ANT PAGERANK ALGORITHM

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Keywords: Search engine, PageRank algorithm, Ranking algorithms, Ant algorithm.

Abstract: The amount of global information in the World Wide Web is growing at an incredible rate. Millions of results are returned from search engines. The rank of pages in the search engines is very important. One of the basic rank algorithms is PageRank algorithm. This paper proposes an enhancement of PageRank algorithm to speed up the computational process. The enhancement of PageRank algorithm depends on using the Ant algorithm. On average, this technique yields about 7.5 out of ten relevant pages to the query topic, and the total time reduced by 19.9 %.

1 INTRODUCTION

The amount of information on the web is growing rapidly, as well as the number of new users inexperienced in the art of web search engine. World Wide Web search engines have become the most heavily used online services, with millions of searches performed each day (B. Davision, 2000). The web search engines use the structure present in hypertext to provide much higher quality search results (S. Brain, 1998).

The rank of pages is an integral component of any search engine. In the context of the web search engines, the role of ranking becomes even more important. The most important researches proposing **Link Analysis Ranking** (A. Borodin, 2005) are found in Kleinberg (J. Kleinberg, 1999), and Brin and Page (S. Brain, 1998).

Link Analysis Ranking can be described as the use of hyperlink structure for the purpose of ranking web documents. Link Analysis Ranking operates on the **graph** representation of hyperlinked web documents. The hyperlink graph is based on the representation of a web page as a **node** and hyperlink between pages as a **directed edge**. The goal of Link Analysis Ranking is to extract this information, and use it to determine a particular weight for every page, and use these weights to rank the web documents. This paper proposes an enhancement to speed up the computation of **PageRank algorithm** (S. Brain, 1998) by using **Ant algorithm**.

The rest of the paper is organized as follows. Section two presents an overview on the Link Analysis Ranking Algorithms. Section three explains Ant System. Section four explains the proposed Ant PageRank algorithm. Section five presents the experimental data preparation. Section six presents experimental results. Finally, Section seven concludes the paper.

2 LINK ANALYSIS RANKING ALGORITHMS

All the Link Analysis Ranking algorithms start with a collection of Web pages to be ranked. These algorithms proceed as follows:

- **Extraction**: extracting the hyperlinks between the pages
- **Construction**: constructing the underlying hyperlink graph. The hyperlink graph is constructed by creating a node for every Web page, and a directed edge for every hyperlink between two pages.
- Calculation of Node Weight: The graph is given as input to the Link Analysis Ranking algorithms. The algorithms operate on the

Zaki Abdo M., Ahmed Ismail M. and Ibraheem Eladawy M. (2009) ANT PAGERANK ALGORITHM.

In Proceedings of the 11th International Conference on Enterprise Information Systems - Artificial Intelligence and Decision Support Systems, pages 74-80

DOI: 10.5220/0001863400740080 Copyright © SciTePress graph, and calculate a weight for each Web page. This weight is used to rank the pages.

In the following two subsections; the **In-degree algorithm** (P. Tasparas, 2004) and **PageRank algorithm** (S. Brain, 1998) will be briefly described as examples of Link Analysis Ranking algorithms.

2.1 In-degree Algorithm

The In-degree algorithm depends on the popularity of pages. The number of pages that link to this page measures the popularity of a page. It ranks pages according to their **in-degree pages** (the number of pages that link to the page).

2.2 The PageRank Algorithm

The algorithm depends on the computation of the *PageRank weight* of all pages in the graph by Eq. (1). The *PageRank weight* of a page A is given as follows:

$$PR(A) = (1 - d) + d\left(\frac{PR(T_1)}{C(T_1)} + \frac{PR(T_2)}{C(T_2)} + \dots + \frac{PR(T_n)}{C(T_n)}\right)$$
(1)

$$= (1-d) + d \sum_{j \in B(A)} \frac{PR(j)}{C(j)}$$

where:

- PR (A) is the PageRank weight of the page A.
- Page A has pages T₁, T₂ ... T_n which point to it, as shown in Figure 1.
- The parameter *d* is a damping factor which can be set between (0, 1); usually set *d* to 0.85.
- C (T) is the number of links going out of page T.
- B (A) the set of nodes that point to node A (Backwards links).



Figure 1: The PageRank graph.

The intuition underlying the In-degree algorithm is that a page has a good weight is a page that is pointed to by many nodes in the graph. Brin and Page (S. Brain, 1998) extended this idea further by observing that not all pages have the same weight. Links from pages of high quality should confer more weight. It is not only important how many pages point to a page, but also what the quality (value of the page weight) of these pages is.

Therefore, Brin and Page (S. Brain, 1998) had proposed a one-level weight propagation scheme, where a page has a good weight is the one that is pointed by many pages have a good weights. Figure 2 shows the PageRank algorithm.

Due to the huge size of actual web, an approximate iterative computation is usually applied to calculate the PageRank weight. This means that each page is assigned an initial starting value and the PageRank weights of all pages are then calculated in several computation circles based on Eq. (1). The minimum PageRank of a page is given by (1 - d); while the maximum PageRank is determined as dN + (1 - d), where N is the number of pages. This maximum PageRank weight can theoretically achieved, only when all web pages solely link to one page, and this page also solely links to itself (Nan Ma, 2008).

PageRank Algorithm

Initialize all PageRank weights to 1 Repeat until the PageRank weights are convergent For every node i

$$PR(i) = (1 - d) + d \sum_{j \in B(i)} \frac{PR(j)}{C(j)}$$



The PageRank algorithm uses a matrix W to represent the hyperlink graph. where:

- Matrix W represents the hyperlink graph
- "1" in row i and column j means that page j points to page i.
- "0" in row i and column j means that page j does not point to page i.
- Matrix W is N * N, where N is the number of nodes.

3 ANT SYSTEMS

Ant System introduced by Dorigo (Dorigo, 1991). Dorigo's artificial ants (called *ants*) have some major differences with real (natural). First, ants have a memory. Second, ants are not completely blind. While natural ants rely on chemical signals to navigate. Finally, due to obvious constraints imposed by the architecture of current computers, artificial ants live in a world where time is discrete (Islam, 2005). In the case of Travel salesman Problem (TSP)

- Ants have a memory; this memory is used to store a list of previously visited cities.
- Ants are not completely blind. Ants are aware of the distance between cites.

To examine the ant algorithms, apply them to the well-known travelling salesman problem (TSP) (E. L. Lawler, 1985)

3.1 Ant System and Travel Salesman Problem (TSP)

Given a set of n towns, the TSP problem can be stated as the problem of finding a minimal length closed tour that visits each town once. The distance between town i and town j is calculated by Euclidean distance equation (2).

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$
 (2)

An instance of the TSP problem is given by a weighted graph (N,E), where N is the set of towns and E is the set of edges between towns, weighted by the distances. assume $b_i(t)(i = 1,...,n)$ is the number of ants in town i at time t. Then the total number of ants is calculated by equation (3)

$$\mathbf{m} = \sum_{i=1}^{n} \mathbf{b}_i(\mathbf{t}) \tag{3}$$

Each ant is a simple agent with the following characteristics:

- When going from town i to town j it lays a substance, called *trail*, on edge (i,j);
- It chooses the town to go to with a probability that is a function of the town distance and of the amount of trail present on the connecting edge.
- Each ant has a data structure, called a *tabu list*. That memorizes the towns already visited up to time t and forbids the ant to visit them again before a tour has been completed. When a tour is completed the tabu list is emptied and the ant is free again to choose its way. The vector containing the tabu list of the k-th ant is tabu_k and tabu_k(s) is the s-th element of the tabu list of

the k-th ant. Let $\tau_{ij}(t)$ be the intensity of trail on edge (i,j) at time t. At each iteration of the algorithm trail intensity becomes

$$\tau_{ij}(t+1) = \rho \cdot \tau_{ij}(t) + \Delta \tau_{ij}(t,t+1)$$
(4)

where ρ is a coefficient such that $(1 - \rho)$ represents the evaporation of trail. The coefficient ρ must be set to a value <1 to avoid unlimited accumulation of trail

$$\Delta \tau_{ij}(t,t+1) = \sum_{k=1}^{m} \Delta \tau_{ij}^{k}(t,t+1)$$
 (5)

where $\Delta \tau_{ij}^{k}(t, t + 1)$ is the quantity per unit of length of trail substance (pheromone in real ants) laid on edge (i,j) by the k-th ant between time t and t+1. The transition probability from town i to town j for the k-th ant is

$$p_{ij}^{k}(t) = \begin{cases} \frac{\left[\tau_{ij}(t)\right]^{\alpha} \cdot \left[\eta_{ij}\right]^{\beta}}{\Sigma_{j \in allowed} \left[\tau_{ij}(t)\right]^{\alpha} \cdot \left[\eta_{ij}\right]^{\beta}} & \text{if } j \in allowed \\ 0 & \text{otherwise} \end{cases}$$
(6)

where **allowed** = {j is not in **tabu**_k} and η_{ij} is the visibility of town j from town i, which is simply the value 1/ d_{ij} . Where α and β are parameters that allow a user to control the relative importance of trail versus visibility. Therefore the transition probability is a tradeoff between visibility (which says that close towns should be chosen with high probability) and trail intensity (that says that if on edge (i,j) there has been a lot of traffic then it is highly desirable, thus implementing the autocatalytic process).

Different choices about how to compute $\Delta \tau^k_{ij}(t, t + 1)$ and when to update the $\tau_{ij}(t)$ cause different Instantiations of the ant algorithm. In the next two sections Dorigo (Dorigo, 1991) present the three approaches. Dorigo (Dorigo, 1991) used as experimental test-bed for ideas, namely Ant-density, Ant-quantity, and Ant-cycle.

3.2 The Ant-density and Ant-quantity Approaches

Initially, two approaches where developed by Dorigo (Dorigo, 2000) to exploit his Ant System heuristic named : **Ant-density** and **Ant-quantity**. In the Ant-density approach a quantity Q1 of trail for every unit of length is left on edge (i,j) every time an ant going from i to j; in the Ant-quantity approach an ant going from i to j leaves a quantity Q2/dij of trail for every unit of length.

Therefore, in the Ant-density approach $\Delta \tau_{ij}^{k}(t, t+1) = \begin{cases} Q_{1} & \text{if } k - \text{th ant goes from i to j between t and } t+1 \\ 0 & \text{Otherwise} \end{cases}$ (7) And in the Ant-quantity approach

 $\Delta \tau_{ij}^{k}(t, t+1) = \begin{cases} \frac{Q_2}{d_{ij}} & \text{if } k-th \text{ ant goes from i to j between t and } t+1\\ 0 & \text{Otherwise} \end{cases}$ (8)

where: Q_1 and Q_2 are the same quantity of pheromone. Thus, an increase in trail intensity on edge (i,j) when an ant goes from i to j is independent of dij in the Ant-density approach and is inversely proportional to dij in the Ant-quantity approach (i.e., shorter edges are made more desirable by ants in the Ant-quantity approach, thus further reinforcing the visibility factor in equation (6)).

3.3 The Ant-cycle Approach

The approach introduced a major difference with respect to the two previous systems. Here $\Delta \tau_{ij}^k$ is not computed at every step, but after a complete tour (n steps). The value of $\Delta \tau_{ij}^k$ (t,t+n) is given by

$$\Delta \tau_{ij}^{k}(t,t+n) = \begin{cases} \frac{Q_3}{L^k} & \text{if } k-\text{th ant uses edge } (i,j)\text{in its tour} \\ 0 & Otherwise \end{cases}$$
(9)

where Q3 is a constant and L_k is the tour length of the k-th ant. The entire Ant-Cycle approach is demonstrated in (Islam, 2005)



Figure 3: The Path of one ant with high in-degree selected from out-degree pages.

4 PROPOSED ANT PAGERANK ALGORITHM

The main objective behind Ant PageRank algorithm is reducing the execution time for computing the PageRank weight of the pages. The PageRank weight of the page depends on the weight of the indegree pages as shown in equation (1). The proposed algorithm locates ants in the pages without in-degree pages, and release the ant move on the graph (as Figure 3) and calculate the PageRank weight of the pages in the tabu of the ant.

The ant moves to one page from out-degree page. This paper introduces novel PageRank approaches to be used the conjunction with the Ant algorithm.

- 1- Ant PageRank approach 1: Locate one ant in every page without in-degree pages. The ant moves to one page from out-degree pages as random selected.
- 2- Ant PageRank approach 2: Locate one ant in every page without in-degree pages. The ant moves to one page from out-degree page as high in-degree page selected.
- 3- Ant PageRank approach 3: Locate two ants in every page without in-degree pages. Every ant move to one page from out-degree page as random selected.

Place N ant on every page without In-degree Pages.

- Approach 1: N=one Ant
- Approach 2: N=one Ant
- Approach 3: N=two Ant
- Every ant will start its tour by selecting the next page to be visiting from out-degree pages as:
 - Approach 1: Random
 - Approach 2: Higher in-degree pages
 - Approach 3: Random
- 3. Add next page visited to the ant's tabu list.
- 4. Repeat step 2 for every ant until the page being visit exists in the ant story (a cycle is found) or visit the page without out-degree Page.
- 5. When step 4 is completed, applying the equation (1) of the element in every ant's tabu list.
- After step 5 is completed the voting determine the best pages
- 7. Return the top ten pages.

Figure 4: Ant PageRank algorithm.

Figure 4 shows the Ant PageRank algorithm. The algorithm divided into three approaches:

- Approach 1: Locate one ant in every page without in-degree and select the next page as random.
- Approach 2: Locate one ant in every page without in-degree and select the next page as

higher in-degree page.

- Approach 3: Locate two ants in every page without in-degree and select the page as random.

The change of the number of ants and the method of selecting the next page of the ant are used to get the best time and high top ten pages for matching of the result in the PageRank algorithm.

5 EXPERIMENTAL DATA PREPARATION

This section presents a brief description of experimental results of the proposed algorithm. The experimental results data are text files (P.Tsaparas, 2008). These text files contain information about queries used in the experimental phase. These text files represent the following 33 queries namely:

"abortion", "affirmative action", "alcohol", "amusement parks", "architecture", "armstrong". "automobile industries", "basketball", "blues", "cheese", "classical guitar", "complexity", "computational complexity", "computational "death penalty", geometry", "genetic", "geometry", "globalization", "gun control", "iraq war", "jaguar", "Jordan", "movies", "national parks". ''net censorship", "randomized algorithms", "recipes", "roswell", "search engines", "shakespeare", "table tennis", "vintage cars", "weather".

Each query represented by three text files named "nodes", "adj_list", and "inv_adj_list". The nodes.txt file is formatted as follows:

- Number of pages
- Information of each page, each page is described as follows [Page ID, http address of the page, page title, number of in-degree pages, Number of out-degree pages]

The *Adjacency List file* contains a list of out-degree page IDs for each page. An example of a page entry can be describes as follow:

1:20 500 6

This means that the page with ID= 1, points to the pages with $IDs = \{20, 500, 6\}$.

The *Inverted Adjacency List file* contains a list of in-degree page IDs for each page. An example of a page entry can be describes as follow:

20:1 5 80 -1

This means that the page with ID =20, is pointed to by the pages with IDs = $\{1, 5, 80\}$.

Table 1: Array of structure to represents the data of the graph.

Node 1 ID	Node 1 Indegree Number	Node 1 Outdegree Number	[] Node 1 Indegree	[] Node 1 Outdegree			
Node 2 ID	Node 2 Indegree Number	Node 2 Outdegree Number	[] Node 2 Indegree	[] Node 2 Outdegree			
				<u></u>			
Node N ID	Node N Indegree Number	Node N Outdegree Number	[] Node N Indegree	[] Node N Outdegree			

The implementation steps of the proposed algorithm are detailed as follow:

- 1- Data preprocessing step.
- 2- Executing proposed algorithm.
 - 3- Comparing the results.

The preprocessing step includes the creation of an *array of structure* to represent the data. The structure is as follows:

- NodeID: a unique identifier for the Page
- NodeIndegreeNumber: Number of in-degree Pages
- NodeOutdegreeNumber: Number of outdegree Pages
- [] NodeIndegree: Array of NodeID of indegree pages
- [] NodeOutdegree: Array of NodeID of outdegree pages

where: Length of the array is number of pages.

Table 1 shows this structure. This structure only needs to store ID of in-degree pages and ID of out-degree pages of each node. This structure replaced by the matrix W_{n*n} used in the classical PageRank algorithm as explained in subsection 2-2.

6 EXPERIMENTAL RESULTS

This section illustrates experiment results of the proposed algorithm. The experiments done by running the proposed algorithm on the 33 query illustrated in section 5. The comparison between the proposed algorithm and the classical PageRank algorithm depends on the following factors:

- Number of memory cells used.
- Number of iteration for each query.
- Total computation time.
- Number of pages matched in top ten pages with the classical PageRank algorithm.

Query	Numbe r of	Memory used (Cell)		Number of iteration			Computation time (Sec)				Number of pages matched with PR alg.			
	Pages (N)	PR alg. (N) ²	Ant PR alg.	PR alg.	Ant PR1	Ant PR2	Ant PR3	PR alg.	Ant PR1	Ant PR2	Ant PR3	Ant PR1	Ant PR2	Ant PR3
abortion	3340	11155600	44574	163660	5147	4287	6952	0.70	0.59	0.55	0.61	8	8	8
affirmative action	2523	6365529	9314	131196	4431	3910	6109	0.41	0.30	0.25	0.30	9	7	10
alcohol	4594	21104836	33342	220512	9642	10575	14248	1.016	0.86	0.80	0.90	8	8	8
amusement parks	3410	11628100	21160	153450	4106	3448	5397	0.66	0.64	0.55	0.61	4	4	4
architecture	7399	54745201	72242	369950	12484	10830	17668	2.58	3.02	4.02	2.97	4	4	5
automobile industries	1196	1430416	6114	52624	1875	1394	2121	0.11	0.09	0.10	0.11	9	8	9
armstrong	3225	10400625	16318	148350	6255	5686	9431	0.55	0.45	0.38	0.48	8	7	8
basketball	6049	36590401	48818	229862	12293	10256	17003	2.06	1.58	1.46	1.58	8	8	8
blues	5354	28665316	48778	256992	10250	8586	14447	1.47	1.25	1.141	1.28	8	8	9
cheese	3266	10666756	23320	150236	6526	5653	8935	0.56	0.55	0.48	0.55	4	6	4
classical guitar	3150	9922500	24088	113400	5314	4509	7326	0.56	0.52	0.44	0.53	5	5	5
complexity	3564	12702096	26962	128304	5139	3857	6542	0.80	0.61	0.53	0.61	7	7	6
computational complexity	1075	1155625	4362	38700	1504	1226	2022	0.09	0.078	0.06	0.11	8	8	8
computational geometry	2292	5253264	16378	38964	3373	2683	4385	0.36	0.30	0.23	0.28	7	7	7
death penalty	4298	18472804	43912	171920	6194	5015	8157	1.36	0.77	0.64	0.78	8	7	8
Genetic	5298	28068804	38522	227814	10581	10041	15229	1.48	1.15	1.03	1.19	8	5	8
Geometry	4326	18714276	26726	211974	7386	6478	10045	1.14	0.84	0.73	0.86	8	8	8
globalization	4334	18783556	34848	216700	6533	5884	9040	1.14	0.89	0.75	0.86	6	5	7
gun control	2955	8732025	23476	153660	4756	3801	6203	0.61	0.45	0.41	0.45	8	8	8
iraq war	3782	14303524	30746	158844	5899	4876	7467	0.84	0.63	0.52	0.63	7	7	7
Jaguar	2820	7952400	16784	141000	5516	4826	7905	0.48	0.36	0.31	0.39	8	8	9
Jordan	4009	16072081	21874	200450	7799	7192	11646	0.73	0.61	0.55	0.66	8	7	8
Movies	7967	63473089	57628	294779	15754	13425	22601	3.45	2.87	2.73	2.86	8	8	7
national parks	4757	22629049	28312	228336	10148	8927	14843	1.18	0.81	0.80	0.86	7	7	8
net censorship	2598	6749604	15776	124704	3927	3402	5059	0.47	0.39	0.36	0.38	8	8	8
randomized algorithms	742	550564	2410	6678	1128	1070	1631	0.0625	0.05	0.14	0.078	9	9	9
Recipes	5243	27489049	36304	272636	10540	9445	15251	1.31	1.13	1.11	1.16	9	7	8
Roswell	2790	7784100	16974	150660	4641	4153	6304	0.5	0.41	0.47	0.42	7	4	7
search engines	11659	135932281	584472	524655	17023	12981	22428	9.73	6.88	6.55	6.77	8	8	7
shakespeare	4383	19210689	27150	219150	8758	8347	12714	0.98	0.72	0.72	0.80	9	7	9
table tennis	1948	3794704	10930	54544	3440	2788	4575	0.25	0.22	0.28	0.23	7	7	7
vintage cars	3460	11971600	25592	148780	4739	3810	6523	0.64	0.64	0.64	0.64	6	4	5
Weather	8011	64176121	69344	376517	18947	14601	27233	3.59	2.94	0.64	3.06	10	8	10
percentage in average		100	0.2	100	4.5	3.9	6.1	100	80.1	78.5	77.3	7.5	6.8	7.5

Table 2: Memory used, total number of iterations, computation time, and number of pages matched for classical PageRank algorithm versus Ant PageRank algorithm approaches.

Table 2 compares the results of both algorithms in terms of memory used, number of iteration, computation time, and number of pages matched with classical PageRank algorithm.

The main contributions of the proposed algorithm are:

- The percentage in average of the memory used in the proposed Ant PageRank algorithm versus conventional PageRank algorithm is 0.2 %. The memory used in the classical PageRank algorithm is N², while the memory used in the proposed algorithm is based on the number of indegree pages and the number of out-degree pages as shown in Table 1.
- The percentage in average of the number of iteration in all pages in the classical PageRank algorithm versus Ant PageRank algorithms are (as shown in Table 2):
 - Ant PageRank algorithm approach 1 reduces the number of iteration by 95.5 %.

- Ant PageRank algorithm approach 2 reduces the number of iteration by 96.1 %.
- Ant PageRank algorithm approach 3 reduces the number of iteration by 93.9 %.
- The average of the computation time in all pages in the classical PageRank algorithm versus Ant algorithms are (as shown in Table 2):
 - Ant PageRank algorithm approach 1 reduces the computation time by 19.9 %.
 - Ant PageRank algorithm approach 2 reduces the computation time by 21.5 %.
 - Ant PageRank algorithm approach 3 reduces the computation time by 18.7 %.
- The average top ten pages results are matched with the classical PageRank algorithm in each approach are (as shown in Table 2):
- Ant PageRank algorithm approach 1 is 7.5 pages.
- Ant PageRank algorithm approach 2 is 6.9 pages.
- Ant PageRank algorithm approach 3 is 7.5 pages.

7 CONCLUSIONS

This paper has proposed the Ant PageRank algorithm. This algorithm has a flexible parameters to control the behaviour of the Ant algorithm. These can be achieved through three approaches, which are proposed to enhance the classical PageRank algorithm. The enhancement is based on two factors, computation time and memory required.

As demonstrated, the top ten pages which are achieved by the proposed algorithm are matched with the top ten pages of the classical PageRank algorithm, with lower execution time, number of memory cells, and number of iteration. Presents

The best experimental is Ant PageRank algorithm experimental 1. This experimental use one ant with random selected. This experimental reduces the percentage in average execution time by 19.9% and matches with 7.5 pages from top ten pages with classical PageRank algorithm.

REFERENCES

- A. Borodin, G. O. Roberts, J. S. Rosenthal, and P. Tsaparas. Link Analysis Ranking Algorithms Theory and Experiments". ACM Transactions on Internet Technologies, ,Vol. 5, No. 1, Pages 231–297, February 2005
- Dorigo Marco, Vittorio Meniezzo, and Alberto Colorni. "Positive Feedback as a Search Strategy", Technical report n91-016. Department of Electronics, Milan Polytechnic Institute, June 1991.
- Dorigo Marco, Eric Bonabeau, and Guy Theraulaz. "Ant Algorithms and Stigmergy", future Generation Computer System, 16(2000), 851-871.
- E.L.Lawler, J.K.Lenstra, A.H.G.Rinnooy-Kan, D.B.Shmoys eds., *The Travelling Salesman Problem*, New York:Wiley, 1985.
- Islam A.Elgawad "Development in web search engine optimization ", M.sc Thesis, Computers and Information - Helwan university, 2005
- J. Kleinberg. "Authoritative sources in a hyperlinked environment". Journal of ACM (JASM), vol. 46, pages 604-632, 1999.
- Nan Ma, Jiancheng Guan, Yi Zhao. "Bringing PageRank to the citation analysis", Information Processing & Management, Volume 44, Issue 2, 2008, Pages 800-810.
- P. Tsaparas (2004), "Link Analysis Ranking", Ph.D Thesis, University of Toronto, March 2004.
- P.Tsaparas(2008)"http://www.cs.toronto.edu/~tsap/experi ments/download/download.html"
- S. Brin and L. Page, "The anatomy of a large-scale hyper textual Web search engine". In Proceedings of the 7th International World Wide Web Conference, Brisbane, Australia, 1998.