

LMI APPROACH FOR AIR-MANAGEMENT IN DIESEL ENGINES USING PDC FUZZY CONTROLLERS

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Abstract: Air management control in a turbocharged diesel engine presents itself as a challenge due to its nonlinear behavior, then classic control techniques are unable to provide the required performance. Hence, it is proposed to design fuzzy controllers based on PDC structure (Parallel Distributed Compensation) using a previously obtained Takagi-Sugeno fuzzy model for the engine. Controller parameters are obtained from a minimization problem subject to LMIs (Linear Matrix Inequalities).

1 INTRODUCTION

A turbocharged diesel engine is a very complex system which must fulfil user requirements (high power, low fuel consumption, flexible driving, etc.), as well as meeting increasingly strict emission standards. These new requirements, together with new environmental constraints (Guzzella and Amstutz, 1998), are forcing continuous improvements in the performance of air management. In order to tackle this tradeoff, a better development of the entire system is needed, especially the air management process. Current turbocharged diesel engines are very complex and nonlinear processes where a large set of variables are involved in the behavior of the whole system. Figure 1 shows a schematic view of a turbodiesel engine, where the most important physical magnitudes involved in the dynamic response of the system are:

- \dot{m}_a : Air mass flow $\left(\frac{Kg}{h}\right)$
- \dot{m}_T : Total collector mass flow $\left(\frac{Kg}{h}\right)$
- VGT : Variable geometry turbine (%)
- EGR : Exhaust gas recirculation. Valve (%)
- \dot{m}_{esc} : Exhaust mass flow $\left(\frac{Kg}{h}\right)$
- \dot{m}_{egr} : EGR mass flow $\left(\frac{Kg}{h}\right)$
- \dot{m}_f : Fuel mass flow $\left(\frac{Kg}{h}\right)$
- p_a : Intake manifold pressure (*bar*)

There are other two variables that affect the behavior of the system in a similar way to the manipulable variables *EGR* and *VGT*. One is the

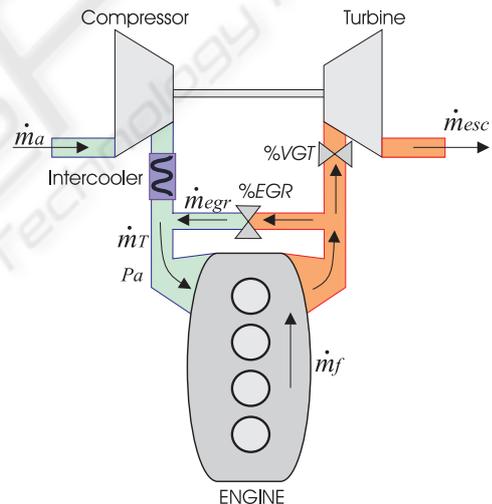


Figure 1: Turbocharged diesel engine.

speed of the engine (*N*) (Nieuwstadt et al., 2000; Kim and Park, 2007), which depends on several variables such as engine torque, load torque, vehicle inertial moment, etc.

The other variable, obviously, is the flow of fuel injected (\dot{m}_f) (Guzzella and Amstutz, 1998) and this determines the *AFR*.

This work proposes to use Takagi-Sugeno (T-S) fuzzy models as mathematical approximation of the turbocharged engines behavior (Takagi and Sugeno, 1985; Khiar et al., 2007; Lee et al., 2007). Next, a fuzzy controller PDC Parallel Distributed

Compensator (PDC) structure is proposed (Sugeno and Kang, 1986; Tanaka and Wang, 2001) based on a fuzzy TS model obtained previously. The control design can be recast as a minimization problem subject to a set of Linear Matrix Inequalities (LMIs) (Boyd et al., 1987). Therefore, the result of the design stage is a fuzzy controller that guarantees closed loop stability as a global approach.

The remainder of this article is organized as follows: *Section 2* provides a mathematical description of T-S fuzzy models and PDC controllers. In *Section 3*, the T-S fuzzy model used is defined. *Section 4* shows the PDC control strategy proposed. In *section 5*, the simulation results obtained by using the proposed control strategy are commented. Finally *section 6* offers the main conclusions.

2 MATHEMATICAL BASE

2.1 T-S Fuzzy Model

The structure of T-S fuzzy models is based on r number of rules composed of two terms: the premise and the consequent of the rule. The premise term describes the degree of fulfillment of each rule for each time step. The consequent term expresses the local dynamics of each fuzzy implication with a linear state space model.

$$\begin{aligned} & \text{RULE } i : \\ \text{IF } & z_1(k) \text{ Is } M_{i1} \ \& \ \dots \ \& \ z_p(k) \text{ Is } M_{ip} \ \text{Then} \\ & X(k+1) = \hat{A}_i X(k) + \hat{B}_i U(k), \end{aligned} \quad (1)$$

$$Y(k) = \hat{C}_i X(k), \quad i = 1, 2, \dots, r$$

Where M_{ij} defines the fuzzy membership functions of the variables $z_p(k)$ which conform the premise term of the fuzzy rules, r is the number of rules in the model and, matrices \hat{A}_i , \hat{B}_i and \hat{C}_i define the state space model for the consequents. Then, the output of the T-S fuzzy model is:

$$X(k+1) = \sum_{i=1}^r h_i(\mathbf{z}(k)) (\hat{A}_i X(k) + \hat{B}_i U(k)) \quad (2)$$

$$Y(k) = \sum_{i=1}^r h_i(\mathbf{z}(k)) (\hat{C}_i X(k)) \quad (3)$$

Where,

$$\mathbf{z}(k) = [z_1(k) \ \dots \ z_p(k)],$$

$$w_i(\mathbf{z}(k)) = \prod_{j=1}^p M_{ij}(z_j(k)), \quad (4)$$

$$h_i(\mathbf{z}(k)) = \frac{w_i(\mathbf{z}(k))}{\sum_{i=1}^r w_i(\mathbf{z}(k))}$$

2.2 Structure of the PDC Controller

During the last decade, a class of numerical optimization problems called linear matrix inequality (LMI) problems has received significant attention (Boyd et al., 1987). These optimization problems can be solved in polynomial time and hence are tractable. For systems and control, the importance of LMI optimization stems from the fact that a wide variety of system and control problems can be recast as LMI problems. One example is presented in (Tanaka and Wang, 2001), where the design problem of PDC controllers expressed in terms of LMIs is handled.

The structure of a PDC fuzzy controller is based on r rules composed of two terms: the premise and the consequent of the rule, and the number of rules and the premise structure is the same as the fuzzy model used for the controller design. The consequent of the PDC is composed of a state feedback law. Therefore, the fuzzy controller design determines these local feedback gains K_i . With the PDC, we have a simple and natural procedure for handling nonlinear control systems (Tanaka and Wang, 2001).

RULE i :

$$\text{IF } z_1(k) \text{ Is } M_{i1} \ \& \ \dots \ \& \ z_p(k) \text{ Is } M_{ip} \ \text{Then} \quad (5)$$

$$U(k) = -\hat{K}_i X(k), \quad i = 1, 2, \dots, r$$

Where M_{ij} defines the fuzzy membership functions of the variables $z_p(k)$ which conform the premise term of the PDC rule, r is the number of rules in the model and, \hat{K}_i are the matrices which feedback the state vector at each rule. Then, the global control action of the PDC controller can be defined as:

$$U(k) = -\sum_{i=1}^r h_i(\mathbf{z}(k)) (\hat{K}_i X(k)) \quad (6)$$

3 T-S FUZZY MODEL FOR THE AIR MANAGEMENT SYSTEM

The fuzzy model used in this article was first introduced in (García-Nieto and Martínez, 2007), where the identification methodology used is based on (Babuska and Verbruggen, 1996) and (Babuska, 1998). The main idea in (García-Nieto and Martínez, 2007) is to apply fuzzy clustering over the space of the variables (Gustafson and Kessel, 1979; Zhao et al., 1994; Martínez and Herrera, 2003; Yu and Li, 2008). The goal is to identify subspaces with similar characteristics where linear submodels will be characterized. Those submodels are part of a global nonlinear model which combines all the linear models using fuzzy rules. The identification method produces

a matrix membership that expresses the degree of fulfillment of each fuzzy rule. Later, the membership function for each antecedent variable is directly obtained from the projection of that membership matrix.

The model introduced in (García-Nieto and Martínez, 2007) has been modified normalising the process variables within -1 and 1 , where the goal of this modification is a better identification accuracy. The fuzzy model of the system only has 3 rules that define the engine behavior throughout its working range:

RULE i :

If $\dot{m}_a(k)$ **Is** D_i **&** $\dot{m}_a(k-1)$ **Is** E_i **&** $p_a(k)$ **Is** F_i **&** $p_a(k-1)$ **Is** J_i **&** $RPM(k)$ **Is** L_i **&** $\dot{m}_f(k)$ **Is** M_i **&** $EGR(k-1)$ **Is** N_i **&** $VGT(k-1)$ **Is** Z_i **Then**

$$\begin{aligned} X(k+1) &= \hat{A}_i X(k) + \hat{B}_i U(k) + \Psi_1 W(k), \\ Y(k) &= \hat{C}_i X(k) \end{aligned} \quad (7)$$

Where

$$X(k) = \begin{bmatrix} \dot{m}_a(k-1) \\ \dot{m}_a(k) \\ p_a(k-1) \\ p_a(k) \\ EGR(k-1) \\ VGT(k-1) \end{bmatrix},$$

$$U(k) = \begin{bmatrix} \Delta EGR(k) \\ \Delta VGT(k) \end{bmatrix}, \quad W(k) = \begin{bmatrix} RPM(k) \\ \dot{m}_f(k) \\ 1 \end{bmatrix},$$

$$Y(k) = \begin{bmatrix} \dot{m}_a(k) \\ p_a(k) \end{bmatrix}, \quad \mathbf{z}(\mathbf{k}) = X(k), \quad (8)$$

$$\begin{aligned} w_i &= D_i(\dot{m}_a(k)) \cdot E_i(\dot{m}_a(k-1)) \cdot F_i(p_a(k)) \cdot \\ &J_i(p_a(k-1)) \cdot L_i(RPM(k)) \cdot M_i(\dot{m}_f(k)) \cdot \\ &N_i(EGR(k)) \cdot Z_i(VGT(k)) \end{aligned} \quad (9)$$

In the appendix, the membership functions for the antecedents variables (see Figure 6) and the state space model matrices are described.

4 PDC FUZZY CONTROLLER DESIGN

Firstly, \hat{A}_i matrices in equation (7) are extended with two integrators. The purpose of this modification is to get rid of steady state error for \dot{m}_a and p_a in the tracking problem.

Secondly, state space feedback matrices are designed requesting three conditions: stability, minimization of the closed loop decay rate and fulfill constraints for control variables and process outputs. Stability and a specific decay rate are guaranteed by applying theorem 10 introduced in (Tanaka and Wang, 2001) and the decay rate definition stated in that reference, which certifies that Lyapunov function decreases exponentially:

$$\begin{aligned} \Delta V(k) &\leq (\alpha^2 - 1)V(k), \quad \alpha < 1, \\ &\Rightarrow V(k+1) \leq \alpha^2 V(k), \end{aligned} \quad (10)$$

Then, considering that the decay rate (α^2) is to be minimized and taking advantage of theorem 10 in (Tanaka and Wang, 2001), it is drawn that it is necessary to solve the following GEVP which comes from the product of β and F :

$$\begin{aligned} &\text{Minimize } \beta \\ &F, M_1, \dots, M_r, Y_0 \\ &\text{Subject to :} \end{aligned} \quad (11)$$

$$\begin{bmatrix} \beta F & F \hat{A}_i^T - M_i^T \hat{B}_i^T \\ \hat{A}_i F - \hat{B}_i M_i & F \end{bmatrix} > 0,$$

$$\begin{bmatrix} \beta F & Aux^T \\ Aux & F \end{bmatrix} \leq 0,$$

$$Aux = \frac{\hat{A}_i F + \hat{A}_j F - \hat{B}_i M_j - \hat{B}_j M_i}{2}, \quad (12)$$

$$\hat{V}_{ij} < 0, \quad i > j \text{ subject to } h_i \cap h_j \neq \emptyset$$

Where

$$\beta = \alpha^2, \quad M_i = \hat{K}_i F, \quad \hat{K}_i = M_i F^{-1}, \quad i = 1, 2, \dots, r \quad (13)$$

Variables h_i and h_j stand for the multivariate membership function of the rule (see equation (7)).

Finally, the LMIs which introduce the constraints for control actions and outputs are to be added in the minimization problem. The theoretical base to insert constraints for this variables is in theorems 11, 12 and 13, from (Tanaka and Wang, 2001). The join of those results derives theorem 4.1.

Theorem 4.1 Assume that $\|X(0)\| < \theta$, where $X(0)$ is unknown but the upper bound is known. The constraints $\|U(k)\|_2 \leq \mu$ and $\|Y(k)\|_2 \leq \gamma$ are enforced at all times $k \geq 0$ if the LMIs

$$\theta^2 I \geq F, \quad (14)$$

$$\begin{bmatrix} F & M_i^T \\ M_i & \mu^2 I \end{bmatrix} \geq 0, \quad (15)$$

$$\begin{bmatrix} F & F \hat{C}_i^T \\ \hat{C}_i F & \gamma^2 I \end{bmatrix} \geq 0, \quad M_i = \hat{K}_i F \quad (16)$$

Concluding, the proposed design is based on solving the conditions (11-12) and (14). However, conditions (11) and (12) are not LMIs but GEVP due to the product of β and F , hence an iterative method to get a solution is needed (Boyd et al., 1987). In this work, those conditions have been solved by using bisection algorithm over β . In particular, this methodology has been applied to the model of the air management process (7), obtaining β as 0.9984. For each value of β conditions (11) and (12) become LMIs, which altogether with LMIs (14) have been solved using the toolboxes Yalmip and LMItoolbox for Matlab (Löfberg, 2004). The corresponding feedback matrices are:

$$\hat{K}_1 = \begin{bmatrix} 0.0677 & -0.2053 \\ -0.2539 & 0.2601 \\ -0.0877 & 1.5966 \\ 0.5705 & -0.6142 \\ 1.0089 & -0.0147 \\ -0.0411 & 1.1141 \\ 0.0717 & -0.0049 \\ -0.0594 & -0.0867 \end{bmatrix}^T \quad (17)$$

$$\hat{K}_2 = \begin{bmatrix} 0.4283 & -0.2427 \\ -0.8365 & 0.2576 \\ 0.2110 & -1.0279 \\ -0.1050 & 2.3448 \\ 1.0076 & -0.0107 \\ -0.0034 & 1.0882 \\ 0.0764 & -0.0040 \\ -0.0277 & -0.0816 \end{bmatrix}^T \quad (18)$$

$$\hat{K}_3 = \begin{bmatrix} 1.2042 & -0.2927 \\ -1.8973 & 0.3474 \\ 0.2409 & -0.2448 \\ -0.7746 & 1.4879 \\ 1.0210 & -0.0070 \\ -0.0062 & 1.0418 \\ 0.0637 & -0.0059 \\ 0.0177 & -0.0779 \end{bmatrix}^T \quad (19)$$

The controller presents a structure equivalent to the model, equation (20) shows the controller designed.

RULE i :

If $\dot{m}_a(k)$ **Is** D_i & $\dot{m}_a(k-1)$ **Is** E_i & $p_a(k)$ **Is** F_i & $p_a(k-1)$ **Is** J_i & $RPM(k)$ **Is** L_i & $\dot{m}_f(k)$ **Is** M_i & $EGR(k-1)$ **Is** N_i & $VGT(k-1)$ **Is** Z_i **Then**

$$U(k) = -\hat{K}_i^* X^*(k) \quad (20)$$

5 RESULTS VALIDATION

Once the controller has been designed, its performance can be checked. However, this is not an easy

task, because it is not possible to modify the air management control strategy included in the electronic control unit (ECU) of the vehicle. These units are restricted by the manufacturer and cannot be modified,¹ and so the control strategy with the physical engine and its ECU cannot be tested. Moreover, the complete engine behavior is not modeled, and so any tracking reference for variables such as N or m_f cannot be determined. For this reason, the goal of the simulation is to decrease the mean value of the air pressure, and track the air mass flow obtained in real tests. If the air pressure is lower with similar levels of air mass flow, then the NOx emissions will decrease since the behavior of the engine in terms of torque should be similar.

To validate the controller, it is necessary to isolate the air management process and create realistic test conditions, which will provide coherent and comparable data to the experimental data from the real engine and its ECU. It is known that the ECU implements the control strategy by defining a reference for \dot{m}_a (related with \dot{m}_f through AFR control) and subject to p_a being within an appropriate working range (see Figure 2). The diagram adopted for the simulation is shown in Figure 3, where the air management process has been isolated from the global process, and \dot{m}_a from the real test is used as the reference to track. The reference for p_a is below that obtained in the experimental test. Under these conditions, it is possible to compare the designed subcontroller with the subcontroller implemented in the ECU.

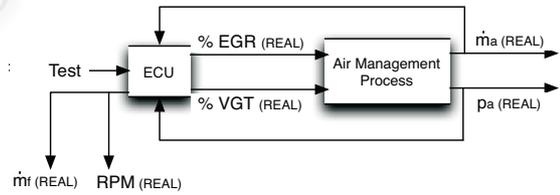


Figure 2: Air management control implemented in commercial ECUs.

Figure 4 shows how \dot{m}_a time evolution manages to track the desired reference. Therefore, the mechanical behavior will be similar to the one obtained when the ECU manages the process. Additionally, it can be seen how p_a is able to track a given pressure profile. In both cases, the time response of the controlled variables (\dot{m}_a p_a), given a change in the references, or in the event of a perturbation, is determined by the decay rate defined during the design procedure. If the decay

¹The ECU implements other controls and additional functions for air management, which are essential for vehicle performance.

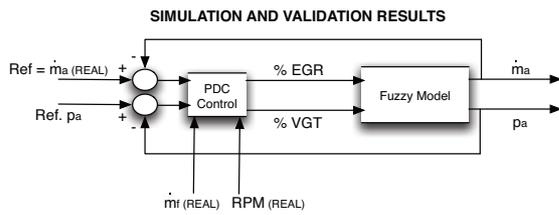


Figure 3: Air management control proposed.

rate had not been used in the design of the controller, the control system would have produced a considerably slower response.

Figure 4 shows that the air pressure when the ECU takes control is higher than the level obtained with the proposed fuzzy controller, although generating more NO_x . It can also be seen that for low demands of \dot{m}_a , the fuzzy controller keeps p_a near the atmospheric pressure, while the ECU provides greater values.

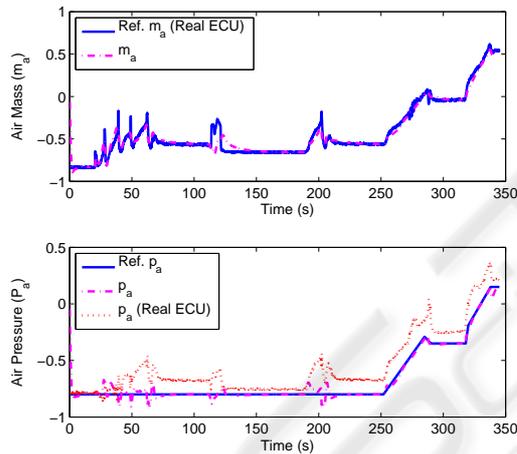
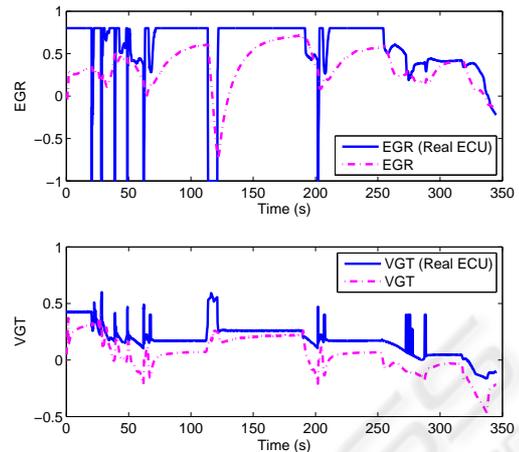

 Figure 4: Response of \dot{m}_a and p_a .

Figure 5 shows how the control actions EGR and VGT , proposed by the designed T-S fuzzy controller, present less transitions between the limits of the valves. This fact could be critical for the actuator life cycle, since a persistent switch between bounds would damage the mechanical parts of the valves.

6 CONCLUSIONS

This article presents the process to design an air management control system for a turbocharged diesel engine. It has been exposed the design of a stabilizer fuzzy controller that provides the fastest response possible considering the constraints for the control actions and outputs. The controller is the result com-


 Figure 5: Control actions EGR and VGT .

ing out from a GEVP problem where additional terms have been added in order to ensure control actions within bounds.

Secondly, the design proposed in this article controls p_a in such a way that the emission of NO_x is reduced.

Finally, the implementation of this controller on open ECUs, where the user can define the controller structure and its parameters, is left as future work. The goal of such an implementation is to test the T-S fuzzy controller with a real vehicle to confirm the simulation results obtained. Moreover, the robustness of the controller will be studied in future works, since possible implementations in real engines must be reliable and durable.

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APPENDIX

- Consequent for Rule 1:

$$\hat{A}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -0.1647 & 0.8911 & -0.0049 & -0.0371 & -0.0279 & 0.1219 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -0.0522 & 0.1016 & 0.7063 & 0.0855 & -0.0027 & 0.0305 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (21)$$

$$\hat{B}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \hat{C}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (22)$$

- Consequent for Rule 2:

$$\hat{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -0.5445 & 1.4793 & -0.2511 & 0.2831 & -0.0082 & -0.0088 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -0.0201 & 0.0272 & -0.6058 & 1.5700 & -0.0024 & 0.0415 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (23)$$

$$\hat{B}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \hat{C}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (24)$$

- Consequent for Rule 3:

$$\hat{A}_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -0.6235 & 1.5663 & -0.0448 & 0.0691 & -0.0100 & -0.0128 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -0.0760 & 0.1241 & -0.1426 & 1.0824 & -0.0009 & 0.0160 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\hat{B}_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \hat{C}_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (25)$$

- Membership functions of the variables $z_p(k)$:

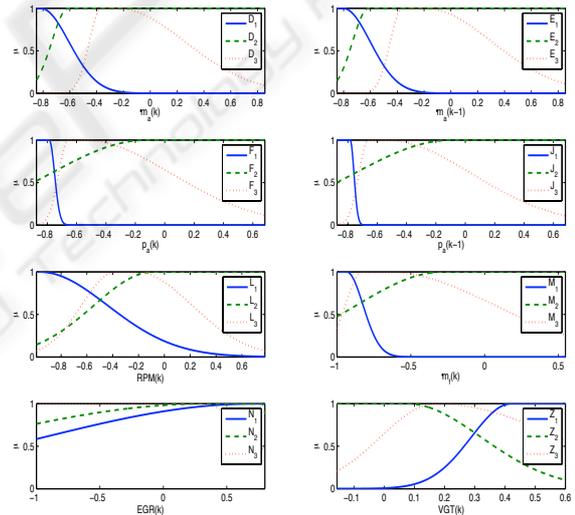


Figure 6: Membership functions of the TS Model.