A COMPARATIVE STUDY BETWEEN CONVENTIONAL AND CONTINUOUS GENETIC ALGORITHMS FOR THE SOLUTION OF CARTESIAN PATH GENERATION PROBLEMS OF ROBOT MANIPULATORS

Za'er Salim Abo-Hammour, Mohammad Suleiman Saraireh
Jordan University, Faculty of Engineering, Mechatronics Department, Amman-Jordan
Mutah University, Faculty of Engineering, Computer Engineering Department, Mutah-Kerak-Jordan

Othman M-K. Alsmadi
Jordan University, Faculty of Engineering, Electrical Department, Amman-Jordan

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Abstract: In this paper, a comparative study between the continuous and the conventional GAs for the solution of Cartesian path generation problems of robot manipulators is performed. The difference between both algorithms lies in the ways in which initialization phase, the crossover operator, and the mutation operator are applied. Generally, the operators of the Continuous Genetic Algorithms (CGA) are of global nature, i.e., applied at the joint's path level, while those of conventional GA are of local nature, i.e., applied at the path point level. It was concluded from the simulations included that CGAs have several advantages over conventional GAs when applied to the path generation problems; first, the joints' paths obtained using the conventional GA are found to be of highly oscillatory nature resulting in very large net joints displacements consuming more energy and requiring more time. This problem is totally avoided in CGA where the resulting joints' paths are smooth. Second, the CGA has faster convergence speed (number of generations required for convergence) than the conventional GA. Third, the average execution time per generation in the conventional GA is two to three times that in the CGA. This is due to the fact that the conventional GA requires a coding process, which is not the case in the CGA. Fourth, the memory requirements of the conventional GA are higher than those of the CGA because the former uses genotype and phenotype representations while the later utilizes only the phenotype representation.

1 INTRODUCTION

Genetic algorithms, GAs, are broadly applicable, general-purpose, generate-and-test optimization methods based on Darwinian principles of biological evolution, that is, "the survival of the fittest" and the genetic operators. They were developed by Holland (Holland, 1975) to study the adaptive process of natural systems and to develop artificial systems that mimic the adaptive mechanism of natural systems.

Conventional genetic algorithms were used by the robotics community for solving the path generation problems of robot manipulators where the inverse kinematics problem is formulated as an optimization problem and is then solved using GAs based on the use of the forward kinematics model of the manipulator. In this regard, Parker et. *al.*1989 introduced genetic algorithms for solving the inverse kinematics problem of redundant manipulators where GAs were used to move a robot to a target location while minimizing the largest joint displacement from the initial position. After that, Davidor proposed a special GA for path generation problem of redundant manipulators (Davidor, 1991). He considered generating robot path as a typical ordered-dependent process and presented a GA model for this problem. The main characteristics of his algorithm are the use of dynamic individuals

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structures and a modified crossover operator called analogous crossover. The goal of the proposed GA is to minimize the accumulative deviation between the generated and the desired path.

CGA has been introduced recently as an alternative and efficient technique for the solution of path generation problems of robot manipulators (Abo-Hammour et *al*, 2002). The CGA is that algorithm which depends on the evolution of curves in one-dimensional space. In general, CGAs use smooth operators and avoid sharp jumps in the parameter values. The algorithm was a contribution to the solution of the inverse kinematics problem of manipulators based on the concept of the minimization of the accumulative path deviation. The effect of various CGA operators and genetic-related control parameters, and the effect of various robot-related parameters on the convergence speed of our proposed methodology for Cartesian path generation was explored in (Abo-Hammour, 2005) and (Abo-Hammour, 2002).

CGAs possess several advantages when applied to path generation problems of robot manipulators (Abo-Hammour et *al*, 2002): first, it can be applied to any general serial manipulator with positional degrees of freedom that might not have any derived closed-form solution for its inverse kinematics. Second, to the authors' knowledge, it is the first singularity-free path generation algorithm that can be applied at the path update rate of the manipulator. Third, extremely high accuracy can be achieved along the generated path almost similar to analytical solutions, if available. Fourth, the proposed approach can be adopted to any general serial manipulator including both non-redundant and redundant systems.

In this paper, a detailed comparative study between conventional and CGAs for the solution of path generation problems of robot manipulators in a free-of-obstacles workspace is performed. This study includes the nature of the joints' paths obtained using both algorithms, the effect of the joints' limits on the solutions obtained using conventional genetic algorithm, the influence of the degree of redundancy and the number of knots along the Cartesian path on the convergence speed of both algorithms, and finally a step by step switching from conventional genetic algorithm to CGA. It is to be noted that both algorithms are based on the concept of the minimization of the accumulative path deviation only; no other objective functions are included in this work.

The organization of the remainder of the paper is as follows: the formulation of the path generation problem for solution by genetic algorithms is described in section 2. Section 3 covers both of the CGA and the conventional genetic algorithm in details. The comparative study between the two algorithms is covered in Section 4. Finally, conclusions are given in Section 5.

2 FORMULATION OF THE PATH GENERATION PROBLEM

Let us consider a robot manipulator with M degrees of freedom and N task space coordinates. Assume that a desired Cartesian path, P_{dc}, is given, the problem is to find the set of joint paths, P_θ, such that the accumulative deviation between the generated Cartesian path, P_{gc}, and the desired Cartesian path, P_{dc}, is minimum. In other words, we are interested in the determination of a set of feasible joint angles, which corresponds to a set of desired spatial coordinates of the end-effector in the task space.

It is to be noted that after the sampling process by N_k samples, P_{dc} and P_{gc} are matrices of dimension N by N_k while P_θ is a matrix of M by N_k dimension. After sampling the geometric path, at the path update rate for best accuracy, the generated values of the joint angles using the genetic algorithm, P_θ, are used by the direct (forward) kinematics model of the robot to obtain the generated Cartesian path given by.

$$P_{gc} = F_k(P_\theta) \tag{1}$$

Where F_k represents the forward kinematics model of the manipulator.

The deviation between the desired Cartesian path, P_{dc}, and the generated Cartesian path, P_{gc}, at some general path point, i, is given as.

$$E(i) = \sum_{k=1}^{N} \left| P_{dc}(k,i) - P_{gc}(k,i) \right| \tag{2}$$

The accumulative deviation between the two paths (desired and generated) depends on whether the initial and final joint angles corresponding to the initial and final configurations of the end-effector are given in advance using the inverse kinematics model of the manipulator or through other numerical technique (fixed end points) or the case in which the initial and final joint angles are not given (free end points). For the fixed end points case, the accumulative deviation between the two paths is given by the formula.

$$E = \sum_{i=2}^{N_k-1} \sum_{k=1}^{N} \left| P_{dc}(k,i) - P_{gc}(k,i) \right| = \sum_{i=2}^{N_k-1} E(i) \tag{3}$$

While for the free end points case, the accumulative deviation between the two paths is given by the formula.

$$E = \sum_{i=1}^{N_k} \sum_{k=1}^{N} \left| P_{dc}(k,i) - P_{gc}(k,i) \right| = \sum_{i=1}^{N_k} E(i) \qquad (4)$$

The fitness function, a nonnegative measure of the quality of individuals, is defined as:

$$F = \frac{1}{1 + E} \qquad (5)$$

The optimal solution of the problem is obtained when the deviation function, E, approaches zero and correspondingly the fitness function, F, approaches unity.

3 GENETIC ALGORITHMS

GAs are based on the triangle of genetic reproduction, evaluation and selection (Goldberg, 1989). Genetic reproduction is performed by means of two basic genetic operators: crossover and mutation. Evaluation is performed by means of the fitness function that depends on the specific problem. Selection is the mechanism that selects parent individuals with probability proportional to their relative fitness. The genetic algorithm used in this work consists of the following steps:

1. ***Initialization.*** An initial population comprising of N_p individuals is randomly generated in this phase.
2. ***Evaluation.*** The fitness, a nonnegative measure of quality used as a measure to reflect the degree of goodness of the individual, is calculated for each individual in the population as given in Equation 6.
3. ***Selection.*** In the selection process, individuals are chosen from the current population to enter a mating pool devoted to the creation of new individuals for the next generation such that the chance of a given individual to be selected to mate is proportional to its relative fitness. This means that best individuals receive more copies in subsequent generations so that their desirable traits may be passed onto their offspring. This step ensures that the overall quality of the population increases from one generation to the next.
4. ***Crossover.*** Crossover provides the means by which valuable information is shared among the population. It combines the features of two parent individuals to form two children individuals that may have new patterns compared to those of their parents and plays a central role in GAs.
5. ***Mutation.*** Mutation is often introduced to guard against premature convergence. Generally, over a period of several generations, the gene pool tends to become more and more homogeneous. The purpose of mutation is to introduce occasional perturbations to the parameters to maintain genetic diversity within the population.
6. ***Replacement.*** After generating the offspring's population through the application of the genetic operators to the parents' population, the parents' population is totally replaced by the offspring's population. This is known as non-overlapping, generational, replacement. This completes the "life cycle" of the population.
7. ***Termination.*** The GA is terminated when some convergence criterion is met. Possible convergence criteria are: the fitness of the best individual so far found exceeds a threshold value, the maximum number of generations is reached, or the progress limit, the improvement in the fitness value of the best member of the population over a specified number of generations is less than some predefined threshold, is reached. After terminating the algorithm, the optimal solution of the problem is the best individual so far found. The block diagram of the genetic algorithm is given in Figure1.

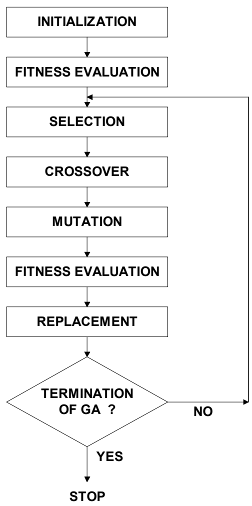


Figure 1: Block Diagram of the Genetic Algorithm.

The conventional genetic algorithm and the CGA used in our work consist of the steps given previously. The evaluation step, selection step, replacement step and the termination step are identical in both algorithms. The differences between both algorithms lie in the initialization phase, the crossover operator, and the mutation operator. These operators have the same goal in both algorithms; the difference lies in the way in which each operator is applied in the corresponding algorithm. These operators are applied at the joint's path level in case of the CGA while they are applied at the path point level in case of conventional genetic algorithm. That is, the operators of the CGA are of global nature while those of conventional genetic algorithm are of local nature. In addition to that, it is to be noted that the conventional genetic algorithm uses the genotype and phenotype data presentations while the CGA uses only the phenotype data presentation. This fact requires a coding process in conventional genetic algorithm, which is not the case in CGA. The CGA is fully described in (Abo-Hammour et *al*, 2002). The reader is kindly asked to read this reference for the complete details about it.

The operators of the conventional genetic algorithm that include the initialization phase, the crossover operator, and the mutation operator are applied at the path point level. In relation to the initialization phase, individuals are generated randomly at the gene level. Conventional crossover involves exchanging genes between each pair of parents selected from mating pool. It is generally applied with relatively high probability of crossover, P_c. Regarding the mutation operator, the bitwise complement mutation is applied in the conventional genetic algorithm at the gene level with some low probability of mutation, P_m. It is realized by performing bit inversion (flipping) on some randomly selected bit positions of children bit strings.

To summarize the evolution process in conventional genetic algorithm, an individual is a candidate solution of the joints' angles; that is, each individual consists of a string of $L=M*N_k*N_s$ genes. Initially, N_p individuals are randomly generated representing the initial population. The population undergoes the selection process, which results in a mating pool among which pairs of individuals are crossed with probability P_c. This process results in an offspring's generation where every individual child undergoes mutation with probability P_m. After that, the next generation is produced according to the replacement strategy applied. This process is repeated till the convergence criterion is met where the $M \times N_k$ parameters of the best individual are the required joints' angles.

4 SIMULATION RESULTS

The CGA and the conventional genetic algorithm were used to solve the Cartesian path generation problem of 2R and 3R planar manipulators. The initial settings of the CGA parameters are as follows: the population size is set to 500 individuals. The rank-based selection strategy is used where the rank-based ratio is set to 0.1. The individual crossover probability is kept at 0.9; the joint crossover probability is also set to 0.9. The individual mutation probability and the joint mutation probability are kept at 0.9. Generational replacement scheme is applied where the number of elite parents that are passed to the next generation is one-tenth of the population. The genetic algorithm is stopped when one of the following conditions is met. First, the fitness of the best individual of the population reaches a value of 0.99; that is the accumulative deviation of the end-effector, *E*, of the best individual is less than or equal to 0.01. Second, the maximum deviation at any path point of the best individual is less than or equal to 0.001. Third, a maximum number of 10000 generations is reached. Fourth, the improvement in the fitness value of the best individual in the population over 1000 generations is less than 0.01. It is to be noted that the first two conditions indicate to a successful termination process (optimal solution is found), while the last two conditions point to a partially successful end depending on the fitness of the best individual in the population (near-optimal solution is reached).

The initial settings of the conventional genetic algorithm parameters are similar to those of the CGA except those related to crossover, mutation and coding process which are as following: the crossover probability is kept at 0.7, the mutation probability is kept at 0.01. The uniform crossover method is used as the algorithm's default crossover method. The required accuracy of the phenotype values is set to 0.001 and binary coding scheme is used.

Due to the stochastic nature of GAs, twelve different runs were made for every result obtained in this work using a different random number generator seed; results are the average values whenever possible.

The selected Cartesian path generation problem is of straight line shape as given by:

$$x_{initial} = 0.0, \quad x_{final} = 0.25$$

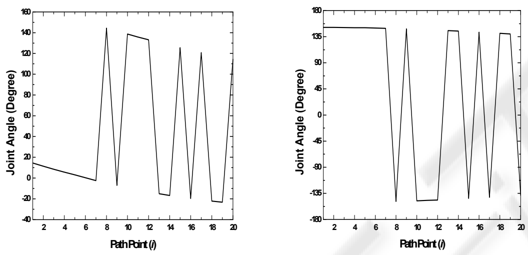
$$P_{dc}(1,i) = X_{dc}(i) = x_{initial} + \frac{x_{final} - x_{initial}}{N_k - 1} * (i-1) \tag{6}$$

$$P_{dc}(2,i) = Y_{dc}(i) = 0.25, \quad 1 \le i \le N_k$$

Two manipulators are used in this work; 2R planar manipulator and 3R planar redundant manipulator. For the 2R manipulator, the link parameters are $L_1=L_2=L=0.5$ meter. For this case, $N=2$, $M=2$, $\theta_{lower}(h)$ =-180° and $\theta_{upper}(h)$ =180° for $h=1,2$. For the 3R planar redundant manipulator, the link parameters are $L_1=L_2=L_3=0.5$ meter. For this case, $N=2$, $M=3$, $\theta_{lower}(h)$ =-180° and $\theta_{upper}(h)$ =180° for $h=1,2,3$.

The number of path points along the Cartesian path, N_k, is set to 20 points. The initial and final joints' angles corresponding to the initial and final configurations of the end-effector along the Cartesian path are not given (i.e., free end points case).

Initially, the conventional genetic algorithm was used to solve the given path generation problem for both manipulators. For the 2R manipulator, the algorithm reaches a fitness value of 0.99 within 50 generations and the average path point deviation is almost 0.0005 meter. The joints' paths for the first and second joints of the 2R manipulator are shown in Figure 2.

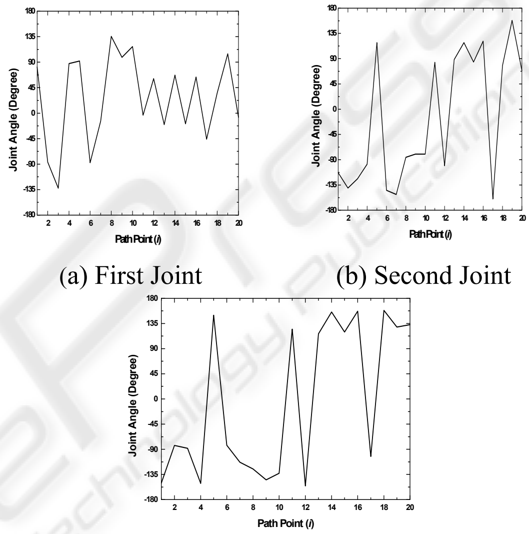


(a) First Joint (b) Second Joint

Figure 2: Joints' Paths of 2R Manipulator Using Conventional Genetic Algorithm for (a) First Joint, and (b) Second Joint.

It is obvious that the resulting solution curves in joint space are highly oscillatory within the given range of the joints' limits. For the given manipulator, there exist two possible solutions for the inverse kinematics problem corresponding to "elbow up" and "elbow down" configurations. It is clear that the resulting solutions for both joints have multiple switching points between these two possible solutions. The switching process from one solution corresponding to one robot configuration to another solution corresponding to other robot configuration results in very large net joints displacements consuming more energy and requiring more time. As a result, while solving such problems, the switching from "elbow up" configuration to the "elbow down" configuration should not be allowed despite the fact that it is still a solution to the problem. Generally, the probability of switching between different solutions increases as the number of feasible solutions of the manipulator increases.

For the 3R planar redundant manipulator, the algorithm reaches a fitness value of 0.99 within 72 generations and the average path point deviation is almost 0.0005 meter. The joints' paths for the first, second and third joints of the 3R manipulator are shown in Figure 3. It is obvious that the resulting joints' paths are highly oscillatory within the range of the joints' limits, which results in large net displacements of the joints.



(a) First Joint (b) Second Joint

(c) Third Joint

Figure 3: Joints' Paths of 3R Manipulator Using Conventional Genetic Algorithm for (a) First Joint, (b) Second Joint, and (c) Third Joint.

The oscillatory behavior of the joints' paths encountered in the conventional genetic algorithm is actually due to the nature of the initialization phase, crossover operator, and mutation operator used in the algorithm. These three operators are applied at the path point level in the conventional genetic algorithm. Conventional initialization phase implies that consecutive path points might have opposite extreme values within the given range of the joint's limits. The problem of oscillatory values among consecutive path points is emphasized when the range of joint's limits is extended as discussed previously. This problem is bypassed in CGA by the use of smooth curves in the initial population that eliminate the possibility of highly oscillating values among the consecutive path points.

Conventional crossover operator results in a jump in the value of the parameter in which the crossover point lies (discontinuity) while keeping

Table 1: Step-by-Step Switching to CGA for the 2R Manipulator.

Initialization Type	Crossover Type	Mutation Type	Avg. Execution Time (Seconds)	Avg. No of Generations	Avg. No. of Switchings
Conventional	Conventional	Conventional	143.99	54	9
Conventional	Conventional	Continuous	160.59	57	7
Conventional	Continuous	Conventional	221.2	78	7
Conventional	Continuous	Continuous	261.44	100	2
Continuous	Conventional	Conventional	117.37	48	1
Continuous	Conventional	Continuous	109.97	49	0
Continuous	Continuous	Conventional	119.09	55	0
Continuous	Continuous	Continuous	99.11	47	0

Table 2: Step-by-Step Switching to CGA for the 3R Manipulator.

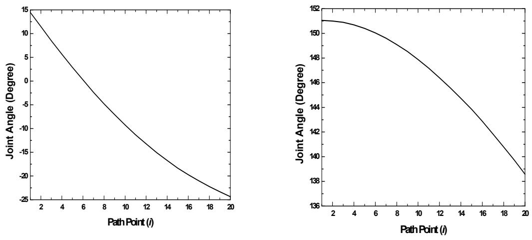
Initialization Type	Crossover Type	Mutation Type	Avg. Execution Time (Seconds)	Avg. No. of Generations	Nature of Joints' Paths
Conventional	Conventional	Conventional	487.72	124	Oscillations With Large Magnitude
Conventional	Conventional	Continuous	390.06	105	Oscillations With Medium Magnitude
Conventional	Continuous	Conventional	295.14	83	Oscillations With Large Magnitude
Conventional	Continuous	Continuous	486.1	144	Oscillations With Medium Magnitude
Continuous	Conventional	Conventional	188.89	53	Oscillations With Small Magnitude
Continuous	Conventional	Continuous	191.2	56	Oscillations With Small Magnitude
Continuous	Continuous	Conventional	181.57	55	Oscillations With Small Magnitude
Continuous	Continuous	Continuous	148.85	49	Smooth Solution Curves

the other parameters the same or exchanged between the two parents. It is clear that each crossing point results in a discontinuity in the joint angles of the obtained children. The worst case obtained regarding the discontinuity of the resulting curves of the children happens in the uniform crossover process. In this scheme, the smoothness of the joint's paths of the parents is completely spoiled since crossover happens at every path point. The solution to the non-smoothness of the resulting joint's paths is through the use of the tangent hyperbolic crossover function used in CGA that results in smooth transition in the joint values of the two parents while generating the two children.

Conventional mutation process changes only the value of the joint angle of the path point in which mutation occurs while keeping other joint angles in the joint's path unchanged. This process results in a jump in the value of the joint angle in which mutation takes place and the overall path will become of oscillatory behavior. The discontinuity in the joint's path depends on the number of mutations that take place in the path and the position of the bit at which mutation takes place; that is, if the mutation bit is leftmost, then the discontinuity will be larger than that of rightmost mutation bit. This problem is solved in CGA by applying the Gaussian mutation function that is of global nature. In our approach, mutation is applied at the joint's path level rather than path point level. As a result, mutation function will start from zero values and increases/decreases slowly till the peak then it will go back to zero values at the other end.

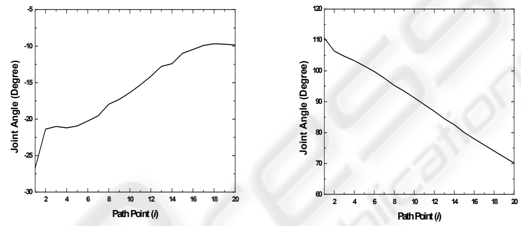
After that, the effect of both versions (conventional and continuous) of the initialization phase, crossover operator and mutation operator on the nature of the joints' paths obtained and the convergence speed of the hybrid algorithm is studied. Table 1 gives the relevant data for the 2R manipulator while Table 2 gives the relevant data for the 3R manipulator. From Table 1, it is clear that the maximum number of switching between the two existing solutions of the inverse kinematics problem for the 2R manipulator happens in case of the conventional genetic algorithm (i.e., conventional types of initialization, crossover and mutation). Furthermore, the initialization phase has the greatest effect on the smoothness/non-smoothness of the solution curves; that is, in case of conventional initialization, the number of switching points is 6 on average while in case of continuous initialization, the number of switching points is 0 on average. It is also clear that as the number of conventional processes decreases, the number of switching points decreases. The minimum execution time and the best convergence speed are achieved using the CGA (i.e., continuous types of initialization, crossover and mutation). Regarding the 3R manipulator, it is clear that the initialization phase has the greatest effect on the smoothness/non-smoothness of the solution curves; that is, in case of conventional initialization, the joints' paths are of oscillatory nature with large or medium magnitude oscillations while in case of continuous initialization, the joints' paths are either smooth or of oscillatory nature with small magnitude oscillations. The minimum execution time and the best convergence speed are achieved using the CGA (i.e., continuous types of initialization, crossover and mutation). For both manipulators, the conventional initialization, continuous crossover and continuous mutation case results in the largest number of generations required for convergence. Regarding the case in which the conventional initialization, continuous crossover and conventional mutation are used which is almost similar to the algorithm proposed by Davidor, it is observed that this hybrid scheme still results in oscillations with large magnitude as shown in Table 2. This is an expected result since the smoothness achieved by the continuous crossover process is disturbed by the conventional mutation process. This goes in agreement with our previous comments about his algorithm that even after the application of the analogous crossover operator, the oscillatory behavior of the joints' paths is not totally avoided due the discontinuities, which might appear in the initialization phase and due to the mutation operator.

The joints' paths for the first and second joints of the 2R manipulator using CGA are shown in Figure 4. It is obvious that the resulting solution curves in joint space are smooth and do not have any switching between the two possible solutions, which results in minimizing the net displacement of the joints. The joints' paths for the first, second and third joints of the 3R manipulator are shown in Figure 5 where similar observations are concluded regarding the smoothness of the solution curves.

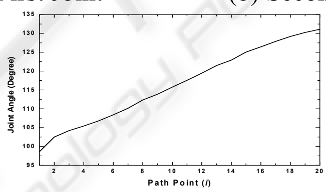


(a) First Joint (b) Second Joint

Figure 4: Joints' Paths of 2R Manipulator Using CGA for (a) First Joint, and (b) Second Joint.



(a) First Joint (b) Second Joint



(c) Third Joint

Figure 5: Joints' Paths of 3R Manipulator Using CGA for (a) First Joint, (b) Second Joint, and (c) Third Joint.

5 CONCLUSIONS

In this work, both of the continuous and the conventional genetic algorithms were used for the solution of the Cartesian path generation problems of robot manipulators.

It was noted that the resulting joints' paths using conventional genetic algorithm have multiple switching points among the possible solutions of the non-redundant manipulators while they are of highly oscillatory nature for the redundant manipulators resulting in very large net displacements of the joints for both systems. This oscillatory behavior in conventional genetic algorithm is actually due to the nature of the initialization phase, crossover operator, and mutation operator used in the algorithm. First, the conventional initialization phase results in consecutive path points that might have opposite extreme values within the given range of the joints' limits. Second, the conventional crossover operator results in a jump in the value of the parameter in which the crossover point lies, while keeping the other parameters the same or exchanged between the two parents. Third, the conventional mutation process changes only the value of the joint angle of the path point in which the mutation occurs while keeping other joint angles in the joint's path unchanged. The resulting discontinuity in the joint's path depends on the number of mutations that take place in the path and the position of the bit at which mutation takes place. These three operators are designed in CGA such that they result in smooth joints' paths from one side and maintain an excellent accuracy along the Cartesian path from the other side. Among the three operators, it was noted that the initialization phase has the greatest effect on the smoothness/non-smoothness of the joints' paths. The convergence speed of the CGA in terms of both the number of generations required for convergence and the average execution time is much superior to that of the conventional genetic algorithm.

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