

MODEL AND APPROACHES FOR TENSION OF PARALLEL STRUCTURES WITH ELASTIC JOINTS FOR MICRO AND NANO MANIPULATORS

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Abstract: In this paper piezo actuated micromanipulators are considered with serial-parallel structure including elastic joints. Such structure allows a preliminary tension of the mechanical system in order to eliminate backlashes and to improve the performance of the piezo-actuators. A kinematics model of a serial-parallel structure for local micro manipulators is build here. A pseudo rigid body approach is used, where elastic joints are modelled as revolute joints. A stiffness model is created to estimate the general stiffness of the manipulator by means of reduction the stiffness of all elastic joints. Two approaches are presented here for preliminary tension of parallel manipulator structure: - deflection from the initial manipulator state by introducing of a driving joints motion during the assembly; - preliminary tensioning of the separate elastic joints. The two approaches considered are experimented on the manipulator for cell injection. The values of the mechanical parameters obtained by preliminary tension of the manipulator are pointed out.

1 INTRODUCTION

Micro and nano manipulators are mostly used in biological and microelectronics research, cellular technology, chemistry and investigation of thin films, in atomic force microscopes and scanning tunnelling microscopes.

There are known micromanipulators with piezo actuators (Fatikow, 1996; Kasper, 1998). Piezo actuated micromanipulators with parallel structure are also developed (Lee, 1999). Robots with parallel structure possess many advantages. Their small workspace in the case of cell manipulations is not a disadvantage, since it is enough for the application considered. Mechanisms with closed kinematic chains (Ionescu, 2002; Guergov, 2005, Prusak, 2009) are suitable for high-precision tasks in 3D space. The high accuracy of such mechanical systems is due to very high structural stiffness.

From the other hand in order piezo-ceramic structures to be with high stiffness they can be realized by parallel or a closed structure which has to be tensed. It is possible to use deformation in elastic joints or antagonistic redundant actuators to achieve tension in closed piezo-ceramic structures with desired degree of freedom (DoF).

To predict the displacements of compliant mechanisms with elastic joints the pseudo-rigid-body-model is commonly used (Zhang, 2002). As a rule it models an elastic joint as a revolute joint with a torsion spring attached. The pseudo-rigid-body method is effective and it simplifies the model of compliant mechanisms. To estimate the mechanism stiffness with elastic joints an analytical model is created out taking into account compliances of elastic joints in all axes. The analytical model is describing the relationship between input and output displacements of the mechanism, (Pham, 2005) or computing the stiffness matrix and estimating the stiffness performances of the robot (Carbone, 2006). To increase accuracy of the stiffness matrix identification alternative methodology is developed using advantages of analytical and numerical techniques (Pashkevich, 2009). Such analytical stiffness models of serial-parallel manipulators with elastic joints are analyzed in order to synthesize desired stiffness of the robot end-effector (Chakarov, 2004).

The objective of this paper is to create a stiffness model and to develop approaches for tension of serial-parallel structures with elastic joints for micro

and nano manipulators with application in cellular technology, microelectronics, chemistry etc.

2 KINEMATIC MODEL OF PARALLEL STRUCTURES FOR LOCAL MICRO AND NANO MANIPULATORS

Investigated structures are serial-parallel structures including basic link 0 and some other links 1, ... , n connected in between in a serial chain. The end-effector M is situated in the end link n of this chain, which moves in a v operation space. The driving chains A₁, ..., A_m, with number m, are attached to the basic link 0 and to the end link n forming parallel chains [Chakarov, 2007] as it is shown in Fig.1. The type of the kinematics joints is not shown in Fig.1, as they can be elastically or kinematically ones.

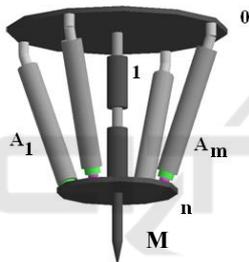


Figure 1: Generalized kinematic scheme of a serial-parallel manipulator.

All joints are modelled as kinematic joints with different number of restrictions, which give 6 DoF for each drive chain. In this way the number DoF of the structure is defined by the number DoF of the serial chain h. Let generalized parameters are accepted to be the parameters of the relative motions in all joints - elastic and non-elastic of the structure, presented by (k x 1) vector

$$\theta = [q; q^1]^T \quad (1)$$

Where

$$q = [q_1, \dots, q_h]^T \quad (2)$$

is an (h x 1) vector of the generalized coordinates in the joints of the main serial chain with h DoF and

$$q^1 = [w; l]^T \quad (3)$$

is a (6m x 1) vector of coordinates in the joints of the actuator chains with number m.

Above

$$w = [w_1, \dots, w_{5m}]^T, \quad (4)$$

is an (5m x 1) vector of coordinates in the passive joints of the actuator chains, and

$$l = [l_1, \dots, l_m]^T, \quad (5)$$

is an (m x 1) vector of coordinates in the motor linear joints of the actuator chains.

Let the Cartesian coordinates of the end effector M are denoted as

$$X = [X_1, \dots, X_v]^T, v \leq 6 \quad (6)$$

The relation between the parameters of the basic serial chain (2) and the parameters of the end effector (6) is known as a direct problem of the kinematics of the serial chain. This problem on the level of displacements and velocities is presented by the equations $X = \Psi(q)$ and

$$\dot{X} = J\dot{q} \quad (7)$$

where $J = [\partial X / \partial q]$ is the (v x h) matrix of Jacoby.

In the parallel structure each closed loop implies the appearance of a connection between the generalized parameters (1). These connections are expressed by 6m scalar functions for the structure including m parallel loops: $\Psi_i(\theta) = 0, i = 1, \dots, 6m$. The differentiation of above equations gives the relation

$$H_q \frac{dq}{dt} + H_w \frac{\partial w}{\partial q} \frac{\partial q}{\partial t} + H_l \frac{\partial l}{\partial q} \frac{\partial q}{\partial t} = 0 \quad (8)$$

The matrix of partial derivations H_q, H_w and H_l with size (6m x h), (6m x 5m) and (6m x m) allows to produce the summarized matrix of the partial derivatives

$$D = \left| \frac{\partial q}{\partial q}; \frac{\partial w}{\partial q}; \frac{\partial l}{\partial q} \right|^T = |E; W; L|^T \quad (9)$$

where E is unitary (h x h) matrix, W is a (5m x h) matrix and L is a (m x h) matrix, or $D = |E; H|^T$.

According to (8) we can reduce the (6m x h) matrix

$$H = |W; L|^T = \frac{\partial q^1}{\partial q} = -|H_w; H_l|^{-1} H_q \quad (10)$$

where $|H_w; H_l|$ is a (6m x 6m) invertible matrix.

Using matrix (10) we have the relations between generalized velocities:

$$\dot{\mathbf{q}}_1 = \mathbf{H}\dot{\mathbf{q}} \quad (11)$$

$$\dot{\mathbf{w}} = \mathbf{W}\dot{\mathbf{q}} \quad (12)$$

$$\dot{\mathbf{l}} = \mathbf{L}\dot{\mathbf{q}} \quad (13)$$

The above equations allow determining the velocities $\dot{\mathbf{q}}_1$ with dimension $6m$ as a function of the generalized velocities $\dot{\mathbf{q}}$ with dimension equal to the DoF h of the structure.

When the number of parameters (5) is equal to the DoF $m = h$, these parameters can be selected as independent parameters. In relations (13) \mathbf{L} is a $(\mathbf{h} \times \mathbf{h})$ matrix and inverse relation is possible:

$$\dot{\mathbf{q}} = \mathbf{L}^{-1}\dot{\mathbf{l}} \quad (14)$$

Equations (7) and (14) allow determining the velocities of end-effector, while equations (11) and (14) - the velocities of passive joints, as function of velocities of linear actuator joints $\dot{\mathbf{l}}$:

$$\dot{\mathbf{X}} = \mathbf{J}\mathbf{L}^{-1}\dot{\mathbf{l}} \quad (15)$$

and

$$\dot{\mathbf{q}}_1 = \mathbf{H}\mathbf{L}^{-1}\dot{\mathbf{l}} \quad (16)$$

By micromanipulations the above equations give the relations between small motions of microactuators $\Delta\mathbf{l}$, small motions of the end-effector $\Delta\mathbf{X}$ and small motions in passive joints $\Delta\mathbf{q}_1$: $\Delta\mathbf{X} = \mathbf{J}\mathbf{L}^{-1}\Delta\mathbf{l}$ and $\Delta\mathbf{q}_1 = \mathbf{H}\mathbf{L}^{-1}\Delta\mathbf{l}$.

3 STIFFNESS MODEL OF SERIAL – PARALLEL STRUCTURES FOR MICRO AND NANO MANIPULATORS

Denote by $\mathbf{P} = [\mathbf{P}_1, \dots, \mathbf{P}_v]^T$ the $(v \times 1)$ vector of the external forces and torques applied to the end-effector, corresponding to Cartesian coordinates (6). Denote by $\mathbf{Q} = [\mathbf{Q}_1, \dots, \mathbf{Q}_h]^T$ the $(h \times 1)$ vector of the generalized forces and torques in the joints of the main chain corresponding to the general coordinates (2). According to the principle of virtual work and equation (7), the connection between forces \mathbf{P} and \mathbf{Q} is as follows:

$$\mathbf{Q} = \mathbf{J}^T \mathbf{P} \quad (17)$$

Denote by $\mathbf{F}_q = [\mathbf{F}_{q1}; \dots; \mathbf{F}_{qh}]^T$ and $\mathbf{F}_w = [\mathbf{F}_{w1}; \dots; \mathbf{F}_{w5m}]^T$ ($h \times 1$) and $(5m \times 1)$ vectors

of the forces and torques in the elastic joints, corresponding to coordinates (2) and (4). Denote by

$\mathbf{F}_1 = [\mathbf{F}_{11}; \dots; \mathbf{F}_{1m}]^T$ the $(m \times 1)$ vector of the driving

forces in the linear joints correspond to the coordinates (5). Above vectors can be summarized in the $(h + 6m) \times 1$ vector of forces and torques, corresponding to the coordinates (1) $\mathbf{F} = [\mathbf{F}_q; \mathbf{F}_w; \mathbf{F}_1]^T$.

According to the principle of virtual work and the equation (12), (13) the relation between forces \mathbf{F} and generalized forces \mathbf{Q} , using summarized matrix (9), is as follows:

$$\mathbf{Q} = \mathbf{D}^T \mathbf{F} \quad (18)$$

$$\mathbf{Q} = \mathbf{F}_q + \mathbf{W}^T \mathbf{F}_w + \mathbf{L}^T \mathbf{F}_1 \quad (19)$$

Equations (17) and (19) produce

$$\mathbf{J}^T \mathbf{P} = \mathbf{F}_q + \mathbf{W}^T \mathbf{F}_w + \mathbf{L}^T \mathbf{F}_1 \quad (20)$$

Differentiation of above equation with respect to parameters (2) and neglect the second partial derivatives, gives

$$\mathbf{J}^T \frac{\partial \mathbf{P}}{\partial \mathbf{X}} \mathbf{J} = \frac{\partial \mathbf{F}_q}{\partial \mathbf{q}} + \mathbf{W}^T \frac{\partial \mathbf{F}_w}{\partial \mathbf{w}} \mathbf{W} + \mathbf{L}^T \frac{\partial \mathbf{F}_1}{\partial \mathbf{l}} \mathbf{L} \quad (21)$$

Considering micromanipulator structure as a system with concentrated compliance in the joints [Chakarov, 2004] gives

$$\mathbf{K} = \mathbf{J}^{-T} [\mathbf{K}_q + \mathbf{W}^T \mathbf{K}_w \mathbf{W} + \mathbf{L}^T \mathbf{K}_1 \mathbf{L}] \mathbf{J}^{-1} \quad (22)$$

where $\mathbf{K} = \partial \mathbf{P} / \partial \mathbf{X}$ is $(v \times v)$ matrix of the Cartesian stiffness of the end effector; $\mathbf{K}_q = \partial \mathbf{F}_q / \partial \mathbf{q}$ is diagonal $(h \times h)$ matrix of the shaft stiffness in the joints of the main serial chain; $\mathbf{K}_w = \partial \mathbf{F}_w / \partial \mathbf{w}$ is diagonal $(5m \times 5m)$ matrix of the shaft stiffness in the passive joints of the driving chains; $\mathbf{K}_1 = \partial \mathbf{F}_1 / \partial \mathbf{l}$ is diagonal $(m \times m)$ matrix of the shaft stiffness in the driving joints.

4 APPROACHES FOR PRELIMINARY TENSIONING OF PARALLEL MICRO MANIPULATORS WITH ELASTIC JOINTS

A preliminary tensioning of the mechanical micromanipulation system is necessary in order to eliminate the backlash and to improve the

performance of the piezo-actuators. When only joints of class five are used for the modeling the mechanical system, the number of all joints is equal to the number k of the generalized system coordinates. In a case when number m of the driving joints is equal to the number of DoF h ($m=h$), then number of all the remaining joints is $(k - h)$.

The following two approaches can be used for tensioning of the manipulator:

- deflection from the initial manipulator state by $m = h$ driving joints motion introduced in the assembly;
- preliminary tensioning of the separate elastic joints with number j , ($k-h \geq j \geq h$).

4.1 Tensioning by Means of Deflection from the Initial State

This can be achieved by means of an assembly deflection δl in the driving joints, which leads to deflection in all the system joints according to (14), (12) and deflection of the end-effector according to (7) defined by the equations:

$$\delta q = L^{-1} \delta l \quad (23)$$

$$\delta w = W \delta q = W L^{-1} \delta l \quad (24)$$

$$\delta X = J \delta q = J L^{-1} \delta l \quad (25)$$

These deflections lead to elastic joints forces defined by the equations:

$$F_q = k_q \delta q = k_q L^{-1} \delta l \quad (26)$$

$$F_w = k_w \delta w = k_w W L^{-1} \delta l \quad (27)$$

where k_q and k_w are stiffness matrices of the passive joints of the basic serial chain and of the driving chains, respectively. The tensioned elastic system according to (19) is in a static equilibrium:

$$Q = F_q + W^T F_w + L^T F_1 = 0 \quad (28)$$

The diagonal matrices k_q and k_w contain non-zero components, responding to elastic joints and zero components responding to kinematic joints. The number of non-zero components j must be bigger or equal to the DoF $j \geq h$ in order to achieve full degree of tension of all the actuators and limbs within the system. Equation (28) allows definition of the forces of the driving joints F in number h as a function of the forces F_q , F_w in number $j \geq h$

$$F_1 = -L^{-T} [F_q + W^T F_w] \quad (29)$$

4.2 Tensioning by Deformations in the Elastic Joints

In the manipulator structure with m driving joints there are $k - m$ passive joints, which can be elastic. Because the driving joints $m = h$, by means of which the piezo-actuators are modelled are hundreds of times more rigid than the elastic manipulator joints, it is accepted that the system has 0 DoF. The tensioning of the elastic joints does not lead to a change in the manipulator position, but only in a change of the internal forces. For the actuator tensioning, the number of the elastic joints j must be bigger than the number of the DoF $j \geq h$. The preliminary joint deformations can be defined by the vectors:

$$\delta q^0 = [\delta q^0_1, \dots, \delta q^0_h]^T \quad (30)$$

$$\delta w^0 = [\delta w^0_1, \dots, \delta w^0_{5m}]^T, \quad (31)$$

where the components of which corresponding to non-elastic (kinematic) joints are equal to 0.

The joint stiffnesses are represented by the diagonal matrices k_q and k_w , which contain non-zero components corresponding to the elastic joints and zero components connected to the kinematic joints.

The preliminary deflections lead to appearance of forces in the elastic joints defined by equalities:

$$F_q = k_q \delta q^0 \quad (32)$$

$$F_w = k_w \delta w^0 \quad (33)$$

The driving elastic joints forces are in a static equilibrium (19):

$$Q = F_q + W^T F_w + L^T F_1 = 0 \quad (34)$$

The upper equation defines the links among all the joint forces and allows the derivation of the driving joints forces in number h as a function of the elastic forces in number $j \geq h$

$$F_1 = -L^{-T} [F_q + W^T F_w] \quad (35)$$

The forces in the driving joints can lead to small deflections in those joints due to the piezo-actuators high stiffness. Those deflections by the two approaches can be defined by the equality:

$$\delta l^0 = \frac{F_1}{k_1} \quad (36)$$

where the diagonal matrix k_1 includes the stiffness of the piezo-actuators, and δl^0 are the deflections in the driving joints after the system tensioning. Resulting deflection of the end effector can be received according to equation (15).

5 NUMERIC EXPERIMENTATION OF TENSIONING APPROACHES IN ROBOTS FOR CELL INJECTION

A robot has been designed to perform automatic cell injection. The cells in the range of 10-30 [μm] are preliminary positioned in a matrix G. Local robot structure has serial-parallel structure with 3 DoF as shown in Fig.2. Base 0, elastic joint J_3 , manipulator body 1, actuator A_3 , and working tool 2 with end-effector M form a serial chain. Actuators A_1 and A_2 are located perpendicularly to the manipulator body 1, and they are linked with the base 0 by means of elastic joints J_1 and J_2 , thus forming parallel chains. The actuators are fixed to the body 1 via universal joints. Parallel structure comprising actuators A_1 and A_2 perform orientation motions, while the actuator A_3 performs injection through the pipette 2 attached to it. The main dimensions of the manipulator are $a_1=a_2=0.073[\text{m}]$, $b_1=0.030[\text{m}]$, $b_2=0.180[\text{m}]$ as shown in Fig.2. Piezoactuators and elastic joints used have parameters specified in Table 1.

Table 1: Parameters of the elements used.

Elements	Travel [μm]	Resolution [nm]	Axial stiffness [N/ μm]	Reduced Axial stiffness [N/ μm]	Angular stiffness [N/rad]
A_1, A_2	30	0.6	27	23.79	
A_3	60	1.2	15	13.95	
J_2, J_3			200		40

The end effector stiffness and the characteristics of the preliminary tensing of the parallel structure can be found. Since the manipulator under consideration is assembled with a special rectangle configuration we can easily derive the scalar equalities for the characteristic components using the matrix equalities (22), (23), (24), (25), (29). Software application based on these matrix equalities is developed using Microsoft Visual Studio.Net Express Edition and C++.

To find an estimation of a stiffness component along axis X, the software application substitutes the respective matrices in equality (22) for $K_w=40[\text{Nm/rad}]$, $J=b_2$, $K_l=23.788[\text{N/m}]$, $W=b_1/a_2$, $K_q=40[\text{Nm/rad}]$, $L=b_1$. Performing the respective calculations, we find for the three additives in (22) that $K_x=1235+209+660778=0.662 \cdot 10^6 [\text{N/m}]$.

Similarly, the rest of the end-effector stiffness components can be found: $K_y=0.662 \cdot 10^6 [\text{N/m}]$, $K_z=13.95[\text{N/m}]$. As seen, the influence of piezo

actuators to the joint stiffness is hundred times larger than the rest of the elements.

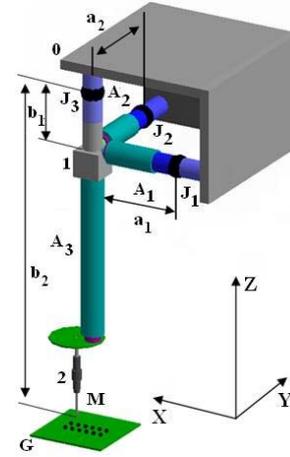


Figure 2: Microrobot for cell injection.

To apply preliminary tension by the actuator, following the approach outlined in Sub-paragraph 3.1., actuators A_1 and A_2 in the parallel structure should deflect by δl_1 and δl_2 , so that the elastic joint deflection J_3 should not exceed $\pm 0.5^\circ$, which is the admissible arbitrary rotation angle. Thus, $\delta q_1=\delta q_2=0.008726[\text{rad}]$.

Using scalar equalities corresponding to (23), (24), (25), (29) as outlined above, we find the components of the elastic joint deflections and those of the actuator tension forces – see Table 2.

To attain preliminary tension in the manipulator as outlined in Sub-paragraph 3.2, the deflections of the three elastic joints J_1 , J_2 and J_3 should be less than or equal to the admissible angles of rotation $\delta w_{11}^0=\delta w_{21}^0=0.008726[\text{rad}]$, $\delta q_1^0=\delta q_2^0=0.008726 [\text{rad}]$.

Considering equality (35) and its scalar forms, we find the tension forces of actuators A_1 and A_2 , $F_{l1} = F_{l2} = -16.42[\text{N}]$. These forces are larger than the tension forces found by applying the first approach, due to the tension of joints J_1 , J_2 that is equal to the admissible limit. Both cases of tension yield small actuator deformations which can be found using equality (36).

Thus, considering the first case, those deformations are $\delta l_1^0 = \delta l_2^0 = -0.572 \cdot 10^{-6}[\text{m}]$, while for the second case we have $\delta l_1^0 = \delta l_2^0 = -0.690 \cdot 10^{-6} [\text{m}]$. The deformations yield deflection of the end effector with respect to its initial position, equal to $\delta X^0=\delta Y^0= -3.432 \cdot 10^{-6}[\text{m}]$ in the first case and to $\delta X^0=\delta Y^0= -4.142 \cdot 10^{-6}[\text{m}]$ for the second case.

Table 2: Calculated values of the mechanical parameters.

[m]	δq_1 [rad]	δq_2 [rad]	δw_{11} [rad]	δw_{12} [rad]	δw_{21} [rad]	δw_{22} [rad]	δX [m]	δY [m]	F_{l1} [N]	F_{l2} [N]
$\delta l_1 = 0.263 \cdot 10^{-3}$	$8.72 \cdot 10^{-3}$	0	0	0	$3.589 \cdot 10^{-3}$	0	$1.572 \cdot 10^{-3}$	0	-13.6	0
$\delta l_2 = 0.263 \cdot 10^{-3}$	0	$8.72 \cdot 10^{-3}$	$3.589 \cdot 10^{-3}$	0	0	0	0	$1.572 \cdot 10^{-3}$	0	-13.6

6 CONCLUSIONS

Piezo actuated micromanipulators with serial-parallel structure including elastic joints are subject of this paper. A kinematic model of the micro manipulators is build using a pseudo rigid body method, where elastic joints are modelled as revolute joints.

A stiffness model is created to estimate the manipulator stiffness by stiffness reduction of all elastic joints. In order to eliminate the backlash and to improve the performance of the piezo-actuators the parallel structure of the manipulator must be tensed preliminary. Two approaches are proposed and presented here for preliminary tension of parallel manipulator structure:

1. Deflection from the initial manipulator state by driving joints motion implemented in the assembly;
2. Preliminary tensioning of separate elastic joints.

The first approach is easier for realization as the deviations are performed only in the driving joints. The elastic joints are tensed to a different degree.

Tensioning of all the elastic joints is realized to the admissible values by the second approach. Thus, the maximal values of the tensioning forces are achieved

The first approach is chosen for the robot developed for cell injection. The actuators A_1 and A_2 are mounted with deviations related to the values shown in the first column of Table 2. The robot with mechanical construction preliminary tensioned experimented achieves motion along the axes X, Y и Z as follows: $\Delta X = \Delta Y = 180$ [μm], $\Delta Z = 60$ [μm]. Minimal displacement obtained experimentally is 30[nm]

Further numeric experiments and tests are under consideration for more stiff elastically joints and structures with arbitrary location of the actuators.

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REFERENCES

- Fatikow, S., Munassypov, R., 1996, An Intelligent Micromanipulation Cell for Industrial and Biomedical Applications Based on a Piezoelectric Microrobot, *5th Int. Conf. on MicroElectro, Opto, Mechanical Systems and Components*, Berlin, 17-19 Sept., pp. 826-828.
- Kasper, R.; Heinemann, W.; Wagner, A., 1998, Modelling and Control for Piezoelectric Actuators for High Speed Applications. *4th Int. Conf. on Motion and Vibration Control*, Vol. 1, ETH Zürich, 231-236.
- Lee, K., Arjunan, S., 1999, A three-degree-of-freedom micromotion in-parallel actuated manipulator, *IEEE Trans. on "Robot. and Autom."*, vol. 7, 238-247.
- Ionescu Fl., Kostadinov, K., 2002, Piezo actuated micro robot for micro and nano manipulations, *ARA Journal, Vol. 2000-2002, Nr.25-27, Montréal*, 98-103.
- Guergov, S., 2005, Multifunctional technological system with complex functions, reconfigurable structure, high level of integration and self-organizing elements. *Proc. of V Int. Congress "Computer Science for Design and Technology"*, STANKIN, Moscow, 31-36.
- Prusak, D., Tadeusz Uhl, 2009, Novel Type of Hybrid 3-DOF Micromanipulator with Piezoelectric Actuators - *Mechanical Construction and Simulations, Solid State Phenomena, Vols. 147-149*, pp25 -30.
- Zhang, W., Zou, J., Watson, L., Zhao, W., 2002, The constant- jacobian method for kinematics of a three DOF planar micro-motion stage, *Journ. of Robotic Systems 19(2)*. 63-72.
- Pham, H., Chen, I., 2005, Stiffness modeling of flexure parallel mechanism, *Precision. Engineering 29*, 467-478.
- Carbone, G., Lim, Hun-ok, Takamishi, A., Ceccarelli, M., 2006, Stiffness analysis of biped humanoid robot, *Mechanism and Machine Theory*, 41, 17-40.
- Pashkevich, A., Chablat, D., Wenger, Ph., 2009, Stiffness analysis of overconstrained parallel manipulators, *Mechanism and Machine Theory*, Vol. 44, 966-982.
- Chakarov, D., 2004, Study of the Antagonistic Stiffness of Parallel Manipulators with Actuation Redundancy, *Mechanism and Machine Theory*, Vol. 39/6, 583-601.
- Chakarov, D., Abed Al-Wahab, M., Kasper, R., Kostadinov, K., 2007, Synthesis of tense piezo structures for lokal mikro & nano manipulations. 8th Magdeburg days of mechanical engineering, Oct.10-11, Magdeburg, pp.173-180.