

SELF-CALIBRATION CONSTRAINTS ON EUCLIDEAN BUNDLE ADJUSTMENT PARAMETERIZATION

Application to the 2 Views Case

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Abstract: During the two last decades, many contributions have been proposed on 3D reconstruction from image sequences. Nevertheless few practical applications exist, especially using vision. We are concerned by the analysis of image sequences acquired during crash tests. In such tests, it is required to extract 3D measurements about motions of objects, generally identified by specific markings. With numerical cameras, it is quite simple to acquire video sequences, but it is very difficult to obtain from operators in charge of these acquisitions, the camera parameters and their relative positions when using a multicamera system. In this paper, we are interested on the simplest situation: two cameras observing the motion of an object of interest: the challenge consists in reconstructing the 3D model of this object, estimating in the same time, the intrinsic and extrinsic parameters of these cameras. So this paper copes with 3D Euclidean reconstruction with uncalibrated cameras: we recall some theoretical results in order to evaluate what are the possible estimations when using only two images acquired by two distinct perspective cameras. Typically it will be the two first images of our sequences. It is presented several contributions of the state of the art on these topics, and then results obtained from synthetic data, so that we could state on advantages and drawbacks of several parameter estimation strategies, based on the Sparse Bundle Adjustment and on the Levenberg-Marquardt optimization function.

1 INTRODUCTION

This paper proposes some simple comparative results concerning the precision of the absolute 3D Euclidean reconstruction we can expected from 2 different pinholes cameras and from matched points between views. The cameras are supposed to have very distinct relative orientations, so that it cannot be considered as a stereovision head; cameras parameters are unknown, except focal lengths that are approximately known from the constructor's data sheet. We focus on the projective reconstruction of 3D points from matched pixels on two views and on the quality of the Euclidean structure estimated without prior geometric information, but imposing constraints/priors on the cameras parameters space. Imposing priors on the parameters may give better theoretical precision rather than fixing parameters. Nevertheless one must take care about parameterization imposed by the self-calibration constraints. We will particularly insist on this point.

This work is motivated by an application about the analysis of video sequences acquired by two or more cameras, typically on a crash test experiments. Two images are presented on Figure 1: they are acquired by two uncalibrated cameras with very different viewpoints. The challenge consists in extracting 3D information from these images recovering in the same estimation process, the intrinsic parameters of the two cameras.

2 CAMERA MODEL

We are looking for the best Euclidean 3D reconstruction we can obtain from m sets of n corresponding images points x_{ij} coming from n 3D point X_i , projected onto the image planes by m distinct pinhole cameras P_j . To reach this goal, we want to estimate a parameter vector p that contain the camera parameters (P_j projection matrix for the camera j) and the 3D points

$$p = (P_j, X_i) \tag{1}$$

Without considering optical distortions, we have a total of $(11m+3n)$ parameters. The parameter vector p gives a predictive camera model

$$u_{ij} : R^{11} \times R^3 \rightarrow R^2 \tag{2}$$

The model is for the 3D point i seen by the camera j :

$$\lambda_{ij} x_{ij} = u_{ij}(p) = P_j X_i \quad i=1\dots n, \quad j=1\dots m \tag{3}$$

The model will impose implicit constraints,

$$c(x_{ij}, u_{ij}) = 0 \tag{4}$$

between underlying feature x_{ij} from noisy measurements of the feature x'_{ij} , and have to be consistent with the feature.

2.0.1 Feature Error Model

Due to the very distinct viewpoints of the cameras, the feature points are selected manually and suppose free from outliers. The observation noise $d(x_{ij})$ induced by this manual selection is assumed to have Gaussian independent and identically distributed terms, with variance s^2 .

$$x'_{ij} = x_{ij} + d(x_{ij}) \tag{5}$$

2.0.2 Cost Function

We have chosen the Maximum Likelihood Estimator (MLE) as decision criterion to estimate the parameters that best fit to the feature error model. MLE gives the global minimum of the inverted log likelihood, taken as a function of the parameters p ,

$$p = \underset{p}{\operatorname{argmin}} J(p) \tag{6}$$

With

$$J(p) = \frac{1}{2\sigma^2} \sum_{i,j=1}^{n,m} \|x_{ij} - u_{ij}(p)\|^2 \tag{7}$$

It is well known that this cost function does generally not have a unique minimum and is very dependent on the initial estimate of the parameters p_0 . These problems occur if it exists a coordinate transformation g of the parameter space P such that

$$J(p) = J(g.p) \tag{8}$$

The set of all such transformation form the group G , called the group of gauge transformation. The set of all parameters such that $p = g.p_0$ (p is geometrically equivalent to p_0) form what is called the leaf Pp_0 associated with p_0 , which is a sub manifold of P . So some constraints have to be imposed on the

parameters set in order to have a unique solution, which minimizes eq.(7), for each connected component of the leaf. However we recall that this will not be a global minimum of J . Moreover, these constraints need to be linear.

2.0.3 Numerical Optimization

The Non Linear Least Square problem defined by MLE eq.(7) will be solved by numerical optimization via a Damped Levenberg-Marquardt algorithm allowing simple bounds constraints on the variables (Gill et al., 1981), also called a Bundle Adjustment procedure (Triggs et al., 2000) as the 3D point coordinates are part of the parameters vector p .

2.1 Counting Argument

If we consider that the m cameras are uncalibrated, the projection matrixes P_j contain $11m$ independent parameters, removing the projective scale factor. These parameters are only defined up to 15 degrees of freedom (noted dof) coordinates transformation T of the projective space P^3 , defining a camera parameter space with $(11m-15)$ essential degrees of freedom (noted edof).

$$\lambda_{ij} x_{ij} = (P_j T)(T^{-1} X_i) \quad i=1\dots n, \quad j=1\dots m \tag{9}$$

That simply means that to have a unique solution on the leaf Pp_0 , the parameters must be constrained by 15 algebraically independent gauge equations c_i . These equations define a sub manifold C in P of co dimension 15 and ensure that there is a unique gauge transformation g that maps a parameter p of Pp_0 to another parameter pc of Pp_0 which respects the constraints.

Now, let us consider the same problem with calibrated cameras: every projection matrix has 6 dof, and are defined only up to a 7 dof similarity transformation of the Euclidean coordinate space, leaving $(6m-7)$ edof for the parameter space. So intuitively, if we want to move from the uncalibrated projective space to the calibrated Euclidean space or goes in the inverse way, we have $(5m-8)$ edof left and 8 constraints more to impose.

2.2 Parameterization

The classical way of representing a perspective camera is to define the following model for a camera projection matrix

$$P_j = K_j E_i \tag{10}$$



Figure 1: Example of a two-cameras application.

where K_i is the 3X3 upper triangular intrinsic matrix (5 dof) that links 3D point coordinates in the camera reference frame to images 2D pixel coordinates,

$$K_i = \begin{bmatrix} f_{iu} & s_i & u_{i0} \\ 0 & f_{iv} & v_{i0} \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

E_i is the extrinsic parameters matrix (6m dof), that links the world coordinates to the cameras ones.

$$E_i = [R_i | t_i] \quad (12)$$

However, this model will impose non linear constraints on the feature instead of linear ones.

2.2.1 Calibrated Case

In the calibrated case, intrinsic parameters are known, leaving a (6m) dof parameterization. So, beginning from an initial parameter vector p_0 , 7 independent constraints have to be imposed on the camera parameters space to have a unique solution among C . Indeed without gauge, Euclidean reconstruction is obtained up to a similarity. A solution is to fix arbitrarily the world coordinate frame to the first camera frame by imposing that

$$E_1 = [I_{3 \times 3} | 0] \quad (13)$$

and to fix the unknown scale of the Euclidean reconstruction (Kanatani and Morris, 2000, Grossman and Victor, 1998) by imposing

$$\|t_2 \dots t_m\| = 1 \quad (14)$$

This is the standard Euclidean gauge. We then parameterize minimally the cameras relative orientations by $6 * m - 7$ free parameters, independent from each other's: $3(m-1)$ Euler angles via the Rodrigues formula and $(3m-4)$ parameters for the normalized multi-camera translation. The constrained parameter space C has $(6m-7)$ edof.

2.2.2 From Calibrated Euclidean Space to Uncalibrated Projective Space

Now, if we want to extent the calibrated Euclidean parameterization defined above to the particular case of uncalibrated cameras, the intrinsic parameters of m cameras must be added into the system parameterization. Beginning with totally unknown intrinsic parameters, the projective cameras are parameterized in a calibrated Euclidean way. The counting argument allows $(5m-8)$ edof to parameterize the intrinsic parameters considering the simple difference between the edof in the Projective and Euclidean cases; equivalently it makes mandatory to have 8 constraints on the $(5m)$ dof of free intrinsic parameters. If we denote f the fixed intrinsic parameters and k the known ones among the m views, we can derive the well known counting process

$$mk + (m-1)f = 8 \quad (15)$$

By the way, we recover the "self-calibration" constraints, which explain the fact that "the intrinsic parameters should be parameterized so that the self-calibration constraints are satisfied" (Pollefeys at al., 1998).

The Euclidean uncalibrated parameterization impose implicit constraints on the projective parameter space via

$$K [R | t] = P T \quad (16)$$

which is directly related with the constraint described by Triggs (Triggs, 1997)

$$P Q P^T = K K^T \quad (17)$$

This constraint is applied on the absolute quadric and is expressed algebraically by the above counting argument.

$$Q = T \text{Diag}(1,1,1,0) T^T \quad (18)$$

We can visualize these 8 constraints, fixing the unknown projective scale, by taking

$$P_{\text{proj}} = [I_{3 \times 3} | 0] \quad (19)$$

so that T and so Q , are parameterized by 8 parameters (5 for intrinsic/absolute conic parameters and 3 for the plane at infinity). It gives a local parameterization of our gauge group G (Triggs, 1998).

$$T = \begin{bmatrix} K & 0 \\ p^T & 1 \end{bmatrix} \quad (20)$$

We have now (11m-15) dof for our set of parameters; it is consistent both with the number of edof of the projective space and with the counting argument rule to well parameterize the intrinsic parameters following the number of views and the priors we have about them. It is also important to mention that the counting argument is only valid for non-critical configurations (configurations that do not permit to locate exactly the absolute quadric in the projective space). These configurations depend either on 3D points parameters (critical surface (Hartley and Zisserman, 2006)), or relative positions between cameras (critical motion sequences as described by Sturm (1997)), or both of them. Some specific approaches have to be developed in such cases.

2.3 Dealing with Parameters Inter Correlations

However, using this natural approach, we are not able to ensure that our essential parameters defined a set of independent parameters. The calibrated Euclidean parameterization provides an independent set of parameters, but in the uncalibrated case, the chosen parameterization gives intercorrelations, as the intrinsic parameters are highly correlated with the extrinsic ones (Shih et al., 1996). For instance, the principal point position is correlated with the camera orientation and the focal length with the translation along the optical axis. In this case, the camera parameters covariance matrix contains some abnormally elevated values. As a consequence, intrinsic gauge constraints imposed by eq.(10), if perturbed by noisy measurements, will greatly impact the Bundle Adjustment as the free parameters will move to compensate this initial error induced by the badly fixed/known ones.

So we could think that a free intrinsic gauge approach, like the one proposed by Malis and Bartoli (Malis and Bartoli, 2001) will greatly improve the

solution. Basically, the authors adopt an elegant method, equivalent to obtain a reduced model of the parameter set by the classical way. If we differentiate eq. (7) with respect to the intrinsic parameters, we set the result to zero, and we solve the resulting equation, then, intrinsic parameters are expressed in terms of the remaining parameters (image point, extrinsic and 3D points). Substituting it into eq. (3) we will obtain a function of the remaining parameters. However, as the intrinsic values are embedded in the free intrinsic gauge parameterization, the self-calibration constraints still have to be needed on remaining parameters leading to the same problem; they are imposed either by Lagrange multipliers or weighting methods.

A method to have a better Euclidean reconstruction is to use some priors about the free parameters during the optimisation, imposing the free parameters to stay between some bounds. In the next section, we propose some comparative results using priors on focal lengths during a Bundle Adjustment process, in the typical standard Euclidean gauge (process noted EBA hereafter). For the 2 view case with distinct cameras, EBA process is applied with a weighting method, with bounds constraints, and using Malis and Bartoli intrinsic free parameterization with artificial weights. Results can be found on Figure 2.

3 TWO VIEWS CASE

Our objective here consists in comparing several ways to calibrate a two-camera system, depending on initial knowledge available on the intrinsic parameters. Seven algorithms are compared.

3.1 Parameter Choice

The number of projective edof authorizes us to parameterize the space parameters with 7 parameters that must ideally be independent, being far from the possible critical configurations for self-calibration and 3D reconstruction: a theoretical study for these ones has been performed by Sturm (Sturm, 1997) for the specific two view case. We recover by the way, the 7 dof of the fundamental matrix (scaled matrix of rank 2), which encapsulates the 2 views epipolar geometry. This is a classical way to show that 2 camera intrinsic parameters can be recovered from images alone by a self-calibration procedure, as it has been shown in a pioneered contribution by Hartley for the focal length in 1993 (Hartley and Zisserman, 2006).

Using the above Euclidean camera model with the Euclidean standard gauge, we can add to our 5 Euclidean parameters space, 2 more parameters chosen among the intrinsic ones to reach the 7 parameters allowed. If we add more parameters, we will neither obtain a good 3D reconstruction nor good estimate of the intrinsic parameters (Bougnoux, 1998, Grossman and Victor, 1998) as the parameterization will not respect the self-calibration constraints and will be over-parameterized.

Let us define two situations: (1) we can suppose that we have estimated the camera calibration parameters by some way, e.g. reading the camera data sheet: we know approximately the focal lengths, and can make the usual square pixel assumption ($s=0$, $f_u = f_v = f$) with (u_0, v_0) at image centres. (2) Using a calibration pattern, we can also consider focal lengths as unknown parameters and fix the other ones to their initial values. So that our set of parameters $p = (pC, pX)$ are the camera parameters $pC = (f_1, f_2, r_x, r_y, r_z, t_x, t_y)$ plus the unknown 3D points $pX = (X_1 \dots X_N)$, with the standard Euclidean gauge and the self-calibration constraint imposed by fixing all the intrinsic parameters apart from the 2 distinct focal lengths.

3.2 Initialisation

First we recover the relative camera orientation with the Essential matrix, via the estimated intrinsic parameters and the fundamental matrix calculated with the Gold Standard Algorithm (projective BA) as described in (Hartley and Zisserman, 2006).

The initial Euclidean reconstruction of the 3D points is obtained with the 2 view optimal methods as described in (Hartley and Zisserman, 2006).

If the true intrinsic are used, we call the obtained reconstruction, the **initial calibrated** reconstruction, and the **initial pre-calibrated** one if initial parameters are only approximated. This provides two initial parameters set (p_{01}, p_{02}) used as initial guesses for the Bundle Adjustment function.

3.3 Algorithms

We recall that the goal is not to recover very precise focal lengths, but to allow a better reconstruction of the scene by adding the 2 focal lengths as parameters during the optimisation process.

First of all, we run the optimization (BA) with the standard Euclidean gauge for the initial calibrated reconstruction and initial calibrated one. Only Extrinsic and 3D points are free during the

optimisation process. We obtain respectively the reconstruction called **EBA calibrated** and **EBA pre-calibrated**. The first one being the best 3D reconstruction we can obtain from noisy measurements without constraint on the structure parameters. These will be our reference data, as the algorithms used to obtain it are well known.

Then, even if results do not appear here, we have verified that if there is only noise on the focal lengths, the free 7 parameters, exactly converge through the exact focal lengths values. This means that self-calibration constraint and Euclidean gauge defined a unique minimum on the parameter set, and not a sub manifold of the parameters set. This result is also true for more cameras as long as the camera parameterization and the Euclidean gauge impose 15 independent gauge constraints, which can easily, be verified experimentally.

Next we investigate the results of our algorithms, when the self-calibration constraints are badly defined by coarse approximations of the intrinsic parameters, so that the correlation between intrinsic and extrinsic parameters (Shih et al., 1996) leads to a set of parameters that have not the required edof.

To control the focal lengths during the BA process, it is assumed that we have probabilistic priors $f = N(f_0, s_f)$ about them. A study of the focal length variability versus optical centre can be found in (Willson and Shafer, 1993).

In the intrinsic free Euclidean BA called **EBA intrinsic free**, these priors are added by imposing a weight value to the cost function of the form $\|f-f_0\|_{sf}$ where the focal is a function of remaining parameters; other constraints coming from the fixed intrinsic parameters are imposed by adding heavily weighted artificial measurements as their variances are supposed to be null.

The same procedure applied to the pre-calibrated case, leads to the reconstruction called **EBA pre-calibrated weighted** reconstruction.

We use then the numerical optimisation described in (section 2.2.3) where focal lengths are subject to linear bounds constraints (Gill et al., 1981) during the non linear least square optimisation eq.(21), leading to the **EBA pre-calibrated bounds** reconstruction.

$$\min_{\substack{p \in \mathbb{R}^{7+3M} \\ i=1..M \\ j=1,2}} J(p) \text{ subject to } |f_{0j} - f_j| \leq \sqrt{2} \quad (21)$$

3.4 Synthetic Image Data

We model the object scene by 20 points randomly created in a sphere of diameter 1000 of centre $(0, 0,$

0). The 20 points agree with the maximum pair of points an operator can reasonably pick up in images pair.

The two modelled cameras are of respective centres C1(1866, 316, 3523) and C2(-3922, 358, 6963) with 2 respective optical axis pointing towards two distinct points of the scene Z1(-0.35, -0.15, -0.92) and Z2(0.51, -0.04, -0.85). The Y axis of each camera is nearly parallel to the ground, in order to model a realistic situation not critical for the self-calibration of the 2 distinct focal lengths (Sturm, 1997). Ground truth is given by the respective camera focal lengths in pixels $f_1 = 1000$ and $f_2 = 2000$ for 500X500 pixels camera images, and the respective Principal points image coordinates (260,240) and (230,220), with square pixel assumptions.

Noise simulation on intrinsic parameters is imposed by choosing the following values for respective camera focal lengths (1100, 1800) and principal points (290,200), (210,280).

For 10 values of the image noise variance, ranging from 0 to 2 pixels, we generate 100 corrupted images from the true one and run the distinct algorithms with the 2 sets of estimated parameters.

To measure the 3D error E_r on the reconstructed scene, which may not be exactly Euclidean, we use the average Horn reconstruction Error (Horn, 1987) that gives the absolute position of the reconstructed 3D points from the true ones, eq.(22).

$$E_r = \frac{1}{n} \sum_{i=1}^n \|X_{\text{reconstructed}} - (sRX_{\text{true}} + t)\| \quad (22)$$

In eq. (23), s , R and t define a similarity of the projective space estimated by linear minimization of the following criterion

$$(s, R, t) = \text{Argmin} \sum_{i=1}^n \|X_{\text{recons}} - (sRX_{\text{true}} + t)\|^2$$

Some insights on the true projective transformation existing between the estimated reconstruction and the true one, have been studied by Bougnoux (Bougnoux, 1998).

3.5 Simulation Results

The best reconstruction, as guessed, is obtained by the **EBA calibrated** and the worst for small to average values of the noise level for the **initial pre-calibrated** case. As expected too, beginning from the two sets of initial parameters, a better 3D reconstruction is provided by the Euclidean BA (with fixed intrinsic parameters).

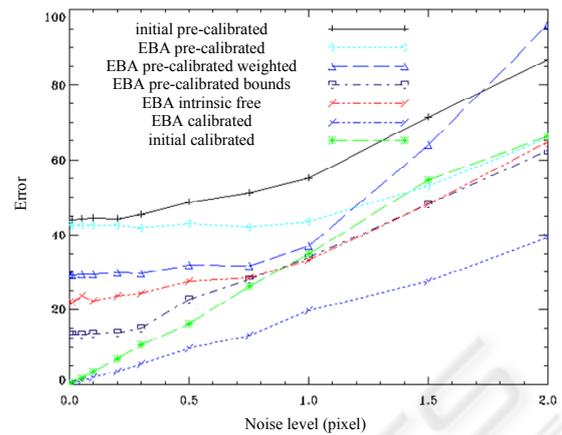


Figure 2: These graphs show the average Horn reconstruction errors for various algorithms applied on our synthetic set of points, generated from 2 distinct views.

We now focus on the interesting case of noisy intrinsic parameters. The **EBA pre-calibrated weighted** gives the worst results. It is basically an intermediate between **EBA pre-calibrated** with fixed focal length (heavy weight) and the one (which is not represented) with totally free focal length (weak weight). As pointed out by Hartley and Silpa-Anan (Hartley, 2002), in their quasi-linear Bundle Adjustment Approach (Bartoli, 2002), weights are difficult to choose optimally, but if there is little noise on intrinsic parameters and on images, then imposing weak bounds will generate the better results. However, for high value of the noise, it performed badly, as the better approach will be to fix the parameter or equivalently, imposed heavy weights to the focal lengths terms, as the correlation between the parameter set will be higher. The same remarks apply to the **EBA intrinsic free**, which performed significantly well. Finally, the better results are obtain with the propose optimization scheme, where the focal length are well controlled during the numerical optimization procedure. As the image noise is increased, the priors approach performs equally but asymptotically, we guess that the better reconstruction will be obtained for the EBA with fixed intrinsic.

4 CONCLUDING REMARKS

This paper points out some of the difficulties that arise when intrinsic cameras parameters are estimated in the same time as the structure and motion parameters via the classical Bundle

Adjustment procedure (sequential quadratic programming).

We have linked the famous self-calibration counting argument to the number of degree of freedom in our parameter set in order to have a minimal parameterization of the projective dof derived from the calibrated Euclidean one. The so defined model implies non linear constraints on the parameters set and leads to interdependencies on the parameters that are difficult to deal with.

The comparative studies in the two views case show that using artificial penalty on the cost function gives good results. Moreover, imposing priors on the focal lengths, even if the initial principal points are far from the true values, leads to correct 3D Euclidean reconstruction when the image noise is quite low. We conclude that for very noisy images with few points (20), the maximum likelihood estimator (MLE) performed better when intrinsic parameters are approximately fixed. To obtain even better results, a search control approach during the step damping of the BA may be helpful. However we see that even with perfect intrinsic parameters, the reconstruction is really dependant on the image noise and quite imprecise. A solution will be to use some constraints coming from the structure to improve the quality of the Euclidean reconstruction.

REFERENCES

- Kanatani, K., D.Morris, D.: Gauges and gauge transformations in 3-D reconstruction from a sequence of images. In Proc. 4th ACCV, (2000)1046-1051
- Pollefeys, M., Koch, R., Van Gool, L.: Self-calibration and metric reconstruction in spite of varying and unknown internal parameters. In Proc. 6th ICCV, (1998)90-96.
- Triggs, B.: Auto calibration and the Absolute Quadric. In Proc. IEEE Computer Vision and Pattern Recognition, (1997)609-614.
- Hartley, R.I.: Euclidean reconstruction from uncalibrated views. In: Mundy, J.L., Zisserman, A and Forsyth, D. (eds.): Applications of Invariance in Computer Vision. Lecture Notes in Computer Science, vol. 825. Springer-Verlag, Berlin Heidelberg New York (1993) 237-256
- Gill, P., Murray, W., Wright, M.: Practical Optimisation, Academic Press, New York, 1981.
- Horn, B. K. P.: Closed-form solution of absolute orientation using unit quaternion. In Journal of the Optical Society of America A, Vol. 4 (1987)629.
- Malis, E., Bartoli, A.: Euclidean Bundle Adjustment Independent on Intrinsic Parameters. Rapport de recherche INRIA, 4377 (2001)
- Hartley, H., Zisserman, A.: Multiple View Geometry in Computer Vision. Cambridge University Press (2000), 3rd printing (2006).
- Shih, S-W., Hung, Y-P., Lin, W-S.: Accuracy Analysis on the Estimation of Camera Parameters for Active Vision Systems. In Proc. 13th ICPR, (1996) 930.
- Grossman, E., Victor, J.S.: The Precision of 3D Reconstruction from Uncalibrated Views. In Proc. BMVC (1998) 115-124.
- Bougnoux, S.: From Projective to Euclidean Reconstruction Under Any Practical Situation, A Criticism of Self-Calibration. In Proc. 6th ICCV, (1998)790-796.
- Willson, R., Shafer, S.: What is the Center of the Image? In Proc. IEEE Computer Vision and Pattern Recognition (1993)670-671
- Hartley, R. I., Silpa-Anan, C.: Reconstruction from two views using approximate calibration. In Proc. 5th ACCV, vol. 1, (2002) 338-343.
- Triggs, B., McLauchlan, P., Hartley, R. I., Fitzgibbon, A.: Bundle Adjustment - A Modern Synthesis. In Visions Algorithms: Theory and Practice, Lecture Notes in Computer Science, vol 1983. Springer-Verlag, Berlin Heidelberg New York (2000) 298-372.
- Bartoli, A.: A Unified Framework for Quasi-Linear Bundle Adjustment. In Proc. 16th ICPR, vol.2 (2002) 560-563.
- Triggs, B.: Optimal estimation of matching constraints. In 3D Structure from Multiple Images of Large-Scale Environments. Lecture Notes in Computer Science, vol. 1506. Springer-Verlag, Berlin Heidelberg New York (1998) 63-77.
- Sturm, P.: Critical Motion Sequences for monocular self-calibration and uncalibrated Euclidean Reconstruction. In Proc. ICCV, (1997)1100-1105.
- Sturm, P.: Critical Motion Sequences for Monocular Self-Calibration and Uncalibrated Euclidean Reconstruction. In Proc. IEEE Conference on Computer Vision and Pattern Recognition (1997)1100.