

# COOPERATIVE UAVS MAPPING COMPLEX ENVIRONMENT USING 2D SPLINEGON

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**Abstract:** This paper presents a novel approach which enables multiple UAVs to efficiently explore an unknown environment and incrementally build the map of the area and its complex shaped obstacles, represented here as concave and convex in shape. The task is achieved by a improved performance of sensor based searching, navigation and mapping of these complex shaped obstacles in an unknown environment. The improved performance is quantified by explicit bounds of navigating the UAVs using an extended Kalman filter and to build the map of the complex shaped obstacles using the 2-D Splinegon. The circle packing search algorithm is used for the completeness of coverage in searching the unknown obstacles regions and the UAVs trajectories are generated by the Dubins path planning algorithm. This novel proposed algorithm results in a robust approach to search and map the obstacles using multiple UAVs that is also computational attractive.

## 1 INTRODUCTION

This paper focusses on swarm of UAVs deployed for a mission of searching an unknown region to detect obstacles and to extract their shape. The circle packing search algorithm (Kershner, 1939), (Guo and Qu, 2005) is implemented where the search is carried out by a sequence of “looks” each of which covers a circle corresponding to the footprint of the sensor on board. This circle packing algorithm covers the plane by packing each circles into the unknown environment. The centers of each of this packed circles represent the way points to be used on path planning of the UAVs. A mission planning algorithm is described which enables the UAVs to switch between the searching mode and the mapping of the detected obstacle. Since the swarm of UAVs fly around an unknown environment, a sense and avoid system is developed so the UAVs autonomously replan their paths when they approach an obstacle or predict intersection of air traffic. Thus the system presented here provides a safe surveillance of unknown areas by swarm of UAVs. Furthermore the proposed mission planning not only enables the swarm of UAVs to switch from searching mode into mapping mode, but also ensures the allocation of the required number of UAVs to map that obstacle with in searched region.

In the fulfillment part of the mapping task, the measurements from laser sensor that are mounted on the UAVs is the only source to construct the map of the detected obstacle. This strongly suggests that the most efficient way in modelling approach should be to define these measurement points as vertices that can form a polygon with line segments. This raises an issue as to how to represent the curved nature of these obstacles. One such promising approach is introduced in this paper that uses a generalization of polygons that produces a set of vertices that are connected by line segments of constant curvature. This is a subset of a class of objects named as splinegons (Dobkin et al., 1988), (Dobkin and Souvaine, 1990).

In the mapping process the fused EKF estimated positions are used with the limited number of measurements (i.e. required number of the the interpolation points) from the laser sensors to build the map. As the vehicles fly around the obstacle, sensors such as laser sensors are used to measure the distance to the obstacles. Out of all these measurements, only a carefully selected number of measurements are chosen which represent the required vertices to construct a simple polygon. The data association algorithm is implemented to select a limited number of vertices and to uniformly distribute these vertices around the obstacle in a computationally efficient way. This se-

lection of maximum number of vertices that can form a polygon is limited from sixteen to twenty vertices. This selection of vertices is accomplished by calculating the length and the curvatures between each of the vertices and to eliminate the vertex that has a minimum length and curvature. The selection process is based on the size and the curved nature of the obstacle. Thus, the constructed polygon is generalised to produce a set of vertices that are connected by line segments of constant curvature called splinegon. This splinegon is a set of vertices that have constant curvature line segments defined with  $C^2$  contact at the vertex points.

1. The prediction of next way points for the search algorithm and prediction of a new way points when obstacle/collision avoidance algorithm is activated.
2. Switching between the search mode into the mapping the unknown obstacle and predict the way points and its orientation to get the measurements.
3. The prediction of the required number of vehicle to accomplish the mapping task (How many vehicle is needed to complete the mission?)
4. If more then one UAV is used for the mapping task,
  - (a) Find a shortest way points for the other UAVs to reach the mapping region.
  - (b) Whether the UAVs needs to exchange the information with one to another, and
  - (c) Identify the state of the obstacle whether it is single or multiple, depending upon the intersection detection between the local updates. Eventually, the global update is performed between the intersection detected UAVs.
5. The prediction of required number of measurement for the mapping from each of the detected obstacles.

This newly proposed algorithm gives a solution that has a highly richness in building the curved nature of the unknown obstacle in an unknown environment. This proposed approach is a computationally attractive one resulting in information driven mission planning and the mapping of unknown environments with limited measurements.

The development of this algorithm is shown in the functional block diagram in figure 1.

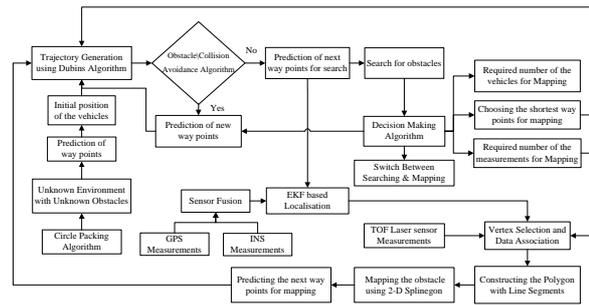


Figure 1: The functional block diagram.

## 2 PATH PLANNING AND CIRCLE PACKING ALGORITHM

The path planning algorithm generates a flyable and safe path to the UAVs to fly from one location to another. The locations are predefined by the way-points. The starting location is called base and the final location to be reached is called target. The Dubins set of paths is used to connect the base and target by a sequence of successive way-points. The base and the target are characterized by the poses, in a set of position coordinates  $(x, y)$  and orientation  $\theta$  of the UAVs. Considered that an initial pose  $P_s(x_s, y_s, \theta_s)$  at the base and a final pose at the target  $P_f(x_f, y_f, \theta_f)$  are given, the path  $r(t)$  connecting the two poses can be a single or a composite curve. A path represented by a curve in  $2D$  is completely determined by its curvature (Kreyszig, 1991) and the maximum curvature bound of the UAV by  $\kappa_{max}$  and other constraints by  $\Pi$ ; then the path planning can be mathematically represented as:

$$P_s(x_s, y_s, \theta_s) \xrightarrow{r(t)} P_f(x_f, y_f, \theta_f), \quad \kappa(t) < \kappa_{max}, \quad \Pi \quad (1)$$

Extending the equation (1) for a group of UAVs, this changes into:

$$P_{si}(x_{si}, y_{si}, \theta_{si}) \xrightarrow{r_i(t)} P_{fi}(x_{fi}, y_{fi}, \theta_{fi}) \\ \kappa_i(t) < \kappa_{i,max} \quad \text{and} \quad \Pi \quad (2)$$

where  $\kappa_i(t)$  is the curvature,  $\kappa_{i,max}$  is the maximum curvature bound of  $i^{th}$  path, and  $i = 1 \dots N$ ,  $N$  is the number of UAVs. This path planning algorithm is integrated with an obstacle and Collision avoidance system to generated the trajectories for each UAVs and to protect the flying UAVs from any collision with the surrounded obstacles or with the on coming UAVs. This algorithm is applied online while the UAVs are in motion.

The circle packing algorithm (Washburn, 1981) is to attempt to cover the given plane by packing the

circles of radius  $R$ . In other words finding a minimum number of circles with the radius of  $R$  to completely cover in the given area of search. This is accomplished by fixing the coverage range or the sensor range are represented as a circle. The key problem is to determine the required number of circles with a radius  $R$  to cover the given area. This in turn produced many solutions to this problem. But the objective is to find an optimal solution to minimise the repeated search. One such algorithm that was reported in (Kershner, 1939) and (Guo and Qu, 2005) is implemented here so as to covert the whole area to perform the searching task.

The solution to optimally place the minimum number of circles can be described as the circle has the radius of  $R_c$  and the area to be covered with this circle is denoted as  $W$ . A pattern is composed of a string of circles with radius  $R_c$  that has to be placed along the vertical line, and the distance between the centers of any of the two adjacent circles is  $\sqrt{3}R_c$ . The  $m$  column of circles are placed that are oriented parallel to the  $Y$ -axis and in the same way the distance between the centers of any of the two adjacent circles is  $1.5R_c$ . The origin  $[x_o, y_o]$  is chosen at the left bottom of the given area  $W$ . This in turn enables to place the  $m$  circles that are parallel to the  $y$ -axis which contains the  $n$  number of circles to completely cover the given area. So the center  $[x_c^{kl}, y_c^{kl}]$  of the  $k^{th}$  row ( $1 \leq k \leq n$ ) and the  $l^{th}$  column ( $1 \leq l \leq m$ ) can be defined viz:

$$[x_c^{kl}, y_c^{kl}] = \begin{cases} [x_o + (l-1)3/2R_c, y_o + (k-1)\sqrt{3}R_c] & \text{if } l \text{ is an odd integer} \\ [x_o + (l-1)3/2R_c, y_o + \sqrt{3}/2R_c + (k-1)\sqrt{3}R_c] & \text{if } l \text{ is an even integer} \end{cases} \quad (3)$$

So, the required number of circles needed in each of the column and row  $m$  and  $n$  can be defined as follows:

$$m = \begin{cases} \text{Int} \left( \frac{x_w}{1.5R_c} \right) + 1, & \text{if } \text{Rem} \left( \frac{x_w}{1.5R_c} \right) \leq \frac{2}{3} \\ \text{Int} \left( \frac{x_w}{1.5R_c} \right) + 2, & \text{if } \text{Rem} \left( \frac{x_w}{1.5R_c} \right) > \frac{2}{3} \end{cases} \quad (4)$$

$$n = \begin{cases} \text{Int} \left( \frac{y_w}{\sqrt{3}R_c} \right) + 1, & \text{if } \text{Rem} \left( \frac{y_w}{\sqrt{3}R_c} \right) \leq \frac{1}{2} \\ \text{Int} \left( \frac{y_w}{\sqrt{3}R_c} \right) + 2, & \text{if } \text{Rem} \left( \frac{y_w}{\sqrt{3}R_c} \right) > \frac{1}{2} \end{cases} \quad (5)$$

Where,  $x_w$  and  $y_w$  is the length of the environment along  $X$ -axis and  $Y$ -axis respectively.  $I$  is an integer number and  $\text{Rem}$  is the remainder of the number, in which  $\text{Rem} = x - \text{Int}(x)$ . So by applying the above

equations the required number of circles for each of the row and column is obtained (Kershner, 1939). The prediction of a set of next way-points to start the mapping task is performed that would generate a set of way points for each UAV from its current location to reach the starting point of the mapping task.

### 3 DEFINITION OF 2D SPLINEGON

A Splinegon with constant curvature line segments can be defined with  $C^2$  contact at the vertices. This implies that the line segments share both a common vertex and that the tangents at the vertices are also the same. In order to ensure  $C^2$  contact between vertices, the line segments must meet both position and tangent end point constraints. A single arc segment between vertices only has one degree of freedom: the arc curvature. This is not enough to be able to match the tangent constraint at both end vertices, as at least two degrees of freedom are necessary. Extra degrees of freedom are thus required to ensure the  $C^2$  constraints were both line segments end vertices can be met. One solution to increasing the degrees of freedom is to introduce an intermediate vertex such that the line segment is replaced by two arc segments of different curvature, as shown in figure 2. Hence two

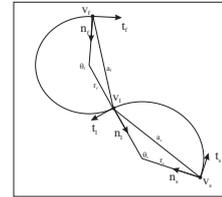


Figure 2: Arc segment with  $C^2$  contact intermediate vertex.

arcs of differing curvatures will connect the UAV vertices via the intermediate vertex. In order to develop the defining equations for such a solution, the intersection of two constant curvature arcs at a point with  $C^2$  contact is considered.

### 4 IMPLEMENTATION AND RESULTS

The primary objective of the current work is to be able to search the given unknown environment with a swarm of UAVs to detect the region of the unknown obstacles and to extract the shape of the obstacle using 2-D Splinegon technique. Initially the

circle packing algorithm is implemented that would pack the required number of circles to fit in the given environment. The search algorithm is employed with the swarm of UAVs that would predict the next way points in online using the neighboring way points or it will follow the given set of POI to detect the obstacles where the path of the vehicle is dictated by the Dubins path planning algorithm. By fixing the detecting sensor range, if any of the UAV is not able to reach any given next way point, or if the obstacle avoidance algorithm is activated so as to prevent the UAV not reaching the given way point then that circle is added into the obstacle region. Once the search is finished, depending upon the search region each of the following requirements are taken by the decision making algorithm.

- Find the area of the obstacle region.
- Find the required number of UAVs to accomplish the mapping task. This can be done based on the area of the obstacle region.
- Generate the way point for each of the UAVs to perform the mapping task.
- Finally, find the shortest way points to each UAVs to reach the starting point of the mapping task from its current location.

As the vehicle moves each of the vehicle is localised with an EKF. At the end of the each cycle (i.e., at the completion of one way point) a local updated map is constructed using the Splinegon technique. In the case of more than one UAV is used in mapping, then an intersection detection algorithm is implemented so as to identify the state of the obstacle and to share the sensor information with the other UAVs. Finally the

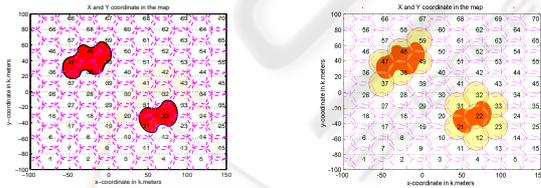


Figure 3: The results of the circle packing algorithm.

global updated map is constructed to get the map of the unknown environment. The circles with radius  $R_c$  which are packed in using circle packing algorithm is shown in figure 2 (a). Then the search algorithm is carried out by giving a set of point of intrust(POI) way points to each of the UAVs so as to find the obstacle region. The shaded circles where the UAVs could not reach are known as the obstacle region which is shown in figure 2 (b). At the end of the search algorithm, the vehicle are switched from searching mode

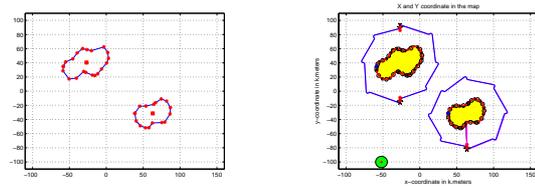


Figure 4: The final updated map using Splinegon.

to the mapping the shape of the complex obstacle. The local update is performed at the end of the each way points. Finally the set of vertices that forms a polygon with line segments in the final update and the global updated map of the given unknown environment is shown in figures 4 (a) and 4 (b).

## 5 CONCLUSIONS

In this paper the authors have described a novel, computationally attractive, approach in estimating the localisation and mapping for curvilinear objects using multiple UAVs. It enables to map obstacles of curvilinear shape, the data association for the networked sensor platforms and the reactive tasking the UAVs. Future work will extend the Splinegon technique to 3D in the robotic network that will enable the flight paths to have even greater flexibility and will enable the complex 3D shapes to be represented by a small set of parameters.

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