

# STRAIN FIELD INTERPOLATION OVER THE SCIARA DEL FUOCO (STROMBOLI VOLCANO) FROM GEODETIC MEASUREMENTS ACQUIRED BY THE AUTOMATIC THEODOROS SYSTEM

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**Abstract:** In this paper we treat two important aspects concerning the automatic monitoring of the ground deformation at the Sciara del Fuoco (SdF), the Stromboli volcano (Italy) steep flank subjects to dangerous landslide events: the developments of suitable software procedures to process observations and the evaluation of both 3D motion maps and 3D strain tensor over the whole investigated area. Ground deformation measured by the monitoring system known as THEODOROS (THEOdolite and Distancemeter Robot Observatory of Stromboli) is often affected by offsets and spikes, due to both malfunctioning and periodical maintenances of the system, and other noise sources making very difficult the interpretation of the ground deformation dynamics. To this purpose a suitable software tool able to reduce these drawbacks was developed. Furthermore, both 3D motion maps and 3D strain tensor are computed in order to provide new useful information aimed to better understanding the complex dynamic of the SdF.

## 1 INTRODUCTION

Stromboli is an active volcano, about 2500 m high above the sea floor. Roughly only the last kilometre of this volcano emerges from the sea, forming an island whose diameter ranges from 2.4 to 5 km. It belongs to the Aeolian Islands and represents the most active volcano of this archipelago. Its conic shape is evidently characterized by a big depression that marks the northwestern flank of the edifice: the Sciara del Fuoco (SdF). On December 28th, 2002, lava flows outpoured from the northern wall of the NE crater and descended into the Sciara del Fuoco, a deep depression marking the NW flank of the volcano edifice. On December 30th, 2002, two landslides occurred on the northern part of the Sciara del Fuoco; they moved a mass in the order of tens of millions of cubic meters both above and below sea level. The landslide produced a tsunami causing significant damage to the eastern coast of the island, reaching the other Aeolian Islands and the Sicilian and southern Italian

coasts. This event led to the upgrading of the ground deformation monitoring system, already existing on the island; the new requirement was the real-time detection of the deformations related to potential slope failures of the SdF. To this purpose the chosen instrument was the Leica TCA 2003 Total Station (TS) equipped with GeoMos software ((Leica, 2002) that allows remote sensor control. The acronym of this system is THEODOROS (THEOdolite and Distancemeter Robot Observatory of Stromboli) (Puglisi et al., 2004). The rest of this paper is organized in the following way. In Sec. II a brief description of the current THEODOROS configuration is given, the interested reader can find more detailed information and a general map of the island with the position of the reflectors in (Puglisi et al., 2004) and (Nunnari et al., 2008). Sec. III reports the approach adopted to pre-processing data; Sec. IV shows the methodology used to compute the strain field; Sec. V reports the case study; finally Sec. VI draws the conclusions of this study.

## 2 BRIEF INTRODUCTION TO THE THEODOROS SYSTEM

The THEODOROS system consists of a remote-controlled Total Station that can be programmed to measure slope distances and angles between the sensor and benchmarks appropriately installed in the SdF area at a specific sampling rate. To ensure a continuous stream of data from the instrument, it requires a constant power supply and a continuous link with the PC controlling the Total Station's activities, installed on the S. Vincenzo Observatory, where the National Department of Civil Protection (DPC) control room is located. The Stromboli volcano eruption of the 27 February 2007 destroyed the THEODOROS benchmarks inside the SdF. A new configuration was designed and new benchmarks were installed on the new fan produced by the lava flow entering the sea. This new topology consists of six reflectors installed outside the SdF, around the Total Station, for the reference system and atmospheric corrections (SENT, BORD, SEMF, SPLB2, CIST and ELIS), nine reflectors for monitoring movements of the lava fan inside the SdF (SDF18, SDF19, SDF20, SDF21, SDF22, SDF23, SDF24, SDF25 and SDF26), two reflectors to monitor the northern border of the SdF (400 and BASTI) and two further reflectors on stable sites to check the stability of the measurements both on short and very long distance measurements (CURV and CRV). Currently the reflectors SDF20 and SDF21 are not working. The sample time indicated as  $t_c$  hereafter is set to be  $t_c = 10$  minutes. Each measurement for each target or reference point provides the instantaneous values of three relevant pieces of information: the slope distance (sd), the horizontal (hz) angle and the vertical angle (ve). Starting from this information, the GeoMos system is able to transform the TS measurement vectors (whose components are sd, hz, ve) into an equivalent vector whose components are expressed in terms of North (N), South (S) and Up (U) with respect to the assumed reference system. In this computation, GeoMos is able to take into account the constraints imposed by the assumption of the reference system. Despite the availability of real-time information, this is not enough to automatically evaluate the state of ground deformation. Indeed the acquired measures are affected by offsets, spikes and noise sources that strongly compromised their interpretation. These drawbacks must be necessary overcome before that suitable quantities related to the ground deformation dynamic can be efficiently computed. In particular in this paper we focus our attention on the problems of offsets and spikes removal, smoothing noisy data and strain tensor evaluation.

## 3 PRE-PROCESSING DATA

The algorithm we propose to remove both spikes and offsets consists of two steps. First the spikes are removed, then attention is focused on offsets. Since the single displacement components (North, East, Up) of each benchmark in the period June 2006 - December 2008 are characterized by a normal distribution, the problem to remove the spikes affecting observations, i.e. the sharp variations of the time series which are generally due to either periodical maintenance or instrument malfunctions, is well solved adopting the standard deviation of observations as reference. Indeed let  $T_{SDF_x}(t)$  be a generic component of the benchmark  $SDF_x$  at time  $t$ , let  $\Delta T_{SDF_x}(t)$  be the difference between two subsequent measures and denoting as  $\sigma$  its standard deviation, the experience gained through the daily monitoring of the SdF suggest us to consider as spikes the  $\Delta T_{SDF_x}(t)$  values falling outside the range covered by one  $\sigma$ .

The offsets affecting observations are essentially due to the maintenances of the THEODORO system. Here it is necessary to distinguish two types of maintenances: periodical maintenance usually carried out every six months, and extra maintenance due to unexpected crash of the system. The offsets related to the periodical maintenance are simply adjusted taking into account the marked sharp variation (jump) visible when the system begins to work. This approach is also suitable for offset due to the crash of the system if the normal functioning of the system is promptly restored. Instead, if the extra maintenance is performed after a sufficiently long time the system crashed, then the offsets removal is not trivial. Indeed, in this case, in order to perform a reliable offsets correction the estimation of the trend of each ground deformation component during the period in which the system was crashed is needed. In order to adjust these kinds of offsets we use the linear trend as shown in figure 2.

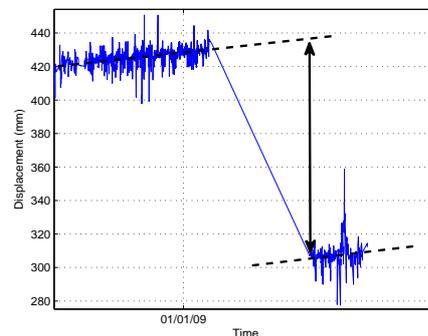


Figure 1: Offsets correction approach based on linear trend.

Although both spikes and offsets removal makes

ground deformations more readable, further processing is needed to reduce noise source affecting data, in particular the thermoelastic effects on ground deformation due to the temperature. To this purpose we have smoothed noisy data with spline functions following the suggestion of the literature (Bilotti et al., 2008), (Ge et al., 2003). A spline function  $s(t)$  is a function defined piecewise by polynomials. This function takes values from an interval  $[a, b]$  and maps them to  $R$ , the set of real numbers. The interval  $[a, b]$  is divided into  $k$  disjoint subintervals  $[t_i, t_{i+1}]$  with  $0 \leq i \leq k-1$  such that  $[a, b] = [t_0, t_1] \cup \dots \cup [t_{k-2}, t_{k-1}]$ . The given  $k$  points  $t_i$  are called knots. The vector  $t = (t_0, \dots, t_{k-1})$  is called a knot vector for the spline. If the knots are equidistantly distributed in the interval  $[a, b]$  the spline is uniform, otherwise it is non-uniform. On each of this subintervals a  $n$ th polynomial is defined and joined with others polynomials at the knot points in such a way that all derivatives up to the  $(n-1)$ th degree are continuous. Within these constraints, the function  $s(t)$  is selected which minimizes:

$$\sum (s(t_i) - x_i)^2 + p \int (s^{(\frac{n+1}{2})}(t))^2 dt \quad (1)$$

where  $(t_i, x_i)$  are the raw data samples and  $s(k)$  denotes the  $k$ th derivative of  $s(t)$ . The weight factor  $p$  is the smoothing parameter whose value must be opportunely chosen to obtain a good compromise between good fit and the smoothness. In figure 2 are shown, respectively, the raw data of the benchmark SdF26 (North component), the data after removing spikes and offsets and finally the spline-smoothing.

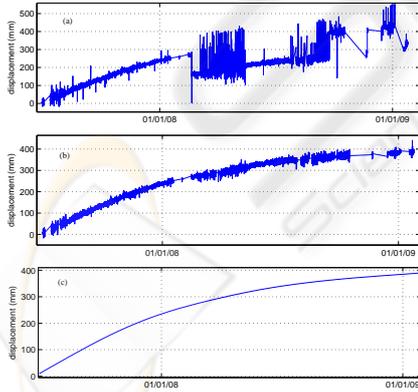


Figure 2: (a) Original SdF26 North component; (b) Spikes and offset removed; (c) Smoothing noise.

## 4 STRAIN INTERPOLATION

In order to compute both 3D displacements map and strain tensor in the area of SdF covered by the

THEODOROS system we use the modified least-square approach introduced by (Shen et al., 1996) and also used by (Pesci and Teza, 2007) and (Teza et al., 2008). Given a point  $P$  having position  $x_0 = (x_{10}, x_{20}, x_{30})$  surrounded by  $N$  experimental points (EPs) whose positions and displacements are respectively  $x(n) = (x_{1(n)}, x_{2(n)}, x_{3(n)})$  and  $u(n) = (u_{1(n)}, u_{2(n)}, u_{3(n)})$  where  $n = 1..N$ , the problem of estimating both the displacements gradient tensor  $H$  and the displacement components  $U_i (i = 1..3)$  of the point  $P$ , according to the infinitesimal strain theory, can be modelled in terms of the following strain interpolation equations:

$$u_{i(n)}(x) = H_{ij} \Delta x_{j(n)} + U_i \quad (i, j = 1..3) \quad (2)$$

where  $\Delta x_{j(n)} = x_{j(n)} - x_{j0}$  are the relative positions of the  $n$ th EP experimental points and the arbitrary point  $P$  and  $H_{ij} = \frac{\partial u_i}{\partial x_j}$  are the elements of the displacement gradient tensor. It can be decomposed in a symmetric and an anti-symmetric part as  $H = E + \Omega$ , where  $E$  is the strain tensor defined as:

$$E = \frac{1}{2} (H_{ij} + H_{ji}) e_i \otimes e_j \quad (3)$$

and  $\Omega$  is the rigid body rotation tensor defined as:

$$\Omega = \frac{1}{2} (H_{ij} - H_{ji}) e_i \otimes e_j \quad (4)$$

Here  $e_i$  is the base vector of the Cartesian reference system and  $\otimes$  is the tensor product. In a compact form the undetermined system of equations (2) can be written as  $Al = u$  where  $A$  is the design matrix simply derivable from equation (2),  $l = [U_1 \ U_2 \ U_3 \ \varepsilon_{11} \ \varepsilon_{12} \ \varepsilon_{13} \ \varepsilon_{22} \ \varepsilon_{23} \ \varepsilon_{33} \ \omega_1 \ \omega_2 \ \omega_3]^T$  is the vector of unknown parameters and  $u = [u(1) \ u(2) \ \dots \ u(n)]^T$  is the observation vector. Assuming a uniform strain field and re-writing the previous linear equation (4) as  $Al = u + e$ , where  $e$  is the residual vector which model the stochastic nature of the estimation problem, a suitable method to solve the system is the Weighted Least Squares (WLS) which gives the expression (5) as a suitable formula to estimate the unknown vector  $l$

$$\hat{l} = (A^T W A)^{-1} A^T W u \quad (5)$$

In (5)  $W$  is the data covariance matrix. Usually  $W$  is assumed to be diagonal (uncorrelated data), i.e. of the form

$$W = \text{diag}(\sigma_{1(1)}^{-2}, \sigma_{2(1)}^{-2}, \sigma_{3(1)}^{-2}, \dots, \sigma_{1(n)}^{-2}, \sigma_{2(n)}^{-2}, \sigma_{3(n)}^{-2}) \quad (6)$$

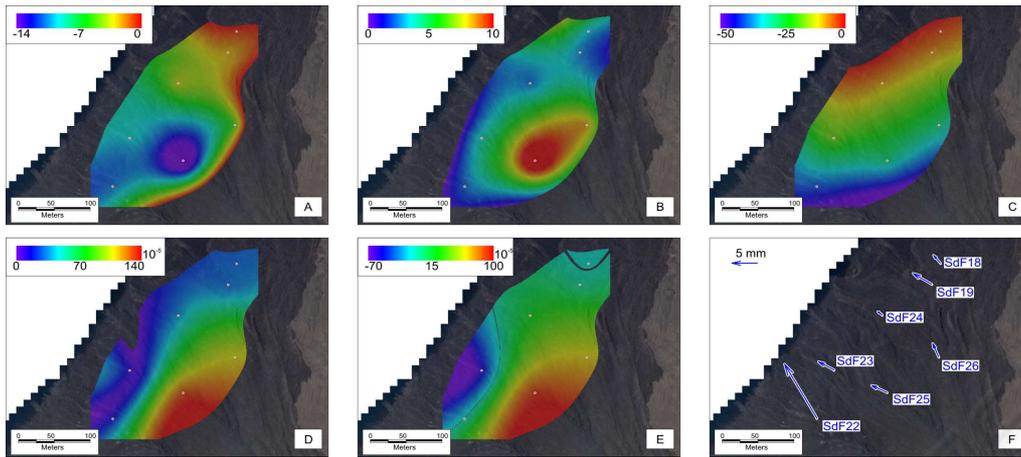


Figure 3: In the frames (a), (b), and (c) are reported the calculated East, North and Up components of displacements respectively. Frames (d) and (e) report the maximum shear strain and the volume variation. Finally, in the frame (f) the displacement vectors of the benchmarks are shown.

where the quantities  $\sigma_{j(n)}$ 's are the standard deviations of the measurements. According to the modified least squares (MLS) approach proposed by (Shen et al., 1996), based on the adjustment of the covariance matrix  $W$ , we use the matrix  $W'$  which is a weighted version of the matrix  $W$  of experimental data. Following the suggestion given by (Shen et al., 1996) and (Teza et al., 2008) the weight function considered here is:

$$W' = W e^{-\frac{d(n)}{d_0}} \quad (7)$$

where  $d(n)$  is the distance between the  $n$ th EP and the arbitrary point  $P$ , and  $d_0$  is a distance-decaying constant defining the "level of locality" of the estimation.

This method, likewise most previous methods (Frank, 1966) and (Prescott, 1976) is used to interpolate the strain among benchmarks of geodetic networks where ground deformations are measured by comparing geodetic surveys.

## 5 AN APPLICATION TO THE SCIARA DEL FUOCO

The benchmarks placed on the lava fan show a general NW-ward motion following the maximum slope of the SdF, with an increasing magnitude from NE to SW (Fig. 3f). This kind of deformation is in good agreement with a seawards motion of the new lava fan, driven by a mainly gravitational dynamics. However, the ground motion is not uniform above the investigated area, showing significant differences in the displacements measured on different benchmarks. In order to analyze the ground deformation pattern

recorded from December 14, 2008 to January 3, 2009 above the deforming lava body, we performed a strain interpolation. Unfortunately the corresponding linear system is undetermined since it implies more unknowns ( $n = 9$ ) than equations ( $m = 7$ ). Therefore the solution is never unique. To this reason we have calculated a basic solution with almost  $m$  non zero components by using the QR factorization with column pivoting. Results are reported in Fig. 3, where the decrease of the horizontal motion (Fig. 3a and b) is evident from benchmark SDF25, that is located on the upper and central area of the fan, towards the coastline and towards North, reaching the minimum values at SDF18 benchmark, located close to the SdF northern rim. The vertical motion (Fig. 3c) shows a more uniform gradient, from a maximum down-lift of about 50 mm at the S-Westernmost benchmark (SDF22) to near 0 at SDF18. A deeper analysis of the overall deformation of the lava fan is allowed by the interpolation of the strain tensor. In Fig. 3d, the distribution of the maximum 3D shear strain is reported, confirming the strongest deformation on the upper area of the lava fan; this is mainly due to the stronger magnitude of horizontal displacements of the southernmost SDF22, SDF25 and SDF26 benchmarks with respect to the northern half of the fan, but also to the relative vertical motion of the two halves of the body. On the upper area, also the volumetric dilatation evidences a maximum expansion (Fig. 3e), mainly imputable to the divergent directions of the displacements affecting SDF25 and SDF26 benchmarks. In addition, a contracting area is detected on the southern coastline of the fan, due to the smaller displacements of the SDF23 and SDF24 benchmarks with respect to the upper ones, while all the northern half of the lava body shows no significant volumetric strain variation

confirming the higher stability of this portion of the fan that is buttressed by the stable northern wall of the SdF.

## 6 CONCLUSIONS

In this paper we have first shown the pre-processing techniques adopted to reduce noise sources affecting ground deformation measures acquired at the Sciarra del Fuoco by the automatic monitoring system referred to as THEODOROS. In particular, due to the gaussian distribution of acquisitions, the problem of spikes removal was simply solved taking into account their standard deviations. The offsets due to the crash of the system have been adjusted based on the evaluation of the linear trend of observations. Finally spline functions have been used to reduce the thermoelastic effects. After these pre-processing steps we have shown the based on infinitesimal strain theory method used to compute both displacements maps and strain field over the area covered by the THEODOROS system. Finally a case study related to the ground motion observed in the period December 2008 - January 2009 was carried out in order to test the proposed methodology. Preliminary results show that the distribution of the maximum 3D shear strain emphasizes the strongest deformation on the upper area of the lava fan. Furthermore the volume variation highlights a contracting area on the southern coastline of the fan. Finally all the northern half of the lava body shows no significant volumetric strain variation confirming the higher stability of this portion of the fan that is buttressed by the stable northern wall of the SdF.

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