, “average”, “good”, and “excellent”. The mapping is standardized; the score intervals corresponding to the grades are presented in Table 1. They also can be described by the crisp sets on Figure 1.



Figure 1: Traditional grades represented as crisp fuzzy sets.

#### 3.2 Fuzzy Set Based Evaluation

In course of the development of the Fuzzy Set Based Evaluation (FUSBE) method the following demands were taken into consideration:

- Although computational complexity may not be an issue owing to the capabilities of the nowadays available computers, the method should be as simple as possible in order to be understandable for both the students and the evaluators; all participants of the evaluation process have to consider it as a fair deal;
- The method should enable for the evaluator to express the vagueness in her or his opinion in form of fuzzy sets in case of each question;
- In case of one-valued scoring (singleton fuzzy sets) the model should lead to the same result as the traditional approach.

In order to fulfil the above mentioned requirements the application of fuzzy arithmetic as score aggregation tool and the use of Centre Of Area (Kóczy & Tikk, 2000) defuzzification method has been selected.

Thus the evaluation process is the following. In case of each question the evaluator determines the fuzzy score by the means of a fuzzy number. Theoretically from the rating model's point of view the set of applicable membership function types is not limited as far as they fulfil the CNF (convex and normal fuzzy set) criteria. However, like any other fuzzy approach based evaluation model FUSBE is practically applicable only when the calculations are

done by a computer. Therefore the cardinality of the selectable membership function types becomes an implementation detail.

For now the piece wise linear membership functions that can be described by a trapezoid (i.e. trapezoid, square, rectangle, triangle, and singleton) are supported by our program. Other fuzzy set shape types like piece wise linear forms with more than four vertices and non-linear forms like bell shaped, sigmoid,  $\Pi$ , L-R, etc. will be included in future versions of the software.

The input of the fuzzy scores does not require any typing. The graphical user interface (GUI) is so designed that the parameters of the fuzzy sets can be set by the help of controls using the mouse. We consider only CNF sets as fuzzy scores of a question. All of the parameters have default values; the evaluation starts with a trapezoid situated at the middle of the scoring interval. In case of the trapezoid shaped membership functions (Figure 2) one needs to specify at most four parameters that define the position ( $a$ ) of the set and the three width values ( $b$ ,  $c$ , and  $d$ ). One modifies the default parameters with trackbars using the mouse (Figure 3).

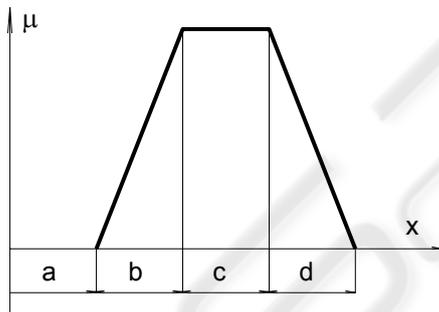


Figure 2: Parameters of a trapezoidal shaped fuzzy score.

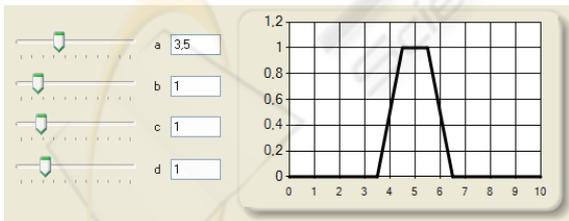


Figure 3: Input of the fuzzy score.

The total fuzzy score is calculated as a sum of the fuzzy scores given to the individual responses. Conform to the theory of the fuzzy arithmetic it is calculated  $\alpha$ -cut wise, where an  $\alpha$ -cut of a fuzzy set is defined by

$$[A]_{\alpha} = \{ x \in X \mid \mu_A(x) \geq \alpha; \alpha \in (0, 1] \} . \quad (11)$$

Thus an  $\alpha$ -cut of the total fuzzy score ( $TFS$ ) will be

$$[TFS]_{\alpha} = \sum_{i=1}^n [FS_i]_{\alpha} , \quad (12)$$

where  $n$  is the number of questions and  $FS_i$  is the fuzzy score of the  $i^{\text{th}}$  question. The calculations are done by the help of the lower and upper endpoints of the  $\alpha$ -cut

$$[TFS]_{\alpha} = \{ x \in X \mid x \geq \inf \{ [TFS]_{\alpha} \} , \quad (13)$$

$$x \leq \sup \{ [TFS]_{\alpha} \} \} ,$$

where

$$\inf \{ [TFS]_{\alpha} \} = \sum_{i=1}^n \inf \{ [FS_i]_{\alpha} \} , \quad (14)$$

$$\sup \{ [TFS]_{\alpha} \} = \sum_{i=1}^n \sup \{ [FS_i]_{\alpha} \} . \quad (15)$$

In the general case the resulting fuzzy set determined as a union of its  $\alpha$ -cuts by

$$TFS = \bigcup_{\alpha=0+}^1 [TFS]_{\alpha} \quad (16)$$

requires a high number of  $\alpha$ -cuts depending on the demanded accuracy of the result. However, in case of trapezoidal shaped membership functions one can simplify the calculations by using the two relevant  $\{ 0+, 1 \}$   $\alpha$  levels.

Owing to the fact that we have been bound to the total score – grades mapping presented in section 3.1 one has to defuzzify the  $TFS$ . FUSBE uses Center Of Area (Kóczy & Tikk, 2000) type defuzzification for this task. Thus the method fulfils all the demands set at the beginning of the section.

## 4 CONCLUSIONS

The evaluation of the students' performance in cases when the process cannot be fully automated contains and will always contain subjective elements that can lead to different scorings depending on the evaluator, on the time of the evaluation, and on other known or unknown factors.

Recently several computational intelligence based methods have been published in order to deal with this subjectivism or to reduce its negative effects. Three of them are presented and examined shortly in the first part of the paper. The second part of the paper introduces a new approach that also possesses a software support.

The method FUSBE is simple, easy-to-understand, and fulfils the conditions demanded on this kind of evaluation approaches. Conform our experience it is accepted by both concerned parties the students and the teachers.

Further research plans cover the development and implementation of a student evaluation method based on fuzzy inference (Kovács, 2006)(Hládek et al., 2008) including the automatic fuzzy model identification (Botzheim et al., 2001)(Gál & Kóczy, 2008) (Precup et al., 2008) as well.

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