

# DETECTION OF A FAULT BY SPC AND IDENTIFICATION

## *A Method for Detecting Faults of a Process Controlled by SPC*

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Abstract: A method for detecting the nature of a fault of a process controlled by SPC ( Statistical Process Control) is presented. The method use the integration of SPC , traditional APC (Automatic Process Control) and the System Identification technique . By a statistical on line control of the parameters of a transfer function and the identification of the transfer function itself, the case of a fault due to a change in the system is recognised. An algorithm called ‘batch control’ for the implementation of the method in a real plant is proposed.

## 1 INTRODUCTION

The objective of Statistical Process Control (SPC) is to detect situation of change of the natural behaviour of a process by monitoring on line the key product variables and detecting the cause of the fault, indicating which variable or group of variables contributes to the signal.

A lot of technique has been developed especially due to the large and different areas where the SPC could be applied.

If, traditionally, the SPC has been developed especially to monitor the complicated processes of chemical plants, after that, the big growth of the information technology in the industries and the large amount of process measures collected in the data base of the control systems, has allowed the implementation of SPC on almost every kind of plant. By the way the common goal for the most application is still to monitor the quality of the process, treating the manufacturing process itself as a black box, of which we know the inputs and outputs, ignoring the others information of the nature of the process.

In fact traditionally SPC and APC (Automatic Process Control) have been developed in parallel and only in the last years there has been works where researchers have made the integration of the two areas ( Tsung,1999).

Another point to remark is that the traditional SPC approach, that is still the most diffused in many kind of industries, is essentially the univariable SPC:

by the implementation of control charts like Shewhart, Cusum, etc.. we look the magnitude of the deviation of each variable independently of all others as they are perfectly independent in the process.

But the being ‘in control’ of a process is essentially a multivariable property : in the modern industrial processes the variables are non independent of one another and only if the simultaneous state of them all is in the joint confidence region defined for the system, we could say that the system is in control : by examining one variable at time with the traditional charts it could be that every variable is in the correct range but the common state is not (Kourti & MacGregor,1994).

For this have been developed multivariable methods that can treat all the variables simultaneously.

The principal is the Hotelling or  $T^2$  statistic : it transforms the state of all the variables in the calculation of the value of a single variable which can be monitor for detecting faults.

If this is a great effort to solve the problem in a very useful way, on the other hand we have now the problem to detect the cause of the fault, the variable responsible.

This paper is organized as follows: in the next section the basic concepts of SPC multivariable, the Hotelling statistic and the interpretation of a  $T^2$  value are recalled. In sections 3 and 4 the main advantages of SPC - APC integration and SPC - System Identification integration are presented.

In section 5 a method for statistically monitoring the points of a system transfer function is presented. In section 6 are given the simulation results for a typical industrial APC. In section 7 an algorithm called ‘batch control’ for the implementation of the technique in some kind of industrial processes is presented. Finally some conclusions are given.

## 2 SPC MULTIVARIABLE

Suppose that our process has 2 measures represented by 2 random variables  $x_1, x_2$  uncorrelated, with mean value  $\mu_1, \mu_2$  and variances  $\sigma_1, \sigma_2$  respectively. Consider the distance of an observation point from the mean point in the plane  $(x_1, x_2)$ . Instead of the usual Euclidean distance we consider the relationship :

$$\frac{(X_1 - \mu_1)^2}{\sigma_1^2} + \frac{(X_2 - \mu_2)^2}{\sigma_2^2} = (SD)^2 \tag{1}$$

This is called ‘statistical distance’ (SD). For an observation the contribution of each coordinate to determining the distance is weighted inversely by its standard deviation, that means that a change in a variable with a small standard deviation will contribute more to the statistical distance than a change in a variable with a large standard deviation. It follows that the statistical distance is a measure of the respect of the statistical behaviour of the two variables.

If they are correlated it will be an additional term in  $x_1, x_2$  and in the plane the curve with constant SD will be an ellipse, eventually tilted according to the correlation sign.

In general if we have a process of  $p$  variables and we have  $n$  observation vector of the  $p$  variables with the system ‘in control’ (or what is called the Historical Data Set (HDS) of the system) we can calculate an estimation of the main vector and of the covariance matrix of the random variables :

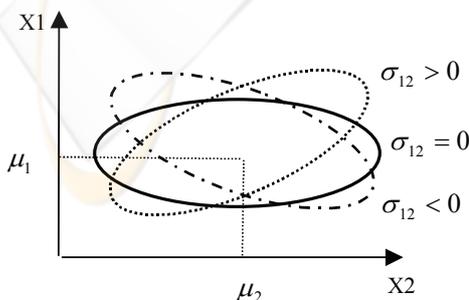


Figure 1: Curves with constant SD for two variables.

$$\mu = \frac{1}{n} \sum_{i=1}^n X_i \tag{2}$$

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2p} \\ \vdots & \vdots & \dots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \dots & \sigma_{pp} \end{bmatrix} \tag{3}$$

For a generic observation we can then compute the quantity :

$$T^2 = (X_i - \mu)^T \Sigma^{-1} (X_i - \mu) \tag{4}$$

This is an univariate quantity that is called Hotelling Statistic or  $T^2$ . It is clear that this is the multivariable generalization of the statistic distance, in words a measure of the closeness (in a statistical way) of the observation to the behaviour of the system expressed by the HDS.

The curves with constant  $T^2$  are then hyper-ellipsoids in the  $p$  dimensional space.

Considering  $T^2$  like a random variable we can see that, in the case of  $\mu$  and  $\Sigma$  estimate by the observations, it follows the distribution of an  $F$  random variable of  $p, n-p$  degrees of freedom.

Let  $\alpha$  be the first type error and let  $F_{(\alpha, p, n-p)}$  be the value  $f$  of  $F \mid P(F \leq f) = 1 - \alpha$  ( $P$  : probability of) we can then calculate an Upper Control Limit (UCL) for  $T^2$  :

$$UCL = \frac{p(n+1)(n-1)}{n(n-p)} F_{(\alpha, p, n-p)} \tag{5}$$

We can say that if we are under the UCL we have a probability  $1 - \alpha$  to say that the system is in control when really it is. We can see that choosing an  $\alpha$  smaller led to have a second type error  $\beta$  bigger, that is a greater probability to not detect a fault when it really exists.

In the industrial processes both errors are important:  $\alpha$  is the representation of the false alarms that can led to stop the production in vain, while  $\beta$ , if large, can led to not detect situations of real out of control.

Generally  $\beta$  is set low because is preferable to have some false alarm than to not detect a fault.

In our examples we have chosen  $\alpha = 0.1$ .

### 2.1 Interpretation of $T^2$ Signals

The  $T^2$  converts a multivariable problem to the calculation of an univariate quantity. But signal interpretation requires a procedure for isolating the responsible of the fault because the contribution could be attributed to individual variables being

outside their allowable range of operation or to a fouled relationship between two or more variables.

Several solutions have been presented for the problem of interpreting a multivariate signal.

One that we show here for example is the MYT decomposition (Mason –Young 2002), that uses an orthogonal transformation to express the  $T^2$  values as two orthogonal equally weighted terms :

$$T^2 = T_1^2 + T_{2,1}^2 \quad (6)$$

$$T_1^2 = (x_1 - \bar{x}_1)^2 / \sigma_1^2 \quad (7)$$

$$T_{2,1}^2 = (x_2 - \bar{x}_{2,1})^2 / \sigma_{2,1}^2 \quad (8)$$

where  $x_{2,1}$  is the estimate of the conditional mean of  $x_2$  for a given value of  $x_1$  and  $\sigma_{2,1}$  is the corresponding estimate of the conditional variance of  $x_2$  for a given value of  $x_1$ . A large value of the first term (called ‘unconditional term’) implies that the observed value of the variable is outside his operational range as was on HDS, while a large values of the second term (‘conditional term’) implies that the observed value of one variable is not where it should be relative to the observed value of the others variables.

By subsequently eliminations of the unconditional terms that signal and iterative decomposition of the conditional terms that signal, it is possible to isolate the variable or group of variables responsible of the fault. We can say that all this methods have in common iterative procedures and sometimes great computational efforts to reach the scope.

### 3 SPC APC INTEGRATION

Combining SPC with APC could be an improvement of the global control of the system.

The scheme is the following :

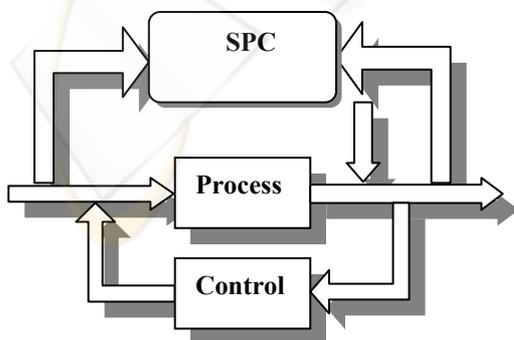


Figure 2: SPC - APC integration.

There is an interaction between SPC and APC : SPC controls not only the variables in and out of the process but also the signals of the control: the result is that the process is no more treated like a black box but the information in the APC are used.

Our procedure uses the information of the transfer function of the system: the change of the system is statistically monitored controlling the parameters of the transfer function

### 4 SPC – SYSTEM IDENTIFICATION

An industrial system is typically formed by automation systems that process the product .

We can divide the entire set of variables of the manufacturing process into ‘process variables’, meaning that they are measures made above the product, (temperature, time of operations etc) and ‘control variables’ (like input - output of the motors, signals of the electronic devices , etc.. ) that reflect the behaviour of the automation systems.

We propose the identification of the transfer functions of the control systems and the statistical monitoring of the transfer functions to detect deviation of the automation system from their normal behaviour .

The identification at the same time allow a better design of the control and a more easily detection of the fault due to the automation systems and not to the process .

### 5 STATISTICAL CONTROL OF A TRANSFER FUNCTION

Consider the simple transfer function of a system G:

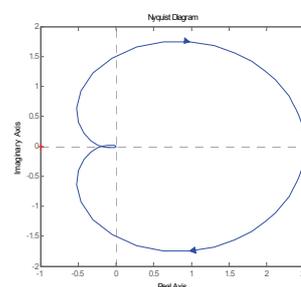


Figure 3: Nyquist plot of a transfer function G.

We can consider the transfer function as a function of 3 parameters : amplitude, phase and

frequency. So we can plot the function in the 3d space of that measures :

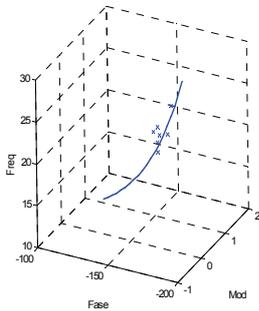


Figure 4: Observations of G points in the space amplitude-phase-frequency.

If we can do measurements of that parameters our observations will be point around the real curve of the transfer function as shown above.

If we plot the curve at constant statistical distance they will be ellipsoid in the 3d space with their major axis tangent at the curve of the G in the point we are sampling it. That tangent is the best linear approximation of the population, because the variables follow the relationship of the G.

So we have that schema for the identification and statistic control of the G:

Make an HDS by collecting observations of the points of the transfer function while the system is in his normal behaviour.

Estimate from the HDS the covariance matrix and the average value of the observations (2) (3).

Set an UCL for the Hotelling statistic chosen a desired  $\alpha$  (5).

Monitoring the T2 control chart build with (4).

## 6 SIMULATION RESULTS

We present the simulation result of a typical industrial APC :

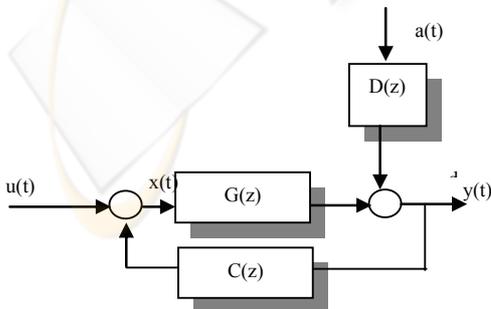


Figure 5: Model for simulations.

formed by a system controlled by a PID controller and disturbed with a coloured noise in the output, according to the typical modelling of an industrial noise.

$$x(t) = -k_p y(t) - k_i \sum_{j=0}^{\infty} y(t-j) - k_d (y(t) - y(t-1))$$

$$D_t = \frac{1 - \theta Z^{-1}}{1 - \phi Z^{-1}} a_t \tag{9}$$

$$G(z) = \frac{0.9z}{z^2 - 1.2z + 0.32}$$

$$a_t \in N(0, \sigma_a)$$

For the on line measurement of the point of the G we have used the technique of the Descriptive Function by inserting a relay with hysteresis h and amplitude A in the control loop.

Leading the system in a controlled oscillation we can see that the relationship

$$G(\omega_c) = -\frac{1}{F} = \frac{\pi Y_c}{4A} e^{j(-\pi + \arcsin \frac{h}{Y_c})} \tag{10}$$

allow to measure a point of the G at the oscillating frequency in amplitude and phase. By varying the hysteresis it is possible to evaluate several points.

We show the result of 50 observations after a change of 10% of the first pole of the system

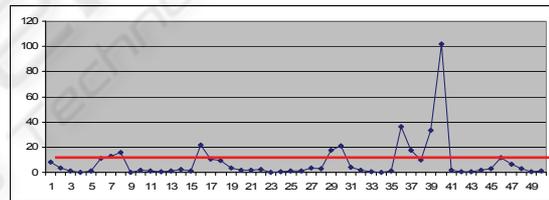


Figure 6:  $T^2$  chart of 50 observation after a change of 10% of the first pole of the system.

The  $T^2$  signals the change in the system with an ARL ( Average Run Length ) of 8.

## 7 PROPOSED BATCH CONTROL ALGORITHM

We have experienced several industrial process for which the production is in two phase : a phase of production , by the working of a material in input , and a phase of wait for the other material to come. In this case we propose to apply the SPC for the 'process variables' during the phase of the effective production ( say 'batch on' ) and to take advantage of the waiting phase ( say 'batch off' ) for the identification of the automation system and applying then the SPC to the 'control variables'. That schema

overcome the problem of detecting if the nature of a fault is in the process or in the automation systems, giving a separation of the two analysis at the origin.

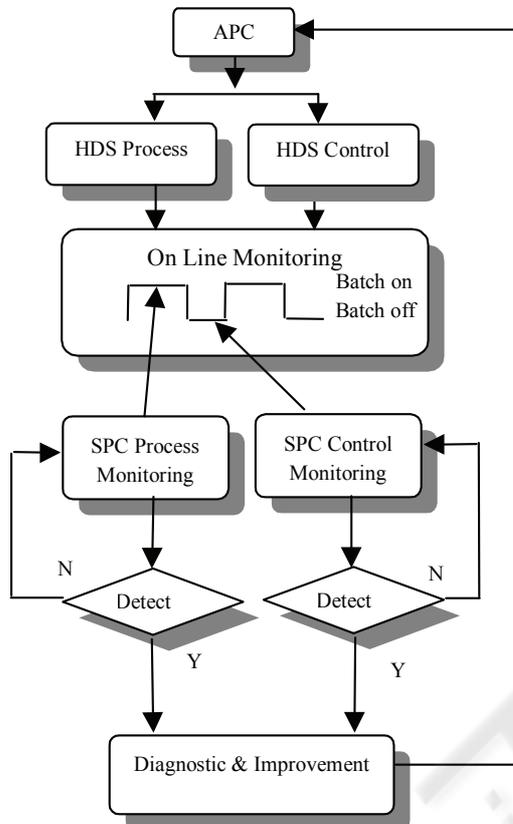


Figure 7: Algorithm of implementation.

When a signal of the control charts is detected we can say if it is due to the process or to a change of the transfer function of one of the automation systems in the manufacturing process. The phase of diagnostic and improvement is able then to correct the process variable signalling ( for ex. adjusting a temperature) or operate directly on the APC that is controlling the automation system changed to compensate for the change, taking advantage of the identification made of the transfer function.

## 8 CONCLUSIONS

In this paper a method for monitoring on line the behaviour of an automation system is presented. The method has the advantage of using the identification of the transfer function of the system, so it can be used on the APC to compensate the changes, and with the advantage for the SPC to provide a direct separation of the possible causes of fault.

Results of simulations are given and finally an implementation schema of the method in a real process is proposed. For further researches we are applying the algorithm proposed in a rolling-mill plant for the production of railways, that has a production cycle on-off like the one described in the paper.

## REFERENCES

- Mason, R.L., Young, J.C., 2002. Multivariate Statistical Process Control with Industrial Application. ASA-SIAM.
- Tsung, F., Apley, D.W., 2002. The dynamic T2 chart for monitoring feedback-controlled processes. *IIE Transactions* (2002) 34,1043-1053 IIE.
- Kourti, T., MacGregor, J.F., 1994. Process analysis, monitoring and diagnosis, using multivariate projection methods. *Chemometrics and Intelligent Laboratory Systems* 28 (1995) 3-21. ELSEVIER.
- Tsung, F., 1999. Improving automatic-controlled process quality using adaptative principal component monitoring. *Qual.Rielab.Engng.Int.* 15:135-142(1999) John Wiley & Sons;Ltd.
- Ljung, L., 1987. System Identification: Theory for the user. Prentice-Hall.