DESIGN METHOD FOR WEDGE-SHAPED FILTERS

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Abstract: We present an analytical design method for a particular class of two-dimensional filters, namely wedge filters. The method relies on a frequency mapping which is applied to a 1D IIR low-pass prototype filter of a desired shape. We used as prototypes a flat-top filter and a Gaussian filter. Such filters have applications in texture analysis based on spatial filtering using various filter banks. In this paper we approached the wedge filter design method, without actually presenting an application in texture classification or other image processing tasks, which are extensively treated in other works.

1 INTRODUCTION

The domain of two-dimensional filters has known a constant development, stimulated by the ever-increasing requirements in different image processing applications. Their design methods, both for analog and digital implementation, are well founded (Dudgeon, 1984). A current design technique for 2D filters is to start with a prototype 1D filter and to transform its impulse response in order to obtain a filter with the desired frequency response. Generally the existing design methods of 2D IIR filters rely to a large extent on 1D analog filter prototypes, using spectral transformations from s to z plane via bilinear or Euler transformations followed by z to (z_1, z_2) transformations (Pendergrass, 1976), (Hirano, 1978), (Harn, 1986).

There are several types of filters with orientation-selective frequency response. They are useful in some image processing tasks like edge detection, motion analysis etc. An important class are steerable filters, synthesized as a linear combination of a set of basis filters (Freeman, 1991). Another important category are Gabor filters, efficiently implemented both in digital and analog versions (Shi, 1998). In (Bamberger, 1991) other types of oriented filters were approached.

A particular class of 2D filters are the so-called wedge filters, due to their symmetric wedge-like shape about the origin in the frequency plane. These filters find interesting application in feature extraction, for instance in texture classification (Randen, 1999). In (Simoncelli and Farid, 1995), (Simoncelli and Farid, 1996), the steerable wedge filters were introduced, which may be used to analyze local orientation patterns in images. In (Coggins, 1985), a bank of four wedge oriented filters was used.

In this work we approach the design of a class of wedge filters in the two-dimensional frequency domain. We will consider a general case of a wedge-shaped filter with a given aperture angle and an imposed orientation angle of its longitudinal axis. For design we will use two different 1D prototype filters, namely maximally-flat and Gaussian. We will consider in both cases only zero-phase filters, generally preferred in image filtering due to the absence of phase distortions. Two ideal wedge filters in the frequency plane are shown in Figure 1. The filter in Figure 1(a) has its frequency response along the axis ω_2. The angle $\sphericalangle AOB = \theta$ will be referred to as *aperture angle*. In Figure 1(b) a more general wedge filter is shown, with aperture angle $\sphericalangle BOD = \theta$, oriented along an axis CC', forming an angle $\sphericalangle AOC = \varphi$ with frequency axis $O - \omega_2$.

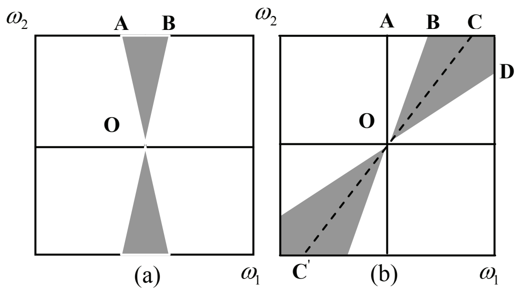

Figure 1: Ideal wedge filters specified in the frequency plane: (a) along the axis ω_2; (b) oriented at an angle φ.

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2 WEDGE FILTER DESIGN USING FREQUENCY TRANSFORMATIONS

Next we present a design method which leads to 2D zero-phase oriented filters from 1D prototypes. Let us consider an 1D recursive low-pass filter prototype frequency response of second order:

$$H_p(\omega) = \frac{b_0 + b_1\omega^2 + b_2\omega^4}{1 + a_1\omega^2 + a_2\omega^4} \tag{1}$$

where usually $b_0 = H_p(0) = 1$.

A wedge filter along the frequency axis ω_2 can be obtained using the 1D–2D frequency transformation (for $\omega_2 \neq 0$) :

$$\omega \rightarrow f(\omega_1, \omega_2) = a \cdot \omega_1 / \omega_2 \tag{2}$$

By a we denoted the coefficient $a = 1/\mathrm{tg}(\theta/2)$, where θ is the aperture angle of the wedge filter, as defined in Figure 1. Replacing in (1) ω by the ratio $a\,\omega_1/\omega_2$, we get the frequency response in ω_1, ω_2 :

$$H(\omega_1, \omega_2) = \frac{b_0\omega_2^4 + b_1a^2\omega_1^2\omega_2^2 + b_2a^4\omega_1^4}{\omega_2^4 + a_1a^2\omega_1^2\omega_2^2 + a_2a^4\omega_1^4} \tag{3}$$

While the frequency mapping (2) is undetermined for $\omega_2 \rightarrow 0$, $\omega_1 \rightarrow 0$, once made the substitution (2) into the prototype function (1), the expression (3) has no undetermination any longer.

At this point we map $H(\omega_1, \omega_2)$ into the complex plane (s_1, s_2) where $s_1 = j\omega_1$, $s_2 = j\omega_2$. Since $\omega_1^2 = -s_1^2$, $\omega_2^2 = -s_2^2$ we get the function $H_S(s_1, s_2)$:

$$H_S(s_1, s_2) = \frac{b_0s_2^4 + b_1a^2s_1^2s_2^2 + b_2a^4s_1^4}{s_2^4 + a_1a^2s_1^2s_2^2 + a_2a^4s_1^4} \tag{4}$$

A little more difficult task is now to find a mapping of $H_S(s_1, s_2)$ into the complex plane (z_1 , z_2). This can be achieved either using the forward or backward Euler approximations, or the bilinear transform, which gives better accuracy.

The bilinear transform is a first-order approximation of the natural logarithm function, which is an exact mapping of the *z*-plane to the *s*-plane. For our purposes the sample interval takes the value $T = 1$ so the bilinear transform for s_1 and s_2 in the complex plane (s_1, s_2) has the form:

$$s_1 = 2\left(\frac{z_1 - 1}{z_1 + 1}\right) \qquad s_2 = 2\left(\frac{z_2 - 1}{z_2 + 1}\right) \tag{5}$$

Substituting s_1 , s_2 in (4), we find after some algebra a function in z_1 and z_2 written in matrix form as:

$$F(z_1, z_2) = \frac{\mathbf{Z}_1 \times [\mathbf{B}] \times \mathbf{Z}_2^T}{\mathbf{Z}_1 \times [\mathbf{A}] \times \mathbf{Z}_2^T} \tag{6}$$

where $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are the vectors:

$$\begin{aligned} \mathbf{Z}_1 &= \begin{bmatrix} z_1^{-2} & z_1^{-1} & 1 & z_1 & z_1^2 \end{bmatrix} \\ \mathbf{Z}_2 &= \begin{bmatrix} z_2^{-2} & z_2^{-1} & 1 & z_2 & z_2^2 \end{bmatrix} \end{aligned} \tag{7}$$

and $\times$ denotes matrix/vector product. Throughout the paper we will use the convenient notion of *template*, borrowed from the field of cellular neural networks (CNNs) (Chua, 1988) to denominate the coefficient matrices corresponding to the numerator and denominator of a 2D filter transfer function $H(z_1, z_2)$. Thus, the templates **B** and **A** can be written as a sum of three separable matrices:

$$\mathbf{B} = b_0 \cdot \mathbf{M}_1^T * \mathbf{M}_2 + b_1a^2 \cdot \mathbf{M}_3^T * \mathbf{M}_3 + b_2a^4 \cdot \mathbf{M}_2^T * \mathbf{M}_1 \tag{8}$$

$$\mathbf{A} = \mathbf{M}_1^T * \mathbf{M}_2 + a_1a^2 \cdot \mathbf{M}_3^T * \mathbf{M}_3 + a_2a^4 \cdot \mathbf{M}_2^T * \mathbf{M}_1 \tag{9}$$

where the vectors are:

$$\begin{aligned} &\mathbf{M}_1 = \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}; \quad \mathbf{M}_2 = \begin{bmatrix} 1 & -4 & 6 & -4 & 1 \end{bmatrix}; \\ &\mathbf{M}_3 = \begin{bmatrix} 1 & 0 & -2 & 0 & 1 \end{bmatrix} \end{aligned} \tag{10}$$

and the operator $*$ denotes outer product of vectors.

In a more general case when the wedge filter axis has an orientation specified by an angle φ (with respect to the axis ω_2), the oriented wedge filter may be obtained by rotating the axes of the plane (ω_1, ω_2) by an angle φ. The rotation is defined by the linear transformation:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} \bar{\omega}_1 \\ \bar{\omega}_2 \end{bmatrix} \tag{11}$$

where ω_1, ω_2 are the original frequency variables and $\bar{\omega}_1, \bar{\omega}_2$ the rotated ones. In this case the 1D to 2D frequency transformation can be written as:

$$\omega \rightarrow f_\varphi(\omega_1, \omega_2) = \frac{a(\omega_1 - \omega_2 \cdot \mathrm{tg}\varphi)}{(\omega_1 \cdot \mathrm{tg}\varphi + \omega_2)} \tag{12}$$

Using the expression above and the bilinear transform, we finally get a mapping of the form:

$$\omega^2 \rightarrow F(z_1, z_2) = a^2 \cdot \frac{\mathbf{z}_1 \times \mathbf{M}_\varphi \times \mathbf{z}_2^T}{\mathbf{z}_1 \times \mathbf{M}_\varphi^{90^0} \times \mathbf{z}_2^T} \tag{13}$$

where $\mathbf{M}_\varphi$ is a 3×3 matrix of the form:

$$\mathbf{M}_\varphi = \begin{bmatrix} (\mathrm{tg}\varphi\text{-}1)^2 & 2(\mathrm{tg}^2\varphi - 1) & (\mathrm{tg}\varphi + 1)^2 \\ -2(\mathrm{tg}^2\varphi - 1) & -4(\mathrm{tg}^2\varphi + 1) & -2(\mathrm{tg}^2\varphi - 1) \\ (\mathrm{tg}\varphi + 1)^2 & 2(\mathrm{tg}^2\varphi - 1) & (\mathrm{tg}\varphi\text{-}1)^2 \end{bmatrix} \tag{14}$$

and $\mathbf{M}_\varphi^{90^0}$ is the matrix $\mathbf{M}_\varphi$ rotated by 90^0.

We apply this frequency transformation directly to the 1D prototype function (3), for ω_1^2 , ω_2^2 and we get the 2D wedge filter transfer function in z_1, z_2:

$$H_\varphi(z_1,z_2)=\frac{\mathbf{Z}_1\times\mathbf{B}_\varphi\times\mathbf{Z}_2^T}{\mathbf{Z}_1\times\mathbf{A}_\varphi\times\mathbf{Z}_2^T} \tag{15}$$

where the 5×5 matrices $\mathbf{A}_\varphi$, $\mathbf{B}_\varphi$ have the form:

$$\mathbf{B}_\varphi=b_0(\mathbf{M}_\varphi*\mathbf{M}_\varphi)^{90^0}+b_1a^2(\mathbf{M}_\varphi*\mathbf{M}_\varphi^{90^0})+b_2a^4(\mathbf{M}_\varphi*\mathbf{M}_\varphi) \tag{16}$$

$$\mathbf{A}_\varphi=(\mathbf{M}_\varphi*\mathbf{M}_\varphi)^{90^0}+a_1a^2(\mathbf{M}_\varphi*\mathbf{M}_\varphi^{90^0})+a_2a^4(\mathbf{M}_\varphi*\mathbf{M}_\varphi) \tag{17}$$

and $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are the vectors given in (7).

Therefore the transfer function $H_\varphi(z_1,z_2)$ in (15) corresponds to a wedge filter with an aperture angle $\theta=2\cdot\mathrm{arctg}(1/a)$ and whose longitudinal axis is tilted about the ω_2 axis in the frequency plane with an angle φ.

Even if this method is straightforward and easy to apply once found the 1D prototype filter, the 2D filter designed in this way, corresponding to the transfer function in z_1, z_2 will present noticeable errors towards the limits of the frequency plane and linearity distortions as compared to the ideal frequency response in (3). This is mainly due to the frequency warping effect introduced by the bilinear transform, which is expressed by the continuous-time to discrete-time frequency mapping:

$$\omega=\frac{2}{T}\mathrm{arctg}\left(\omega_a\frac{T}{2}\right) \tag{18}$$

where ω is the frequency of the discrete-time filter and ω_a the frequency of the continuous-time filter. In order to correct this error we will next apply a pre-warping, using the inverse of the mapping (18). Since for our purposes we can take $T=1$, in the frequency transformation (12) we will substitute the following mappings:

$$\omega_1\to2\mathrm{tg}\left(\frac{\omega_1}{2}\right)\qquad\omega_2\to2\mathrm{tg}\left(\frac{\omega_2}{2}\right) \tag{19}$$

In dealing with the nonlinear mappings (19), a polynomial or rational approximation would be more suitable. One of the most efficient rational approximations (best tradeoff between accuracy and approximation order) is the Chebyshev-Padé approximation. Using it we obtain:

$$\mathrm{tg}\left(\frac{\omega}{2}\right)\cong\frac{\omega\cdot\left(0.5-0.008439\cdot\omega^2\right)}{\left(1-0.1\cdot\omega^2\right)}=g(\omega) \tag{20}$$

very accurate on a frequency range close to $[-\pi,\pi]$. Using (12) we obtain the frequency transformation which includes frequency pre-warping for ω_1, ω_2:

$$\omega\to f_{\varphi P}(\omega_1,\omega_2)=\frac{a\left(\mathrm{tg}(\omega_1/2)-\mathrm{tg}(\omega_2/2)\cdot\mathrm{tg}\varphi\right)}{\left(\mathrm{tg}(\omega_1/2)\cdot\mathrm{tg}\varphi+\mathrm{tg}(\omega_2/2)\right)} \tag{21}$$

Substituting in (21) $\mathrm{tg}(\omega/2)$ by the rational approximation $g(\omega)$ we get a rational expression in ω_1 and ω_2 for the frequency transformation $\omega\to f_{\varphi P}(\omega_1,\omega_2)$. Then as previously we map $f_{\varphi P}(\omega_1,\omega_2)$ into the complex plane (s_1,s_2) and finally we get using bilinear transform the frequency mapping written again in matrix form:

$F:\mathbb{R}\to\mathbb{C}^2$, $\omega\to F(z_1,z_2)$

$$F(z_1,z_2)=\frac{\mathbf{Z}_1\times\left[\mathbf{B}_{P\varphi}\right]\times\mathbf{Z}_2^T}{\mathbf{Z}_1\times\left[\mathbf{A}_{P\varphi}\right]\times\mathbf{Z}_2^T} \tag{22}$$

The templates corresponding to the numerator and denominator, of size 4×4, are expressed as:

$$\mathbf{B}_{p\varphi}=\mathbf{M}_1-\mathrm{tg}\varphi\cdot\mathbf{M}_1^{90^0}\qquad\mathbf{A}_{p\varphi}=\mathrm{tg}\varphi\cdot\mathbf{M}_1+\mathbf{M}_1^{90^0} \tag{23}$$

where $\mathbf{M}_1^{90^0}$ is the matrix $\mathbf{M}_1$ rotated clock-wise by 90^0 which is numerically given by:

$$\mathbf{M}_1=\begin{bmatrix}-1&1\\-1&1\end{bmatrix}*\begin{bmatrix}0.559283&1.081434&0.559283\\0.915190&1.769619&0.915190\\0.559283&1.081434&0.559283\end{bmatrix} \tag{24}$$

The elements of $\mathbf{M}_1$ result from combinations of the coefficients occurring in the expression of $g(\omega)$ in (20). Finally we obtain the 1D to 2D frequency transformation written in the matrix form:

$$\omega^2\to F(z_1,z_2)=a^2\cdot\frac{\mathbf{z}_1\times\mathbf{B}\times\mathbf{z}_2^T}{\mathbf{z}_1\times\mathbf{A}\times\mathbf{z}_2^T} \tag{25}$$

where matrices $\mathbf{B}=\mathbf{B}_{p\varphi}*\mathbf{B}_{p\varphi}$ and $\mathbf{A}=\mathbf{A}_{p\varphi}*\mathbf{A}_{p\varphi}$ resulted by convolution are of size 7×7.

We can apply this frequency transformation directly to the 1D prototype function (1) and we obtain the 2D wedge filter transfer function in z_1 and z_2:

$$H_{W\varphi}(z_1,z_2)=\frac{\mathbf{Z}_1\times\mathbf{B}_{\mathbf{W}\varphi}\times\mathbf{Z}_2^T}{\mathbf{Z}_1\times\mathbf{A}_{\mathbf{W}\varphi}\times\mathbf{Z}_2^T} \tag{26}$$

where the vectors $\mathbf{Z}_1$ and $\mathbf{Z}_2$ have the form:

$$\mathbf{Z}_1=[z^N\quad z^{N-1}\quad\ldots\quad z\quad1]$$
$$\mathbf{Z}_2=[z^N\quad z^{N-1}\quad\ldots\quad z\quad1] \tag{27}$$

with $N=12$; the 13×13 matrices $\mathbf{A}_{\mathbf{W}\varphi}$, $\mathbf{B}_{\mathbf{W}\varphi}$ are:

$$\mathbf{B}_{\mathbf{W}\varphi}=b_0(\mathbf{A}*\mathbf{A})+b_1a^2(\mathbf{A}*\mathbf{B})+b_2a^4(\mathbf{B}*\mathbf{B}) \tag{28}$$

$$\mathbf{A}_{\mathbf{W}\varphi}=\mathbf{A}*\mathbf{A}+a_1a^2(\mathbf{A}*\mathbf{B})+a_2a^4(\mathbf{B}*\mathbf{B}) \tag{29}$$

As an important remark, even if the filter templates result relatively large, this is the price paid for ensuring a good linearity of the wedge filter shape in the frequency plane. The frequency pre-warping has therefore increased the filter order. However, the filter large-size templates result as a discrete convolution of small size matrices (3×3, 5×5) and consequently can be considered partially separable. At least the numerator of the general prototype (1) may have real roots, therefore it can be factorized, which implies convolution of smaller size matrices.

3 FILTER PROTOTYPES

3.1 Maximally-flat Filter Prototype

Let us consider a maximally-flat 1D IIR prototype filter with the frequency response:

$$H_p(\omega)=\frac{0.887175-0.269975\cdot\omega^2+0.018905\cdot\omega^4}{1-0.600346\cdot\omega^2+5.332057\cdot\omega^4} \quad (30)$$

which is plotted in Fig.2(a) in the range $\omega\in[-\pi,\pi]$. Using the method described before, let us design a wedge filter with an aperture angle $\theta=0.2\pi$ and oriented at an angle $\varphi=\pi/5$. For these values we get $a=\mathrm{tg}(\theta/2)$=0.3249 and $\mathrm{tg}\varphi=0.7265$. The frequency response and contour plot for these parameters are shown in Fig.4.

3.2 Gaussian Filter Prototype

Another type of wedge filters may use Gaussian-shaped filters as 1D prototypes. Next we will find an efficient rational approximation for a Gaussian frequency response:

$$G(\omega)=\exp\left(-\sigma^2\omega^2/2\right) \quad (31)$$

The parameter σ gives the Gaussian selectivity. We look for a rational approximation of $G(\omega)$ as a ratio of polynomials in $\cos\omega$:

$$G(\omega)=e^{-\frac{\omega^2\sigma^2}{2}}\cong\frac{B(\omega)}{A(\omega)}=\sum_{m=0}^{M}b_m\cos(m\omega)\Big/\sum_{n=0}^{N}a_n\cos(n\omega) \quad (32)$$

where $\omega\in(-\pi,\pi)$, $a_0=1$. The degrees of the numerator and denominator (M, N) may be not necessarily equal.

One of the most efficient rational approximation (best tradeoff between accuracy and approximation order) is the Chebyshev-Padé rational approximation. The coefficients are usually determined numerically using a symbolic calculation software. In the expression of $G(\omega)$ in (31) we make the change of frequency variable:

$$x=\cos(\omega)\Leftrightarrow\omega=\arccos(x) \quad (33)$$

Then we find a Chebyshev-Padé approximation of

$$G_1(\omega)=\exp\left(-\sigma^2\arccos^2(\omega)/2\right) \quad (34)$$

as a rational function of the intermediate variable x. We return to the original variable ω, then finally obtain a rational function in $\cos\omega$. For $\sigma=2$, a second-order approximation is accurate enough:

$$G_2(\omega)=e^{-2\omega}\cong\frac{0.018+0.02749\cdot\cos\omega+0.01092\cdot\cos2\omega}{1-1.231918\cdot\cos\omega+0.288144\cdot\cos2\omega} \quad (35)$$

Using usual trigonometric identities, $G_2(\omega)$ is finally put into the factorized form:

$$G_2(\omega)=\frac{0.03067\cdot(\cos\omega+0.89692)\cdot(\cos\omega+0.36239)}{(1-1.73057\cdot\cos\omega+0.80955\cdot(\cos\omega)^2)} \quad (36)$$

This frequency response is plotted in Fig.2(b).

Using a symbolic computation software (MAPLE etc.) we can derive an accurate rational approximation (Chebyshev- Padé) of the cosine function in the range $[-\pi\sqrt{2},\pi\sqrt{2}]$:

$$\cos\omega\cong\frac{1-0.447754\omega^2+0.018248\omega^4}{1+0.041694\omega^2+0.002416\omega^4} \quad (37)$$

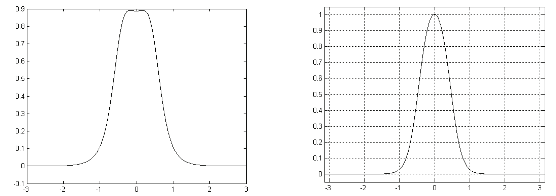
Substituting in (36) the expression of $\cos\omega$, we get:

$$G_2(\omega)=0.38833\cdot\frac{\begin{pmatrix}(1.89692-0.41036\cdot\omega^2+0.02041\cdot\omega^4)\\(1.3624-0.43264\cdot\omega^2+0.01912\cdot\omega^4)\end{pmatrix}}{\left(1+0.837\cdot\omega^2+2.5201\cdot\omega^4\right)^2} \quad (38)$$

We can separate $G_2(\omega)$ into two factor functions of the form (1) and we can apply the same design procedure as before in order to obtain either a wedge filter oriented along one of the axis ω_1, ω_2 or along an axis tilted with a given angle φ about one of the axis. Using the frequency mapping $\omega^2\rightarrow F(z_1,z_2)$ given by (25), we finally obtain a transfer function in z_1, z_2. This filter has a Gaussian cross-section with every vertical plane perpendicular to its longitudinal axis. Since the numerator and denominator are factorized, the filter templates result as a convolution of smaller size matrices. The Gaussian wedge filter with the same parameters $\theta=0.2\pi$ and $\varphi=0.2\pi$ is shown in Fig.5. In Fig.3 a flat-top and a Gaussian wedge filter with $\theta=0.15\pi$ and $\varphi=0$ are displayed.

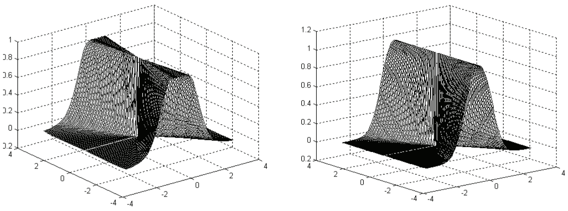
4 CONCLUSIONS

We proposed a design method for 2D IIR zero-phase wedge filters, oriented along a specified direction. They are based on a 1D low-pass prototype with an imposed frequency response, for instance flat-top and Gaussian. A 1D to 2D frequency mapping function is derived which is applied to the 1D prototype to obtain the 2D filter.
The distortions introduced by bilinear transform are compensated through a pre-warping along both axes



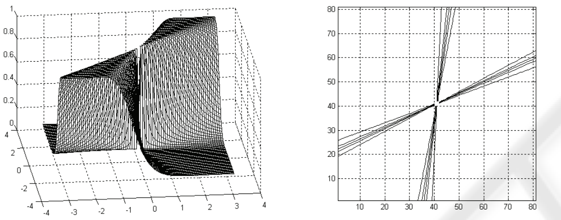
$-\pi$ (a) π $-\pi$ (b) π

Figure 2: 1D IIR prototype filters: (a) maximally flat; (b) Gaussian-shaped.



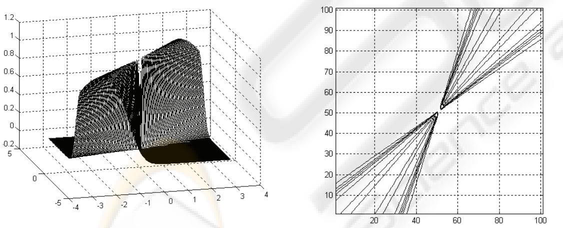
(a) (b)

Figure 3: (a) Flat-top wedge filter and (b) Gaussian wedge filter with $\theta = 0.15\pi$ and $\varphi = 0$.



(a) (b)

Figure 4: Oriented flat-top wedge filter with $\theta = 0.2\pi$ and $\varphi = 0.2\pi$: (a) frequency response; (b) contour plot.



(a) (b)

Figure 5: Oriented Gaussian wedge filter with $\theta = 0.2\pi$ and $\varphi = 0.2\pi$: (a) frequency response; (b) contour plot.

ω_1, ω_2. The efficient Chebyshev-Padé rational approximation is also used. The proposed design method is direct and does not involve any numerical optimization techniques. Further research on the topic may combine the analytical and numerical methods to design more efficient filters and also to obtain an efficient implementation of this class of filters.

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